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Sovereign Risk, Bank Risk and Central Banking
Session 1: Fiscal Outlook and Sustainability
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BANCO DE MÉXICO

A Quantitative Model of Default

- *There are many aspects to the recent sovereign crisis in the Euro zone. Our objective is to capture some of the most salient features.^{1/} In particular, to show the kind of elements a policy maker considers when assessing the possibility of a default.*
- *We posit a general equilibrium model of sovereign debt. Initially, we capture a development of debt imbalances, followed by a sequence of adverse shocks to income which eventually leads to a default.*
- *We follow Aguiar and Gopinath (2006), Arellano (2008), and Cuadra, Sánchez, and Sapriza (2010).*
- *The model qualitatively resembles the fiscal and debt situation which various economies in the Euro zone have recently faced.*

^{1/} Ramos-Francia and Cuadra (2013). “A senseless debate about austerity and growth.” Mimeo, Banco de México.

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$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t^\alpha G_t^{1-\alpha})^{1-\sigma}}{1-\sigma}$$

$$C_t = (1 - T_t)y_t$$

- *The representative agent's utility depends on the expected consumption and public expenditures' paths.*
- *The agent takes the tax rate and public expenditure as given.*
- *The income follows a Markov process with conditional pdf $Q(y'|y)$, i.e., an endowment economy.*
- *A benevolent government collects taxes and makes public expenditures.*

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$$V^c(B, y) = \text{Max}_{T, G, B'} \left\{ \frac{(C^\alpha G^{1-\alpha})^{1-\sigma}}{1-\sigma} + \beta \sum_{y'} V_0(B', y') Q(y'|y) \right\}$$

s.t.

$$G = Ty + B - q(B', y)B'$$

$$C = (1 - T)y$$

- *The government may contract non-contingent debt in foreign financial markets.*
- *If the government does not default, it has access to credit markets. Thus, it maximizes utility subject to the households' budget constraint, and its own budget constraint, deciding how much to tax, spend, and borrow.*

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$$V^d(y) = \text{Max}_{T_d, G_d} \left\{ \frac{(C_d^\alpha G_d^{1-\alpha})^{1-\sigma}}{1-\sigma} + \beta \sum_{y'} [\mu V_0(0, y') + (1-\mu)V^d(y')] Q(y'|y) \right\}$$

s.t.

$$G_d = T_d h(y)$$

$$C_d = (1 - T_d)h(y)$$

- *If the government defaults, it loses access to international markets and faces an output loss, but avoids the debt service. It still has to decide taxes and expenditures.*
- *In this case, the tax on income is the only instrument to finance public expenditures. The output in autarky is represented by $h(y) = (1-\eta)y$, which is lower than y .*

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$$V_0(B, y) = \max\{V^c(B, y), V^d(y)\}$$

- *To make the decision whether to default or not, the government compares:*
 - ✓ *The cost of repayment given the disutility of reducing the current consumption to repay the non-contingent loan; against,*
 - ✓ *The cost of temporarily being excluded from international markets, that is, the foregone benefits of consumption smoothing and output loss given the state of autarky.*

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$$q = \frac{(1 - \lambda(B', y))}{1 + r_f}$$

- *The bond price is derived from the following two conditions:*
 - ✓ *Foreign creditors are risk neutral and have access to an international risk free bond market with a rate r_f .*
 - ✓ *Perfect competition among creditors implies a zero profits condition.*
- *Hence, the bond price q reflects the probability of default, $\lambda(B', y)$.*

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Model's Parameters

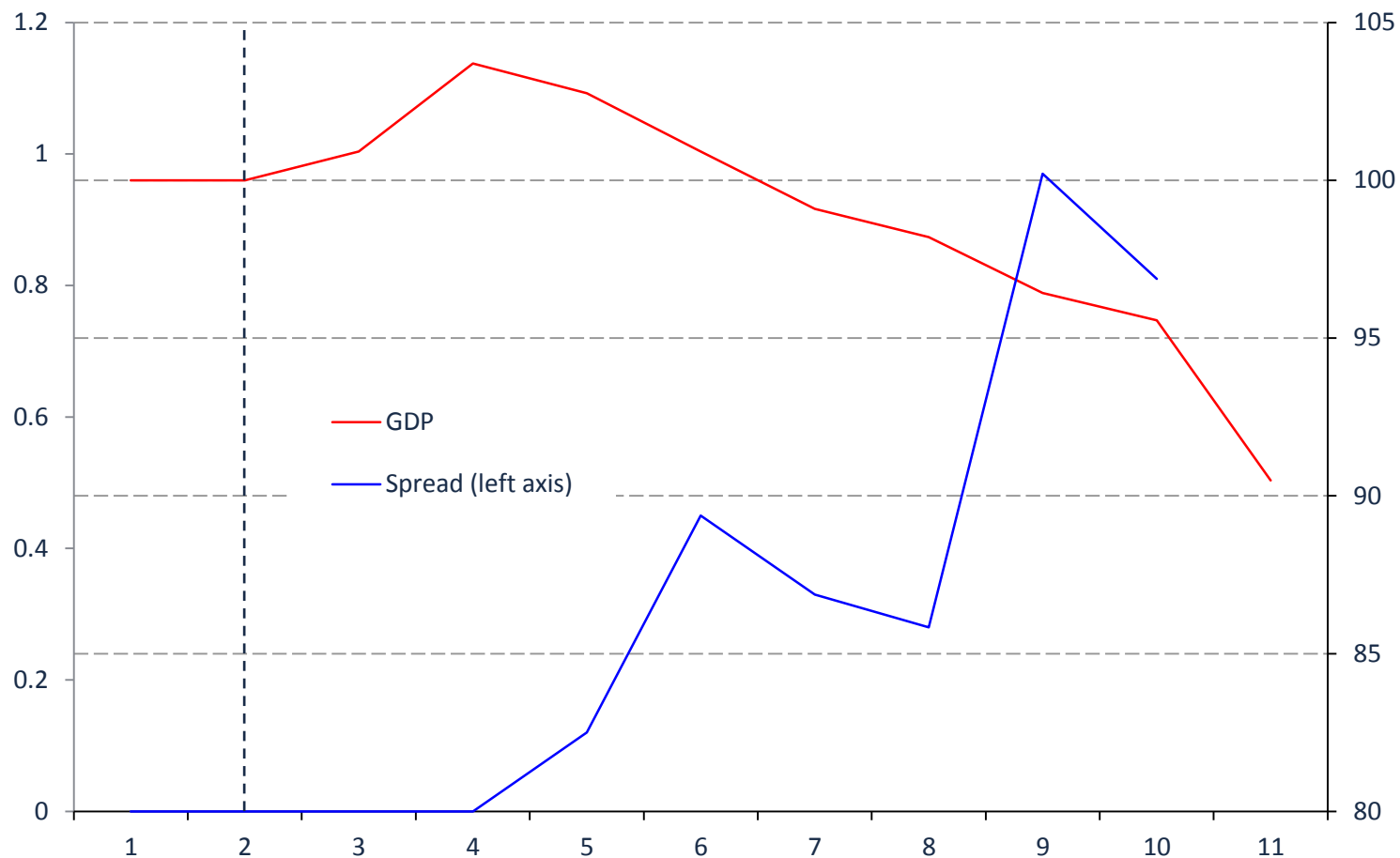
		Value	Source
Risk Aversion	σ	2.00	Standard in the literature
Discount Factor	β	0.95	Standard in the literature, annual frequency
Consumption Weight	α	0.70	Cuadra, Sánchez, and Sapriza (2010)
Re-entry Probability	μ	0.10	Arellano (2008)
Output Loss in Autarky^{1/}	η	0.02	Aguiar and Gopinath (2006)
Endowment Process's	ρ_y	0.90	Aguiar and Gopinath (2006)
Parameters	σ_y	0.02	

1/ i.e. $h(y) = (1-\eta)y$

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GDP vs. Spread

GDP Index period 1 = 100, Spread Percent

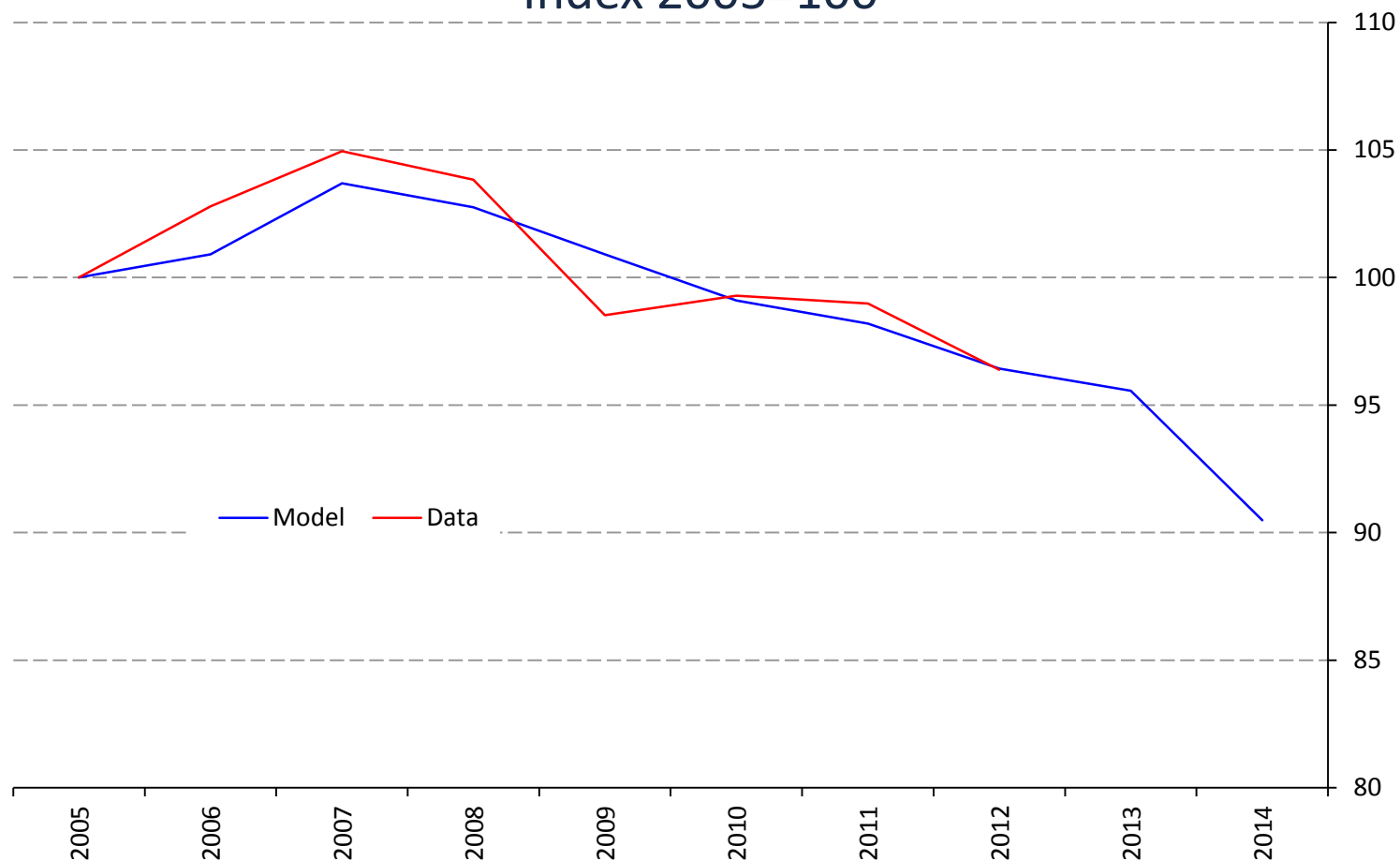


Source: Own calculations based on the model's calibration.

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Output from Data vs. Output from the Calibrated Model ^{1/}

Index 2005=100

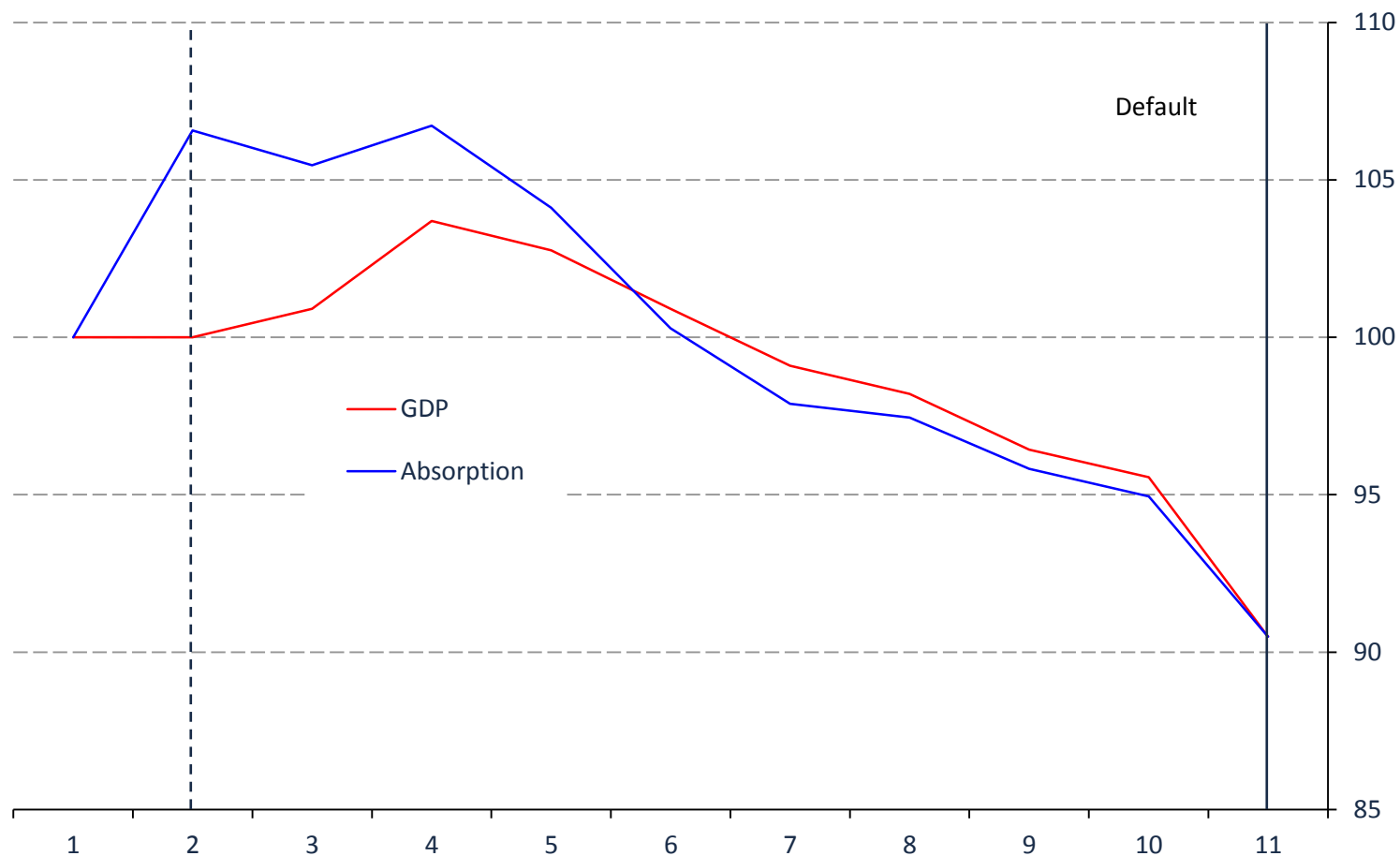


1/ Output from data refers to the sum of the (seasonal adjusted) real GDP of Portugal, Ireland, Italy, and Greece.
Source: Haver Analytics and own calculations based on the model.

A Quantitative Model of Default

GDP vs. Absorption

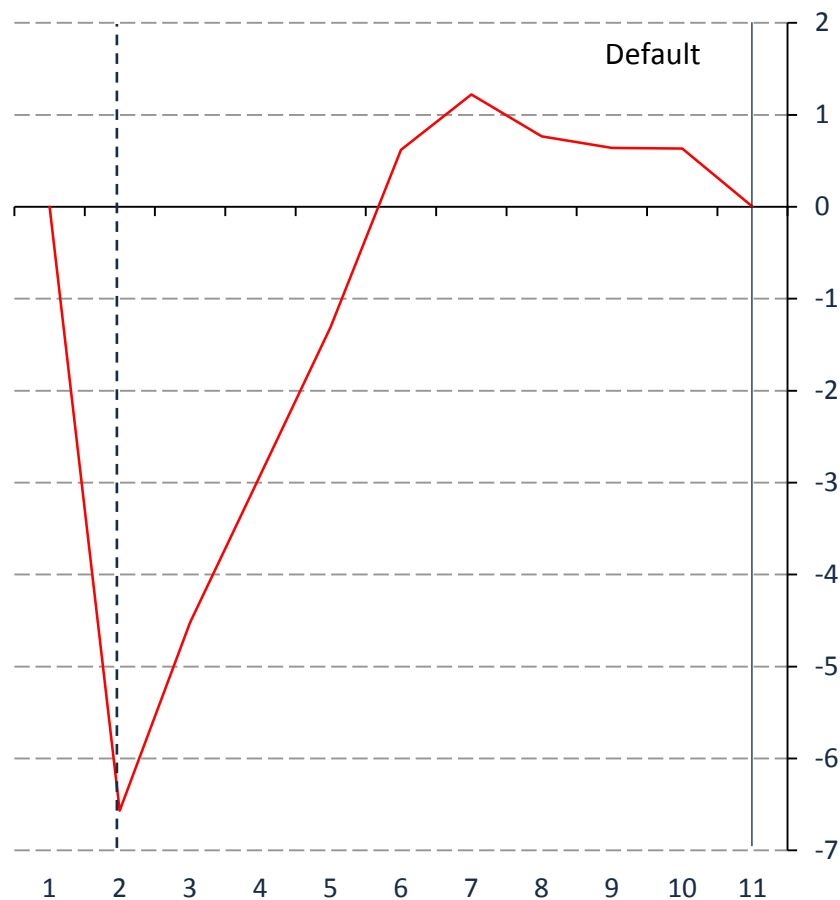
Index (Period 1 = 100)



Source: Own calculations based on the model's calibration.

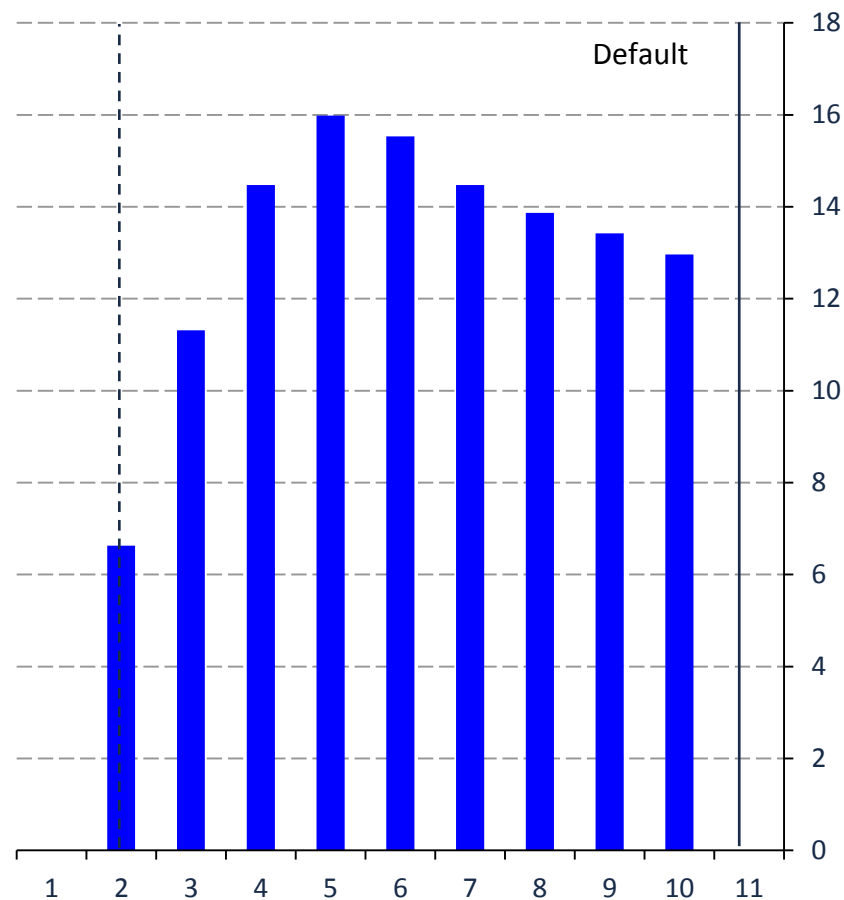
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Fiscal Balance
Percent of GDP



Source: Own calculations based on the model's calibration.

Debt Service
Percent of GDP



Source: Own calculations based on the model's calibration.

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- *The official aid provided by multilateral institutions has changed the relative prices of a decision to default and, thus, diminished the incentives to do so.*
- *Official aid, which in the model can be thought of as an increase in the endowment's level:*
 - ✓ *Positions the recipient country away from default;*
 - ✓ *Lowers the cost of its debt service; and,*
 - ✓ *Diminishes the cost of additional borrowing.*
- *Crucially, official aid has to be large enough so a default is avoided. Yet, if it is greater than what is necessary then the excess becomes a zero-sum game.*

Have the changes in relative prices been sufficient?

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- *The model does not capture several other important aspects.*
 - ✓ *For instance, the externalities and spillovers once a country defaults.*
 - ✓ *A potential banking crisis ensued from a sovereign default episode.*
 - ✓ *The redenomination uncertainty if a country exits the Euro.*

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References

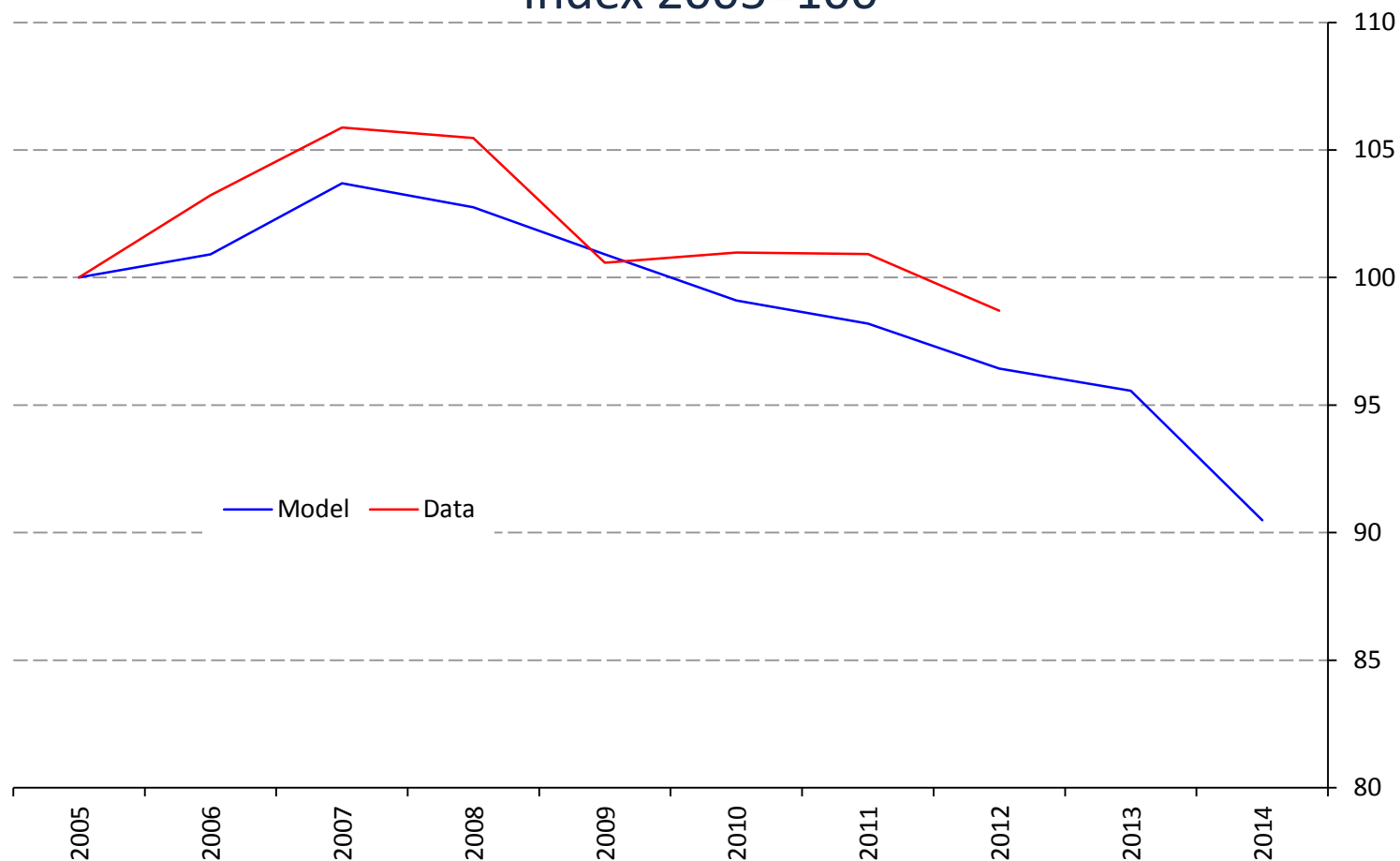
1. Aguiar, Mark and Gita Gopinath (2006). “Defaultable debt, interest rates, and the current account.” *Journal of International Economics*, Vol. 69(1).
2. Arellano, Cristina (2008). “Default risk and income fluctuations in emerging economies.” *American Economic Review*, Vol. 98(3).
3. Cuadra, Gabriel, Juan Sánchez, and Horacio Sapriza (2010). “Fiscal policy and default risk in emerging markets.” *Review of Economic Dynamics*. Vol. 13(2).
4. Ramos-Francia, Manuel and Gabriel Cuadra (2013). “A senseless debate about austerity and growth.” *Mimeo, Banco de México*.

Appendix

A Quantitative Model of Default

Output from Data vs. Output from the Calibrated Model ^{1/}

Index 2005=100



1/ Output from data refers to the sum of the (seasonal adjusted) real GDP of Portugal, Ireland, Italy, Greece, and Spain.
Source: Haver Analytics and own calculations based on the model.

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Notation

C_t consumption at time t with access to financial markets.

C_d consumption at time t in autarky.

G_t public expenditure at time t with access to financial markets.

G_d public expenditure at time t in autarky.

T_t tax rate at time t with access to financial markets.

T_d tax rate at time t in autarky.

y_t income at time t .

B_t non-contingent debt at time t .

q_t sovereign bond price at time t .

β subjective discount factor.

σ coefficient of risk aversion.

$1/\sigma$ intertemporal elasticity of substitution.

α consumption weight.

μ re-entry probability.

η output loss in autarky, i.e. $h(y) = (1-\eta)y$.

ρ_y, σ_y endowment process's parameters.

$y_{t+1} = \rho_y y_t + \sigma_y z_{t+1}$ where z_{t+1} is an iid $N(0,1)$.

Variables

Parameters

Process



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