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Arnulfo Rodríguez
Banco de México

Fidel González
Sam Houston State University

Jesús R. González García
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Uncertainty about the Persistence of Cost-Push Shocks and the Optimal Reaction of the Monetary Authority[*]

Arnulfo Rodríguez[†]
Banco de México

Fidel González[‡]
Sam Houston State University

Jesús R. González García[§]
IMF

Abstract

In this paper we formalize the uncertainty about the persistence of cost-push shocks using an open economy optimal control model with Markov regime-switching and robust control. The latter is used in only one of the regimes producing relatively more persistent cost-push shocks in that regime. Conditional on being in the regime with relatively less persistence, we obtain two main results: a) underestimating the probability of switching to the regime with relatively more persistent cost-push shocks causes higher welfare losses than its overestimation; and b) the welfare losses associated with either underestimation or overestimation of such probability increase with the size of the penalty on inflation deviations from its target.
Keywords: Model uncertainty, Robustness, Markov regime-switching, Monetary policy, Inflation targeting.
JEL Classification: C61, E61

Resumen

En este documento formalizamos la incertidumbre de la persistencia de choques cost-push al usar un modelo de control óptimo para una economía abierta con transiciones de Markov y control robusto. Este último es usado únicamente en uno de los regímenes para producir choques cost-push más persistentes en ese régimen. Condicionando a estar en el régimen con relativamente menor persistencia, obtenemos dos resultados principales: a) la subestimación de la probabilidad de transitar al régimen con choques cost-push relativamente más persistentes ocasiona pérdidas de bienestar mayores que su sobreestimación; y b) las pérdidas de bienestar asociadas ya sea con la subestimación o la sobreestimación de tal probabilidad se incrementan con el tamaño del castigo sobre las desviaciones de la inflación de su objetivo.
Palabras Clave: Incertidumbre de modelo, Robustez, Transición de regímenes de Markov, Política monetaria, Inflación por objetivos.

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[†] Dirección General de Investigación Económica. Email: arodriguez@banxico.org.mx.

[‡] Department of Economics and International Business, Sam Houston State University. Email: fidel_gonzalez@shsu.edu.

[§] Statistics Department, IMF. Email: jgonzalezgarcia@imf.org.

1. Introduction

One of the main concerns of monetary policy is the uncertainty about the persistence of cost-push shocks. For instance, in 2004 a surge in the global demand of commodities (or primary goods) increased their international price, prompting a cautious behavior of many central banks in the face of inflationary pressures throughout the year. Figure 1 shows the increase in commodity prices during 2004. In the Mexican case, the cautious approach to price shocks by the monetary authority was due to several factors. First, the direct impact of higher commodity prices on inflation. Second, the uncertainty about the evolution of commodity prices in the future. Third, the possibility of second round effects of the aforementioned shocks on the process of price formation. Finally, the possibility of undesirable effects on inflation derived from the combination of continuing increases in commodity prices and the recovery experienced by the global economy.[1]

Commodity Prices (2000=100)

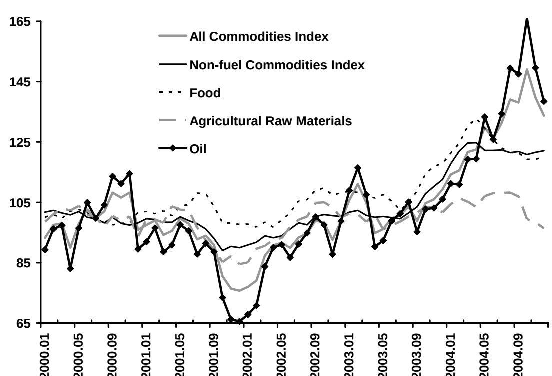

Figure 1. World Commodity Prices 2000-2004

[1] The possibility of persistent effects of the shocks observed in 2004 was highlighted in the Summary of the Quarterly Inflation Report October-December 2004 published by Banco de México in January 2005.

In this paper, we develop a formal framework to obtain the optimal policy of an inflation-targeting monetary authority in the presence of uncertainty about the persistence of cost-push shocks.[2] We allow the economy to randomly alternate between two regimes that only differ in the degree of persistence of cost-push shocks. The possibility of sudden changes in the persistence of cost-push shocks is given by introducing robust control in only one of the regimes of the Markov chain process. Following Hansen and Sargent (2003), robust control in one regime is specified by introducing a set of additive distortions to the cost-push process which generates more persistent shocks than the non-robust regime. This combination of Markov regime-switching and robust control is applied to an open economy model for the Mexican economy. We obtain the welfare losses conditional on being in the regime with relatively less persistent shocks. In the evaluation of the monetary policy rule, we compare recklessness and caution losses. Recklessness losses occur when the monetary authority underestimates the probability of switching to the regime with relatively more persistent shocks. On the other hand, cautionary losses take place when the monetary authority overestimates the aforementioned probability.

Our investigation suggests that monetary authorities in this environment should err on the side of caution. We find that a cautious monetary authority delivers lower welfare losses than a reckless one when it is possible to switch to the regime with relatively more persistent cost-push shocks. Moreover, we show that both recklessness and caution losses increase with the penalty on inflation deviations from its target.

Previous literature on robust control finds that optimal monetary policy generally commands a stronger response of the interest rate to fluctuations in target variables, such as inflation and the output gap when comparing to the case of no uncertainty. In particular, Becker et al. (1994) produce an algorithm for robust optimal decisions with stochastic nonlinear models

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[2] For this paper purposes, the underlying factors affecting this type of uncertainty are indistinguishable.

applied to the United Kingdom. Tetlow and von zur Muehlen (2001a) explore two types of Knightian model uncertainty to explain the difference between estimated interest rate rules and optimal feedback descriptions of monetary policy.[3] Tetlow and von zur Muehlen (2001b) deal with robust control by using three different ways of modeling misspecification in order to explain the inflationary phenomena of the 1970s in the United States. Rustem et al. (2001) compare policy recommendations for worst-case scenarios with those of the robust control approach in inflation targeting regimes. Stock (1999), Onatski and Stock (2002), and Giannoni (2002) study a type of uncertainty reflected on the values of coefficients of the linear equations of a structural model. Walsh (2004) concludes that the problems arising from unexpected shocks become more serious if the shocks last longer. Consequently, central bankers who desire a robust policy will react to all inflation shocks as if they were going to be more persistent. Markov chain processes in optimal control problems have been the subject of recent interest. Zampolli (2006) combines optimal control and Markov regime-switching and finds more cautious optimal monetary policies in the presence of abrupt changes in one multiplicative parameter. Blake and Zampolli (2004) extend those results to find the optimal time-consistent monetary policy for models with forward-looking variables.

To the authors´ knowledge, there have not been previous studies combining Markov regime-switching and robust control to obtain the optimal policy in the presence of uncertainty about the persistence of cost-push shocks. Conditional on being in the regime with relatively less persistence, we find two main results: 1) underestimating the probability of switching to the regime with relatively more persistent cost-push shocks causes higher welfare losses than its overestimation; and 2) the losses associated with the underestimation and overestimation of such probability increase with the penalty on inflation deviations from its target. These results argue in

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3 These authors talk about the notion of Knightian uncertainty when the best guess of the true model is flawed in a serious but unspecificable way.

favor of caution over recklessness when it is possible to switch to the regime with relatively more persistent cost-push shocks.

The remainder of this paper is organized as follows. In Section 2, we set up the optimal control problem with unstructured regime shifts. Section 3 shows the procedure to compute the optimal solution to the problem with Markov regime-switching and robust control. Section 4 presents the open economy model for the Mexican economy. Section 5 describes the procedure to find a reasonable level of robustness. In Section 6, we obtain the recklessness and caution losses for different preference parameters of the monetary authority. Finally, Section 7 presents our conclusions.

2. Optimal control problem with unstructured regime shifts

In this model, the policy maker is a monetary authority with inflation targeting. Moreover, at any given point the economy can alternate between two regimes. The probability of shifting regimes is given by a first order Markov chain process. In regime 1 the policy maker is uncertain about her cost-push process and cannot assign probabilities to alternative sets of cost-push specifications. In order to deal with Knightian model uncertainty, the policy maker uses robust control and introduces an autocorrelated distortion in the cost-push process in the form of a new control variable, ω_{t+1}. The value of ω_{t+1} depends on the next period regime, and ultimately on the history of state variables.[4] This produces a key difference between the two regimes: cost-push shocks are more persistent in regime 1 than in regime 2. However, the distortion needs to be bounded or it will produce infinite damage to the policy maker. The bound on ω_{t+1} is chosen outside the model and it is inversely associated with the "free" parameter of robust control, θ. An increase in θ decreases

[4] Since the distortion ω_{t+1} is turned off in regime 2, the cost-push process does not become relatively more persistent in such regime –i.e. there is no Knightian model uncertainty in regime 2.

the degree of the Knightian model uncertainty and the persistence of the cost-push shock in regime 1. When $\theta \to \infty$ the Knightian model uncertainty disappears and the cost-push shocks persistence is the same in both regimes. Moreover, given that in regime 1 the policy maker faces Knightian model uncertainty, the regime shift is unstructured – i.e. there is no change in a particular parameter when there is regime-switching.

The policy maker's problem is an infinite horizon Quadratic Linear Problem (QLP) that consists of choosing the nominal interest rate to minimize the nominal interest rate variability and the deviations of the inflation and output gap variables from their respective targets. The quadratic loss function can be expressed as follows:

$$L_t = E_t \sum_{k=0}^{\infty} \beta^k \left[(1-\phi)\left[144\alpha(\pi_{t+k} - \pi^*)^2 + (1-\alpha)x_{t+k}^2 \right] + \phi(i_{t+k} - i_{t-1+k})^2 \right] \tag{1}$$

Moreover, the policy maker introduces a fictitious "evil" agent who tries to maximize such deviations in regime 1 by making the cost-push process relatively more persistent. In addition, the policy maker faces a set of constraints and regime switching. The unstructured regime shifts are derived from changing the value of the robust control "free" parameter. Formally, the robust control problem consists of choosing $\mathbf{u}_{it}^*$ to extremize the quadratic criterion function.[5] Since the Riccati equations for the QLP result from first-order conditions, and the first-order conditions for extremizing a quadratic criterion function match those of an ordinary (non-robust) QLP with two controls (see Hansen and Sargent, 2003, pp. 29-30), the optimal control problem with unstructured regime shifts can be written as follows:

$$\mathbf{x}_{1t}^{'}\mathbf{V}_{it}\mathbf{x}_{1t} + d_{it} = \min_{\mathbf{u}_{it}^*}\max \left[\mathbf{x}_{1t}^{'}\tilde{\mathbf{Q}}_{it}\mathbf{x}_{1t} + \mathbf{2x}_{1t}^{'}\tilde{\mathbf{U}}_{it}\mathbf{u}_{it}^* + \mathbf{u}_{it}^{*\prime}\tilde{\mathbf{R}}_{it}\mathbf{u}_{it}^* + \beta\mathbf{E}_t\left(\mathbf{x}_{1t+1}^{'}\mathbf{V}_{it+1}\mathbf{x}_{1t+1} + d_{it+1}\right)\right] \tag{2}$$

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5 Extremization refers to minimizing the criterion function with respect to the original control variables and maximizing it with respect to ω_{k+1} which is a function of the next period's regime.

subject to the system equations[6]

$$\mathbf{x}_{1t+1} = \widetilde{\mathbf{A}}_{jt}\mathbf{x}_{1t} + \widetilde{\mathbf{B}}_{jt}\mathbf{u}^*_{it} + \mathbf{C}_1\boldsymbol{\varepsilon}_{t+1}\ , \quad \mathbf{x}_{1t} \ \text{given}, i = 1,\ 2 \ \text{and} \ j = 1, 2 \tag{3}$$

where $\mathbf{x}_{1t}$ is n1 x 1 vector of predetermined state variables for period t; β is the discount factor (0 < β ≤ 1); $\mathbf{V}_{it}$ is the value function in the current regime i for period t. The new control $\mathbf{u}^*_{it}$ is a (m+n1) x 1 vector of control variables and the model distortions in regime i for period t of the following form:

$$\mathbf{u}^*_{it} = \begin{bmatrix} \mathbf{u}_{it} & (m \times 1) \\ \boldsymbol{\omega}_{it+1} & (n_1 \times 1) \end{bmatrix} \tag{4}$$

where $\boldsymbol{\omega}_{it+1\ (n_1 \times 1)}$ is a (n1 x 1) vector of model distortions for period t that depend on the next period's regime. In addition the new matrices in the objective function and in the system of equations are given by the following equations:

$$\widetilde{\mathbf{Q}}_{it} = \mathbf{Q}_{11} + \mathbf{Q}_{12}\mathbf{D}_{it} + \mathbf{D}_{it}'\mathbf{Q}_{21} + \mathbf{D}_{it}'\mathbf{Q}_{22}\mathbf{D}_{it} \tag{5}$$

$$\widetilde{\mathbf{R}}_{it} = \mathbf{R}_{it} + \mathbf{G}_{it}'\mathbf{Q}_{22}\mathbf{G}_{it} + \mathbf{G}_{it}\mathbf{U}_2 + \mathbf{U}_1'\mathbf{G}_{it} \tag{6}$$

$$\widetilde{\mathbf{U}}_{it} = \mathbf{Q}_{12}\mathbf{G}_{it} + \mathbf{D}_{it}'\mathbf{Q}_{22}\mathbf{G}_{it} + \mathbf{U}_1 + \mathbf{D}_{it}'\mathbf{U}_2 \tag{7}$$

$$\widetilde{\mathbf{A}}_{jt} = \mathbf{A}_{11} + \mathbf{A}_{12}\mathbf{D}_{jt} \tag{8}$$

$$\widetilde{\mathbf{B}}_{jt} = \mathbf{B}_1 + \mathbf{A}_{12}\mathbf{G}_{jt} \tag{9}$$

The regimes are defined as follows:

$$r_{t+1} = \begin{cases} 1 & \text{if cost-push shocks are relatively more persistent} \\ 2 & \text{if cost-push shocks are relatively less persistent} \end{cases}$$

[6] Some of the auxiliary matrices are defined in Appendix A.

The regime r_{t+1} is assumed to follow a fist order Markov chain process with the following transition matrix:

$$P = \begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix} \tag{10}$$

where $p = \Pr\{r_{t+1} = 2 \mid r_t = 1\}$ and $q = \Pr\{r_{t+1} = 1 \mid r_t = 2\}$ $\forall$ t =1,2,3…

Thus, p is the probability that the economy alternates from the relatively more persistent to the relatively less persistent cost-push shock process and q represents exactly the opposite type of probability. These probabilities represent the uncertainty about the type of regime in the next period. We assume that the regime of the economy r_{t+1} is revealed only at the end of period t, after the policy action has been decided. That is, when the policy maker chooses the policy rule, r_t is known but r_{t+1} is still uncertain.

3. Optimal solution with unstructured regime shifts

Solving the optimal control problem with unstructured regime shifts is equivalent to finding a contingent policy rule $\mathbf{u}_{\mathbf{it}}^*$. By adapting the Giordani and Söderlind's (2004) discretion solution to the presence of Markov regime-switching given by Equations (2)-(4) and (10), we obtain the following solution:

$$\mathbf{u}_{\mathbf{it}}^* = -\tilde{\mathbf{F}}_{\mathbf{it}}\mathbf{x}_{\mathbf{1t}} \tag{11}$$

where

$$\begin{aligned}\tilde{\mathbf{F}}_{\mathbf{1t}} = \mathbf{inv}(&\tilde{\mathbf{R}}_{\mathbf{1t}} + \beta\tilde{\mathbf{B}}_{\mathbf{1t}}'(1-p)\tilde{\mathbf{V}}_{\mathbf{1t}}\tilde{\mathbf{B}}_{\mathbf{1t}} + \beta\tilde{\mathbf{B}}_{\mathbf{2t}}'p\tilde{\mathbf{V}}_{\mathbf{2t}}\tilde{\mathbf{B}}_{\mathbf{2t}}) \\ &* (\tilde{\mathbf{U}}_{\mathbf{1t}}' + \beta(\tilde{\mathbf{B}}_{\mathbf{1t}}'(1-p)\tilde{\mathbf{V}}_{\mathbf{1t}}\tilde{\mathbf{A}}_{\mathbf{1t}} + p\tilde{\mathbf{B}}_{\mathbf{2t}}'\tilde{\mathbf{V}}_{\mathbf{2t}}\tilde{\mathbf{A}}_{\mathbf{2t}}))\end{aligned} \tag{12}$$

$$\begin{aligned}\tilde{\mathbf{F}}_{\mathbf{2t}} = \mathbf{inv}(&\tilde{\mathbf{R}}_{\mathbf{2t}} + \beta\tilde{\mathbf{B}}_{\mathbf{1t}}'q\tilde{\mathbf{V}}_{\mathbf{1t}}\tilde{\mathbf{B}}_{\mathbf{1t}} + \beta\tilde{\mathbf{B}}_{\mathbf{2t}}'(1-q)\tilde{\mathbf{V}}_{\mathbf{2t}}\tilde{\mathbf{B}}_{\mathbf{2t}}) \\ &* (\tilde{\mathbf{U}}_{\mathbf{2t}}' + \beta(\tilde{\mathbf{B}}_{\mathbf{1t}}'q\tilde{\mathbf{V}}_{\mathbf{1t}}\tilde{\mathbf{A}}_{\mathbf{1t}} + (1-q)\tilde{\mathbf{B}}_{\mathbf{2t}}'\tilde{\mathbf{V}}_{\mathbf{2t}}\tilde{\mathbf{A}}_{\mathbf{2t}}))\end{aligned} \tag{13}$$

combining Equations (11)-(13) with Equation (3), we obtain

$$\mathbf{x_{2t}} = \mathbf{K_{it}} \mathbf{x_{1t}} \text{, with } \mathbf{K_{it}} = \mathbf{D_{it}} - \mathbf{G_{it}} \mathbf{\tilde{F}_{it}} \tag{14}$$

Hence, the value functions for regime 1 and 2 are given by Equations (15) and (16), respectively:

$$\begin{aligned} \mathbf{\tilde{V}_{1t}} &= \mathbf{\tilde{Q}_{1t}} - \mathbf{\tilde{U}_{1t}} \mathbf{\tilde{F}_{1t}} - \mathbf{\tilde{F}_{1t}}' \mathbf{\tilde{U}_{1t}}' + \mathbf{\tilde{F}_{1t}}' \mathbf{\tilde{R}_{1t}} \mathbf{\tilde{F}_{1t}} \\ &+ \beta((\mathbf{\tilde{A}_{1t}} - \mathbf{\tilde{B}_{1t}} \mathbf{\tilde{F}_{1t}})'(1-p) \mathbf{\tilde{V}_{1t}} (\mathbf{\tilde{A}_{1t}} - \mathbf{\tilde{B}_{1t}} \mathbf{\tilde{F}_{1t}}) + p(\mathbf{\tilde{A}_{2t}} - \mathbf{\tilde{B}_{2t}} \mathbf{\tilde{F}_{2t}})' \mathbf{\tilde{V}_{2t}} (\mathbf{\tilde{A}_{2t}} - \mathbf{\tilde{B}_{2t}} \mathbf{\tilde{F}_{2t}})) \end{aligned} \tag{15}$$

$$\begin{aligned} \mathbf{\tilde{V}_{2t}} &= \mathbf{\tilde{Q}_{2t}} - \mathbf{\tilde{U}_{2t}} \mathbf{\tilde{F}_{2t}} - \mathbf{\tilde{F}_{2t}}' \mathbf{\tilde{U}_{2t}}' + \mathbf{\tilde{F}_{2t}}' \mathbf{\tilde{R}_{2t}} \mathbf{\tilde{F}_{2t}} \\ &+ \beta((\mathbf{\tilde{A}_{1t}} - \mathbf{\tilde{B}_{1t}} \mathbf{\tilde{F}_{1t}})' q \mathbf{\tilde{V}_{1t}} (\mathbf{\tilde{A}_{1t}} - \mathbf{\tilde{B}_{1t}} \mathbf{\tilde{F}_{1t}}) + (1-q)(\mathbf{\tilde{A}_{2t}} - \mathbf{\tilde{B}_{2t}} \mathbf{\tilde{F}_{2t}})' \mathbf{\tilde{V}_{2t}} (\mathbf{\tilde{A}_{2t}} - \mathbf{\tilde{B}_{2t}} \mathbf{\tilde{F}_{2t}})) \end{aligned} \tag{16}$$

The solution to the algorithm to Equations (11)-(16) is shown in Appendix A. Such solution incorporate the standard solutions when we set $p = q = 0$. In this special case, we would obtain the solution to two optimal control problems, one corresponding to regime 1 and the other to regime 2 under the assumption that each regime will be there permanently.

4. Unstructured regime shifts in an open economy model

We used the open economy model for the Mexican economy in Roldán-Peña (2005). This model takes into account the dynamic homogeneity property as well as some parameters restrictions which reflect some assumptions about long-term values for the real interest rate and the real exchange rate.[7] The endogenous variables are the output gap (x_t), core inflation (π_t^c), and the real exchange rate (tcr_t). Headline inflation and the change in the nominal exchange rate are denoted by π_t and Δtcn_t, respectively. The equations for the endogenous variables, headline inflation and the purchasing power parity are shown below. The superscript "US" in a variable denotes its value for the United States which is considered exogenous.

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[7] For estimation methods and samples used see Roldán-Peña (2005).

$$x_t = b_0 + b_1 x_{t-1} + b_2 E_t\{x_{t+1}\} + b_3 r_{t-1} + b_4 x_{t-1}^{US} + b_6 \Delta gto_{t-2} + u_t \tag{17}$$

$$\pi_t^c = a_1 E_t\{\pi_{t+1}^c\} + a_2 x_t + a_3\left(\Delta tcn_{t-2} + \pi_{t-2}^{US}\right) + a_4 \Delta sal_{t-1} + g_t \tag{18}$$

$$tcr_t = c_0 tcr_{t-1} + c_1 tcr_{t-4} + c_2\left(E_t\{tcr_{t+1}\} + \left(\frac{1}{1200}\right)\left(r_t^{US} - r_t\right)\right) + v_t \tag{19}$$

$$\pi_t = w_c \pi_t^c + w_{nc} \pi_t^{nc} \tag{20}$$

$$\Delta tcn_t + \pi_t^{US} = \Delta tcr_t + \pi_t \tag{21}$$

where g_t, u_t and v_t represent the error terms of the core inflation, output gap and real exchange rate specifications, respectively. [8] They are defined as autoregressive AR(1) processes as follows:

$$g_t = \rho_g g_{t-1} + \hat{g}_t \tag{22}$$

$$u_t = \rho_u u_{t-1} + \hat{u}_t \tag{23}$$

$$v_t = \rho_v v_{t-1} + \hat{v}_t \tag{24}$$

The exogenous variables are the non-core inflation (π_t^{nc}), the change in wages (Δsal_t) and the change in government spending (Δgto_t), given by the following equations:

$$\pi_t^{nc} = d_0 + d_1 \pi_{t-1}^{nc} + w_t \tag{25}$$

$$\Delta sal_t = e_0 + e_1 \Delta sal_{t-1} + \chi_t \tag{26}$$

$$\Delta gto_t = f_0 + f_1 \Delta gto_{t-1} + y_t \tag{27}$$

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[8] The distortion ω_{t+1} affects g_t in a way that makes it relatively more persistent. However, since this distortion is Knightian in nature, it does not affect the autoregressive parameter of the cost-push process.

where w_t, χ_t and y_t represent the error terms of the non-core inflation, change in wages and change in government spending specifications, respectively. They are assumed to follow an autoregressive AR(1) process:

$$w_t = \rho_w w_{t-1} + \hat{w}_t \quad (28)$$

$$\chi_t = \rho_\chi \chi_{t-1} + \hat{\chi}_t \quad (29)$$

$$y_t = \rho_y y_{t-1} + \hat{y}_t \quad (30)$$

The external exogenous variables (US inflation, output gap and interest rate, defined as π_t^{US}, x_t^{US} and i_t^{US} respectively) are determined as a vector autoregression VAR(2):

$$\pi_t^{US} = \alpha_0 + \alpha_1 \pi_{t-1}^{US} + \alpha_2 \pi_{t-2}^{US} + \alpha_3 x_{t-1}^{US} + \alpha_4 x_{t-2}^{US} + \alpha_5 i_{t-1}^{US} + \alpha_6 \pi_{t-2}^{US} + \delta_t \quad (31)$$

$$x_t^{US} = \beta_0 + \beta_1 \pi_{t-1}^{US} + \beta_2 \pi_{t-2}^{US} + \beta_3 x_{t-1}^{US} + \beta_4 x_{t-2}^{US} + \beta_5 i_{t-1}^{US} + \beta_6 \pi_{t-2}^{US} + \varepsilon_t \quad (32)$$

$$i_t^{US} = \gamma_0 + \gamma_1 \pi_{t-1}^{US} + \gamma_2 \pi_{t-2}^{US} + \gamma_3 x_{t-1}^{US} + \gamma_4 x_{t-2}^{US} + \gamma_5 i_{t-1}^{US} + \gamma_6 \pi_{t-2}^{US} + \eta_t \quad (33)$$

where δ_t, ε_t and η_t represent the error terms of the US inflation, output gap and interest rate specifications, respectively, and are defined as autoregressive AR(1) processes:

$$\delta_t = \rho_\delta \delta_{t-1} + \hat{\delta}_t \quad (34)$$

$$\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \hat{\varepsilon}_t \quad (35)$$

$$\eta_t = \rho_\eta \eta_{t-1} + \hat{\eta}_t \quad (36)$$

The state-space representation of the model is:

$$\begin{bmatrix} \mathbf{x}_{1t+1} \\ E_t\{\mathbf{x}_{2t+1}\} \end{bmatrix} = \mathbf{A} \begin{bmatrix} \mathbf{x}_{1t} \\ \mathbf{x}_{2t} \end{bmatrix} + \mathbf{B}\mathbf{u}_{1t}^{*} + \begin{bmatrix} \mathbf{C}_1 \boldsymbol{\varepsilon}_{t+1} \\ \mathbf{0} \end{bmatrix} \quad (37)$$

where the vector of predetermined state variables $\mathbf{x}_{1t}$ is given by the following equation:[9]

[9] The term *cte* denotes a constant term.

$$\mathbf{x_{1t}} = \begin{bmatrix} cte, \pi_t^*, \pi_t^{nc}, \Delta sal_t, \Delta gto_t, \pi_{t-1}^c, \pi_{t-1}^{nc}, x_{t-1}, tcr_{t-1}, \Delta sal_{t-1}, \Delta gto_{t-1}, \pi_{t-2}^c, \pi_{t-2}^{nc}, \\ tcr_{t-2}, \Delta sal_{t-2}, \Delta gto_{t-2}, \pi_{t-3}^c, \pi_{t-3}^{nc}, tcr_{t-3}, tcr_{t-4}, \pi_t^{US}, x_t^{US}, i_t^{US}, \pi_{t-1}^{US}, x_{t-1}^{US}, i_{t-1}^{US}, g_t, w_t, \\ u_t, v_t, \chi_t, y_t, \delta_t, \varepsilon_t, \eta_t, g_{t-1}, w_{t-1}, i_{t-1} \end{bmatrix} \tag{38}$$

and the vector of non-predetermined state variables $\mathbf{x_{2t}}$ has the components shown in the following equation:

$$\mathbf{x_{2t}} = \left[\pi_t^c, x_t, tcr_t\right] \tag{39}$$

The control variable vector $\mathbf{u_{it}}$ is given by the following:

$$\mathbf{u_{it}^*} = \begin{bmatrix} i_{it} \\ \boldsymbol{\omega}_{\mathbf{it+1}\ (n_1 \times 1)} \end{bmatrix} \tag{40}$$

5. Selection of the Robust Control 'Free' Parameter

The formulation of robust control used in this paper requires the value of the 'free' parameter, θ, to come from outside the model. The main purpose is to find reasonable values of the 'free' parameter to prevent the policy maker from appearing catastrophist instead of cautious. We follow Hansen and Sargent (2003) and use the detection error probability theory to choose reasonable values of θ. In particular, the objective is to find values of θ for which it is statistically difficult to distinguish between the reference and the distorted model. This way, extremely pessimistic cases are ruled out.

The procedure consists of obtaining two types of probabilities: i) the probability of choosing the reference model when the data were generated by the distorted model and ii) the probability of choosing the distorted model when the data were generated by the reference model. The average of these two probabilities is the probability of making an error in the detection of the model – i.e. the detection error probability. Note that if there is no robustness $(\theta \to \infty)$ the reference and the distorted model are the same and the detection error probability is 0.5. On the other hand, when the level of robustness is infinite the detection error probability is zero. Hansen and Sargent (2003) recommend the use of θ associated with detection error probabilities between 0.1 and 0.2 which correspond to confidence intervals of 95% and 90%, respectively.

We use the code of Giordani and Soderlind (2004) to obtain the detection error probabilities. The detailed procedure is shown in Appendix B. We solved the problem for a time horizon of 150 months (T=150) and 1,000 simulations. Each simulation represents a random draw of the additive noise. We decided to use $\theta = 325$, which produces a detection error probability of 0.2 when α = 0.5, for a couple of reasons: i) it corresponds to a confidence interval of 90% and ii) produces important differences between the policy rules of the reference and distorted model. We find that in our model a detection error probability higher than 0.2 does not produce important differences between the policy rules.

In order to observe the implied persistence of $\theta = 325$, we obtain the impulse-response functions of the output gap, core inflation and the nominal interest rate to a one-standard-deviation cost-push innovation. Figure 2 shows the impulse-response functions for regimes 1 and 2, assuming the absence of a Markov chain process between regimes.[10] In regime 1, the impulse-response functions of the nominal interest rate, core inflation and output gap reveal a relatively more persistent cost-push process than in regime 2.

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[10] Only for the purpose of illustrating the differences between regimes 1 and 2, the initial state variables were set to zero.

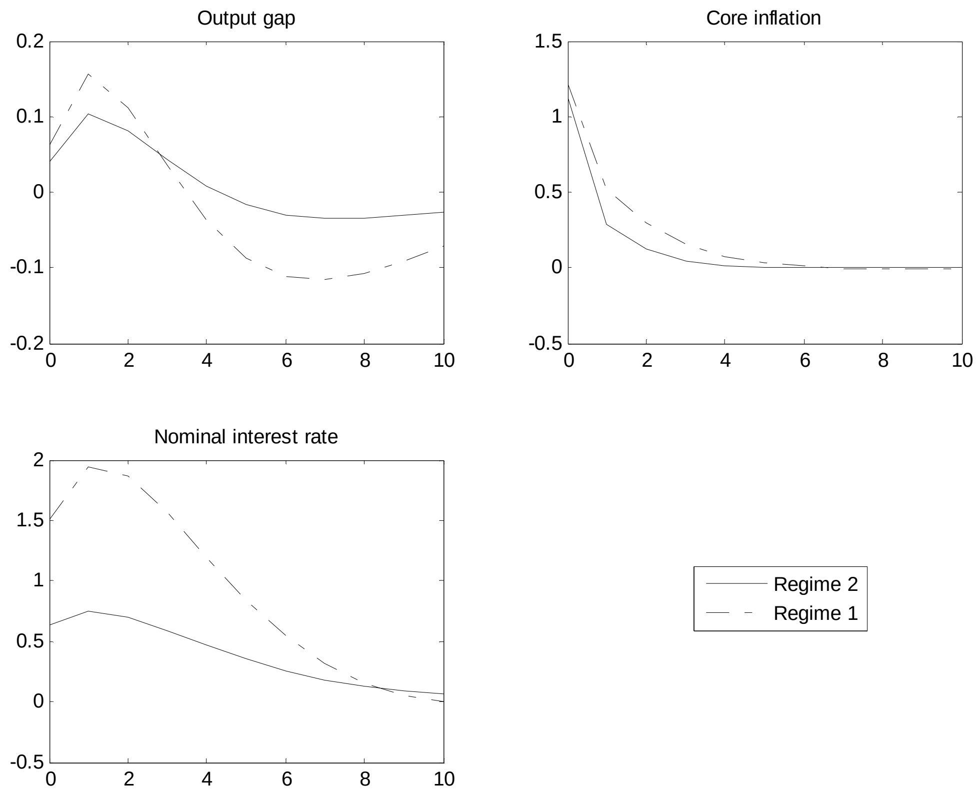

Figure 2. Impulse-Response Functions to a One-Standard-Deviation Cost-Push Innovation

6. Caution versus recklessness losses

The welfare analysis done in this paper is conditional on being in regime 2 and considers the possibility of switching to regime 1. The approach used in this work to deal with the Knightian uncertainty faced by the policy maker follows Zampolli (2006). First, we assume that the policy maker does not know the true transition probability q, but chooses a transition probability $\hat{q}$. Second, we fix the transition probability p or, equivalently, the expected duration of regime 1, which is $\frac{1}{p}$ periods. Finally, we obtain the losses associated with all the pairs $(\hat{q}, q)$. Losses are normalized with respect to $(\hat{q}, q)$ = (0,0) and are conditional on being in regime 2. For every q, minimal losses occur when $\hat{q} = q$.

In order to evaluate and characterize the optimal policy rule we define recklessness and caution losses. Recklessness losses are defined as the welfare losses that occur when the policy maker underestimates the probability of switching to regime 1, that is, when $\hat{q} < q$. On the other hand, caution losses are the welfare losses that takes place when the policy maker overestimates the probability of switching to regime 1, that is, when $\hat{q} > q$. Finally, recklessness and caution losses are defined as the sum of losses for which $\hat{q} < q$ and $\hat{q} > q$, respectively. The following table shows the recklessness and caution losses.

Table 1. Recklessness and Caution Losses.

		LOSSES		
		α = 0.1	α = 0.5	α = 0.9
CAUTION	p = 0.25	55.01	55.08	55.44
	p = 0.5	55.01	55.08	55.72
	p = 0.75	55.01	55.09	56.23
RECKLESSNESS	p = 0.25	55.27	56.83	65.00
	p = 0.5	55.27	56.85	69.29
	p = 0.75	55.27	56.87	82.65

Table 1 shows that recklessness losses are always higher than caution losses. This result argues in favor of caution over recklessness in the formulation of monetary policy when it is possible to transit to the regime with relatively more persistent cost-push shocks. Moreover, both types of losses are non decreasing with p and α. Since the losses are conditional on being in regime 2, higher values of p produce more frequent switches from regime 1 to regime 2. Finally, the difference between recklessness and caution losses increases with α.

Figure 3 shows the losses for all $(\hat{q}, q)$ pairs for different preference parameters and values of p.[11] The transition probabilities chosen by the policy maker are on the y-axis and the true transition probabilities on the x-axis. First, it can be seen that all losses substantially increase when $q = 1.0$ regardless of $\hat{q}$. This occurs because regime 2, for all $q < 1.0$, strongly prevails in the weighted matrix $\mathbf{R}_{2t}$ given by Equation (A-24).

11 We decided to use the middle value of the range used by Favero and Milani (2005) for the interest rate smoothing parameter ϕ. Other values for this parameter do not change the qualitative results.

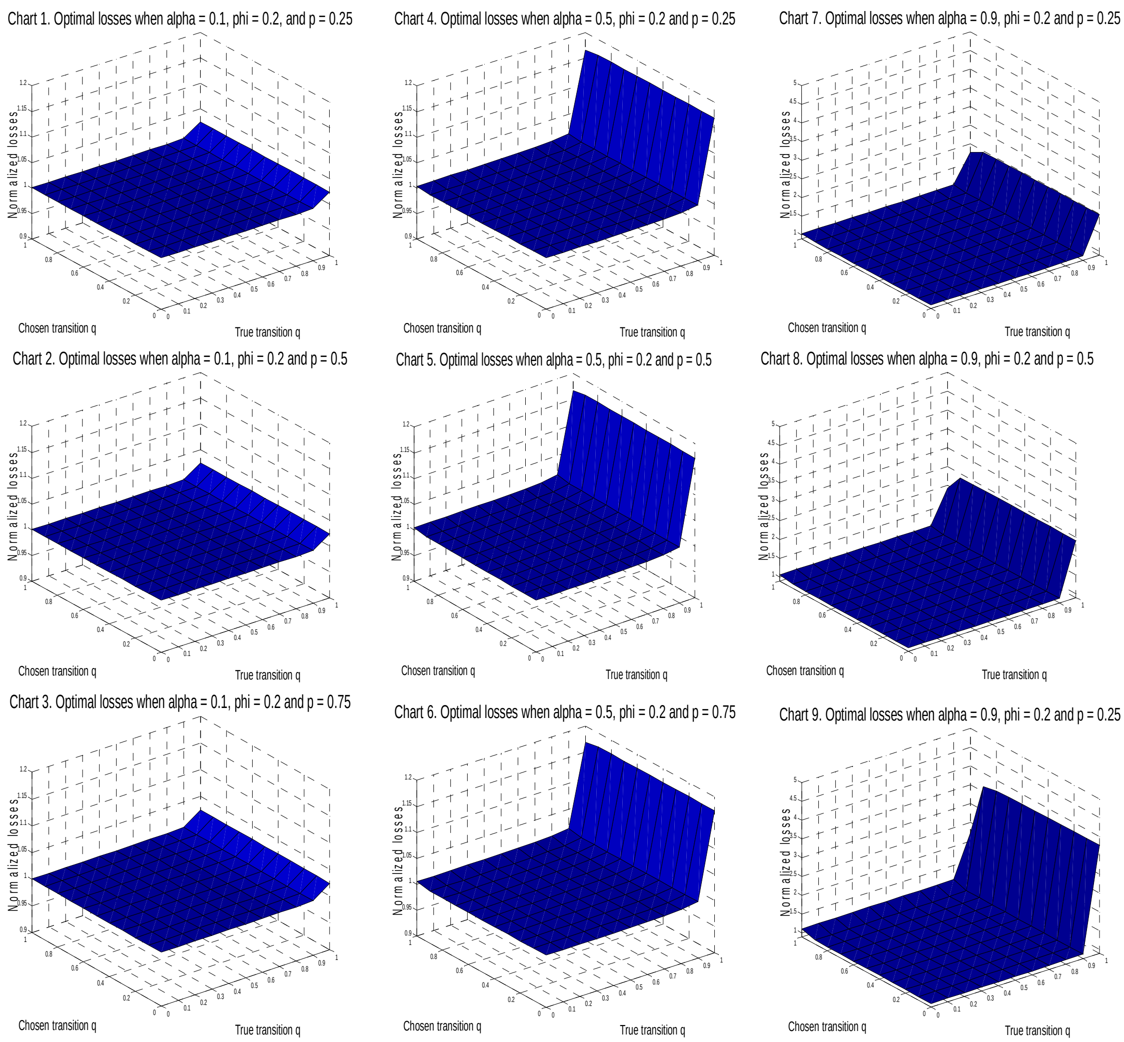

Figure 3. Losses associated with all the pairs $(\hat{q}, q)$ conditional on being in regime 2

Second, a horizontal comparison of the charts shows that losses increase with the preference parameter α.[12] At first, this result seems counterintuitive. However, the detection error probabilities obtained for $\theta = 325$ decreased with α. In other words, the "evil" agent is able to do more damage when the policy maker increases the penalty on the only variable subject to the distortions.[13] Moreover, the charts show that recklessness losses substantially increase when the true

[12] It is worth mentioning that the scale of Charts 7-9 is different from the rest's.

[13] Indeed, the system was no longer controllable for $\alpha = 1.0$ and some combinations of $(\hat{q}, q, p)$.

transition probability $q = 1.0$. On the other hand, caution losses do not substantially increase when the true transition probability $q = 0.0$.

7. Conclusions

In this paper we develop a framework to obtain the optimal policy response in the presence of uncertainty about the persistence of cost-push shocks. We allow the economy to randomly alternate between two regimes that only differ in the degree of persistence of cost-push shocks. We model the possibility of sudden changes in such persistence by using robust control in one of the regimes of the Markov chain process. This combination of Markov regime-switching and robust control is applied to an open economy model for the Mexican economy. We obtain the welfare losses conditional on being in the regime with relatively less persistent shocks. In the evaluation of a monetary policy rule, we compare recklessness and caution losses. The former occurs when the monetary authority underestimates the probability of switching to the regime with relatively more persistent shocks. The latter occurs when the monetary authority overestimates such probability.

To the authors' knowledge, no previous study has combined Markov regime-switching and robust control. Conditional on being in the regime with relatively less persistence, such combination delivers the following results: 1) underestimating the probability of switching to the regime with relatively more persistent cost-push shocks causes more welfare losses than its overestimation; and 2) the losses associated with the underestimation and overestimation of such probability increase with the penalty on inflation deviations from its target. These results argue in favor of caution over recklessness when it is possible to switch to the regime with relatively more persistent cost-push shocks.

Appendix A

In this appendix we show the solution algorithm for the optimal policy under discretion with Markov regime-switching.

$n = n_1 + n_2$ where n_1 and n_2 represent the number of predetermined and non-predetermined variables, respectively (A-1)

$con = 32,500$ con is the value taken by θ in regime 2 (A-2)

$$\theta = 325 \tag{A-3}$$

$$\beta = 0.99585 \tag{A-4}$$

$$\mathbf{A_{11}} = \mathbf{A}(\mathbf{1}: n_1, \mathbf{1}: n_1) \tag{A-5}$$

$$\mathbf{A_{12}} = \mathbf{A}(\mathbf{1}: n_1, (n_1 + \mathbf{1}): n) \tag{A-6}$$

$$\mathbf{A_{21}} = \mathbf{A}((n_1 + \mathbf{1}): n, \mathbf{1}: n_1) \tag{A-7}$$

$$\mathbf{A_{22}} = \mathbf{A}((n_1 + \mathbf{1}): n, (n_1 + \mathbf{1}): n) \tag{A-8}$$

$$\mathbf{Q_{11}} = \mathbf{Q}(\mathbf{1}: n_1, \mathbf{1}: n_1) \tag{A-9}$$

$$\mathbf{Q_{12}} = \mathbf{Q}(\mathbf{1}: n_1, (n_1 + \mathbf{1}): n) \tag{A-10}$$

$$\mathbf{Q_{21}} = \mathbf{Q}((n_1 + \mathbf{1}): n, \mathbf{1}: n_1) \tag{A-11}$$

$$\mathbf{Q_{22}} = \mathbf{Q}((n_1 + \mathbf{1}): n, (n_1 + \mathbf{1}): n) \tag{A-12}$$

$$\mathbf{B_1} = \mathbf{B}(\mathbf{1}: n_1, :) \tag{A-13}$$

$$\mathbf{B_1^*} = \left[\mathbf{B_1}\ \mathbf{C_1}\right] \tag{A-14}$$

$$\mathbf{B_2} = \mathbf{B}(n_1 + \mathbf{1}: n, :) \tag{A-15}$$

$$\mathbf{U_1} = \mathbf{U}(\mathbf{1}: n_1, :) \tag{A-16}$$

$$\mathbf{U_2} = \mathbf{U}(n_1 + \mathbf{1}: n, :) \tag{A-17}$$

The Bellman equation for the optimization can be written

$$\mathbf{x}_{\mathbf{1t}}'\mathbf{V}_{\mathbf{it}}\mathbf{x}_{\mathbf{1t}} + d_{it} = \min_{\mathbf{u}_{\mathbf{it}}^*}\max\left[\mathbf{x}_{\mathbf{1t}}'\widetilde{\mathbf{Q}}_{\mathbf{it}}\mathbf{x}_{\mathbf{1t}} + \mathbf{2}\mathbf{x}_{\mathbf{1t}}'\widetilde{\mathbf{U}}_{\mathbf{it}}\mathbf{u}_{\mathbf{it}}^* + \mathbf{u}_{\mathbf{it}}^{*\prime}\widetilde{\mathbf{R}}_{\mathbf{it}}\mathbf{u}_{\mathbf{it}}^* + \beta\mathbf{E}_t\left(\mathbf{x}_{\mathbf{1t+1}}'\mathbf{V}_{\mathbf{it+1}}\mathbf{x}_{\mathbf{1t+1}} + d_{it+1}\right)\right]$$

$$s.t.\ \mathbf{x}_{1t+1} = \widetilde{\mathbf{A}}_{jt}\mathbf{x}_{1t} + \widetilde{\mathbf{B}}_{jt}\mathbf{u}^*_{it} + \mathbf{C}_1\boldsymbol{\varepsilon}_{t+1}\,,\quad \mathbf{x}_{1t}\ \text{given},\ i=1,2\ \text{and}\ j=1,2. \tag{A-18}$$

where the matrices with a tilde (~) are defined as

$$\mathbf{D}_{1t} = (\mathbf{A}_{22} - \mathbf{K}_{1t}\mathbf{A}_{12})^{-1}(\mathbf{K}_{1t}\mathbf{A}_{11} - \mathbf{A}_{21}) \tag{A-19}$$

$$\mathbf{G}_{1t} = (\mathbf{A}_{22} - \mathbf{K}_{1t}\mathbf{A}_{12})^{-1}(\mathbf{K}_{1t}\mathbf{B}_1 - \mathbf{B}_2) \tag{A-20}$$

$$\mathbf{R}_{1t} = (1-p)\begin{bmatrix}\mathbf{R}_{k\times k} & \mathbf{0}_{k\times n_1}\\ \mathbf{0}_{n_1\times k} & -\theta\,\mathbf{I}_{n_1}\end{bmatrix} + p\begin{bmatrix}\mathbf{R}_{k\times k} & \mathbf{0}_{k\times n_1}\\ \mathbf{0}_{n_1\times k} & -con\,\mathbf{I}_{n_1}\end{bmatrix} \tag{A-21}$$

$$\mathbf{D}_{2t} = (\mathbf{A}_{22} - \mathbf{K}_{2t}\mathbf{A}_{12})^{-1}(\mathbf{K}_{2t}\mathbf{A}_{11} - \mathbf{A}_{21}) \tag{A-22}$$

$$\mathbf{G}_{2t} = (\mathbf{A}_{22} - \mathbf{K}_{2t}\mathbf{A}_{12})^{-1}(\mathbf{K}_{2t}\mathbf{B}_1 - \mathbf{B}_2) \tag{A-23}$$

$$\mathbf{R}_{2t} = q\begin{bmatrix}\mathbf{R}_{k\times k} & \mathbf{0}_{k\times n_1}\\ \mathbf{0}_{n_1\times k} & -\theta\,\mathbf{I}_{n_1}\end{bmatrix} + (1-q)\begin{bmatrix}\mathbf{R}_{k\times k} & \mathbf{0}_{k\times n_1}\\ \mathbf{0}_{n_1\times k} & -con\,\mathbf{I}_{n_1}\end{bmatrix} \tag{A-24}$$

$$\widetilde{\mathbf{A}}_{1t} = \mathbf{A}_{11} + \mathbf{A}_{12}\mathbf{D}_{1t} \tag{A-25}$$

$$\widetilde{\mathbf{B}}_{1t} = \mathbf{B}_1 + \mathbf{A}_{12}\mathbf{G}_{1t} \tag{A-26}$$

$$\widetilde{\mathbf{Q}}_{1t} = \mathbf{Q}_{11} + \mathbf{Q}_{12}\mathbf{D}_{1t} + \mathbf{D}_{1t}'\mathbf{Q}_{21} + \mathbf{D}_{1t}'\mathbf{Q}_{22}\mathbf{D}_{1t} \tag{A-27}$$

$$\widetilde{\mathbf{U}}_{1t} = \mathbf{Q}_{12}\mathbf{G}_{1t} + \mathbf{D}_{1t}'\mathbf{Q}_{22}\mathbf{G}_{1t} + \mathbf{U}_1 + \mathbf{D}_{1t}'\mathbf{U}_2 \tag{A-28}$$

$$\widetilde{\mathbf{R}}_{1t} = \mathbf{R}_{1t} + \mathbf{G}_{1t}'\mathbf{Q}_{22}\mathbf{G}_{1t} + \mathbf{G}_{1t}\mathbf{U}_2 + \mathbf{U}_1'\mathbf{G}_{1t} \tag{A-29}$$

$$\widetilde{\mathbf{A}}_{2t} = \mathbf{A}_{11} + \mathbf{A}_{12}\mathbf{D}_{2t} \tag{A-30}$$

$$\widetilde{\mathbf{B}}_{2t} = \mathbf{B}_1 + \mathbf{A}_{12}\mathbf{G}_{2t} \tag{A-31}$$

$$\widetilde{\mathbf{Q}}_{2t} = \mathbf{Q}_{11} + \mathbf{Q}_{12}\mathbf{D}_{2t} + \mathbf{D}_{2t}'\mathbf{Q}_{21} + \mathbf{D}_{2t}'\mathbf{Q}_{22}\mathbf{D}_{2t} \tag{A-32}$$

$$\widetilde{\mathbf{U}}_{2t} = \mathbf{Q}_{12}\mathbf{G}_{2t} + \mathbf{D}_{2t}'\mathbf{Q}_{22}\mathbf{G}_{2t} + \mathbf{U}_1 + \mathbf{D}_{2t}'\mathbf{U}_2 \tag{A-33}$$

$$\widetilde{\mathbf{R}}_{2t} = \mathbf{R}_{2t} + \mathbf{G}_{2t}'\mathbf{Q}_{22}\mathbf{G}_{2t} + \mathbf{G}_{2t}'\mathbf{U}_2 + \mathbf{U}_1'\mathbf{G}_{2t} \tag{A-34}$$

The first order conditions of (A-18) with respect to $\mathbf{u}^*_{it}$ are

$$\mathbf{u}^*_{it} = -\widetilde{\mathbf{F}}_{it}\mathbf{x}_{1t} \tag{A-35}$$

$$\widetilde{\mathbf{F}}_{\mathbf{1t}} = \mathbf{inv}(\widetilde{\mathbf{R}}_{\mathbf{1t}} + \beta\widetilde{\mathbf{B}}_{\mathbf{1t}}'(1-p)\widetilde{\mathbf{V}}_{\mathbf{1t}}\widetilde{\mathbf{B}}_{\mathbf{1t}} + \beta\widetilde{\mathbf{B}}_{\mathbf{2t}}'\, p\widetilde{\mathbf{V}}_{\mathbf{2t}}\widetilde{\mathbf{B}}_{\mathbf{2t}}) \\ *(\widetilde{\mathbf{U}}_{\mathbf{1t}}' + \beta(\widetilde{\mathbf{B}}_{\mathbf{1t}}'(1-p)\widetilde{\mathbf{V}}_{\mathbf{1t}}\widetilde{\mathbf{A}}_{\mathbf{1t}} + p\widetilde{\mathbf{B}}_{\mathbf{2t}}'\widetilde{\mathbf{V}}_{\mathbf{2t}}\widetilde{\mathbf{A}}_{\mathbf{2t}})) \tag{A-36}$$

$$\widetilde{\mathbf{F}}_{\mathbf{2t}} = \mathbf{inv}(\widetilde{\mathbf{R}}_{\mathbf{2t}} + \beta\widetilde{\mathbf{B}}_{\mathbf{1t}}'\, q\widetilde{\mathbf{V}}_{\mathbf{1t}}\widetilde{\mathbf{B}}_{\mathbf{1t}} + \beta\widetilde{\mathbf{B}}_{\mathbf{2t}}'(1-q)\widetilde{\mathbf{V}}_{\mathbf{2t}}\widetilde{\mathbf{B}}_{\mathbf{2t}}) \\ *(\widetilde{\mathbf{U}}_{\mathbf{2t}}' + \beta(\widetilde{\mathbf{B}}_{\mathbf{1t}}'\, q\widetilde{\mathbf{V}}_{\mathbf{1t}}\widetilde{\mathbf{A}}_{\mathbf{1t}} + (1-q)\widetilde{\mathbf{B}}_{\mathbf{2t}}'\widetilde{\mathbf{V}}_{\mathbf{2t}}\widetilde{\mathbf{A}}_{\mathbf{2t}})) \tag{A-37}$$

Combining with (A-18) gives

$$\mathbf{x}_{\mathbf{2t}} = \mathbf{K}_{\mathbf{it}}\mathbf{x}_{\mathbf{1t}}, \text{ with } \mathbf{K}_{\mathbf{it}} = \mathbf{D}_{\mathbf{it}} - \mathbf{G}_{\mathbf{it}}\widetilde{\mathbf{F}}_{\mathbf{it}} \text{ and} \tag{A-38}$$

$$\widetilde{\mathbf{V}}_{\mathbf{1t}} = \widetilde{\mathbf{Q}}_{\mathbf{1t}} - \widetilde{\mathbf{U}}_{\mathbf{1t}}\widetilde{\mathbf{F}}_{\mathbf{1t}} - \widetilde{\mathbf{F}}_{\mathbf{1t}}'\widetilde{\mathbf{U}}_{\mathbf{1t}}' + \widetilde{\mathbf{F}}_{\mathbf{1t}}'\widetilde{\mathbf{R}}_{\mathbf{1t}}\widetilde{\mathbf{F}}_{\mathbf{1t}} \\ + \beta((\widetilde{\mathbf{A}}_{\mathbf{1t}} - \widetilde{\mathbf{B}}_{\mathbf{1t}}\widetilde{\mathbf{F}}_{\mathbf{1t}})'(1-p)\widetilde{\mathbf{V}}_{\mathbf{1t}}(\widetilde{\mathbf{A}}_{\mathbf{1t}} - \widetilde{\mathbf{B}}_{\mathbf{1t}}\widetilde{\mathbf{F}}_{\mathbf{1t}}) + p(\widetilde{\mathbf{A}}_{\mathbf{2t}} - \widetilde{\mathbf{B}}_{\mathbf{2t}}\widetilde{\mathbf{F}}_{\mathbf{2t}})'\widetilde{\mathbf{V}}_{\mathbf{2t}}(\widetilde{\mathbf{A}}_{\mathbf{2t}} - \widetilde{\mathbf{B}}_{\mathbf{2t}}\widetilde{\mathbf{F}}_{\mathbf{2t}})) \tag{A-39}$$

$$\widetilde{\mathbf{V}}_{\mathbf{2t}} = \widetilde{\mathbf{Q}}_{\mathbf{2t}} - \widetilde{\mathbf{U}}_{\mathbf{2t}}\widetilde{\mathbf{F}}_{\mathbf{2t}} - \widetilde{\mathbf{F}}_{\mathbf{2t}}'\widetilde{\mathbf{U}}_{\mathbf{2t}}' + \widetilde{\mathbf{F}}_{\mathbf{2t}}'\widetilde{\mathbf{R}}_{\mathbf{2t}}\widetilde{\mathbf{F}}_{\mathbf{2t}} \\ + \beta((\widetilde{\mathbf{A}}_{\mathbf{1t}} - \widetilde{\mathbf{B}}_{\mathbf{1t}}\widetilde{\mathbf{F}}_{\mathbf{1t}})'\, q\widetilde{\mathbf{V}}_{\mathbf{1t}}(\widetilde{\mathbf{A}}_{\mathbf{1t}} - \widetilde{\mathbf{B}}_{\mathbf{1t}}\widetilde{\mathbf{F}}_{\mathbf{1t}}) + (1-q)(\widetilde{\mathbf{A}}_{\mathbf{2t}} - \widetilde{\mathbf{B}}_{\mathbf{2t}}\widetilde{\mathbf{F}}_{\mathbf{2t}})'\widetilde{\mathbf{V}}_{\mathbf{2t}}(\widetilde{\mathbf{A}}_{\mathbf{2t}} - \widetilde{\mathbf{B}}_{\mathbf{2t}}\widetilde{\mathbf{F}}_{\mathbf{2t}})) \tag{A-40}$$

Following Giordani and Söderlind (2004), the algorithm involves iterating until convergence ('backwards in time') on (A-19)-(A-40). It should be started with a symmetric positive definite $\mathbf{V}_{\mathbf{it+1}}$ and some $\mathbf{K}_{\mathbf{it+1}}$. If $\widetilde{\mathbf{F}}_{\mathbf{it}}$ and $\mathbf{K}_{\mathbf{it}}$ converge to constants $\widetilde{\mathbf{F}}_{\mathbf{i}}$ and $\mathbf{K}_{\mathbf{i}}$, the dynamics of the model are

$$\mathbf{x}_{\mathbf{1t+1}} = \mathbf{M}_{\mathbf{i}}\mathbf{x}_{\mathbf{1t}} + \mathbf{C}_{\mathbf{1}}\boldsymbol{\varepsilon}_{\mathbf{t+1}}, \text{ where } \mathbf{M}_{\mathbf{i}} = \mathbf{A}_{\mathbf{11}} + \mathbf{A}_{\mathbf{12}}\mathbf{K}_{\mathbf{i}} - \mathbf{B}_{\mathbf{1}}^{*}\widetilde{\mathbf{F}}_{\mathbf{i}}\ , \tag{A-41}$$

$$\begin{bmatrix} \mathbf{x}_{\mathbf{2t}} \\ \mathbf{u}_{\mathbf{it}}^{*} \end{bmatrix} = \mathbf{N}_{\mathbf{i}}\mathbf{x}_{\mathbf{1t}}, \text{ where } \mathbf{N}_{\mathbf{i}} = \begin{bmatrix} \mathbf{K}_{\mathbf{i}} \\ -\widetilde{\mathbf{F}}_{\mathbf{i}} \end{bmatrix} \text{ and } \mathbf{K}_{\mathbf{i}} = \mathbf{D}_{\mathbf{i}} - \mathbf{G}_{\mathbf{i}}\widetilde{\mathbf{F}}_{\mathbf{i}} \tag{A-42}$$

Appendix B

In this appendix we show the detailed procedure to obtain the detection error probability for our model. We follow the procedure shown in Hansen and Sargent (2003). We start by defining the reference model as R and the distorted model as D. Model R is represented by the state-space representation given by the system equation (37) in the text whereas model D is the distorted model. The latter differs from the former because the cost-push process represented by equation (22) now has the additive distortion ω_{t+1}. The additive noise is given by the vector ε_{t+1}. The likelihood of a sample for model *i* given that the data is generated by model *j* is denoted by L_{ij}, where $j \neq i$ and $i = \text{R, D}$. The likelihood ratio is defined as follows:

$$r_i \equiv \log \frac{L_{ii}}{L_{ij}} \tag{B-1}$$

The probability of making a mistake in the detection of a model given that the data was generated by model *i* is given by the following equation:

$$p_i = \Pr(mistake \mid i) = frec(r_i \leq 0) \tag{B-2}$$

The probability of making a mistake in the detection of a model is the average of the probability of making a mistake when the data was generated either by R or D:

$$p(\theta) = \frac{1}{2}(p_R + p_D) \tag{B-3}$$

In order to find $p(\theta)$ we need to obtain p_R and p_D. We first find p_R using the following five steps:

1. Generate a sample of T = 150 observations for the state variable in the reference model R. That is, we obtain the optimal trajectory for the state variables in the finite horizon model of T periods.
2. We use Giordani and Soderlind (2004) specification that assumes the distribution of the additive errors to be $N(\mathbf{0},\mathbf{I})$. In other words the residuals have an identity variance-covariance matrix. We obtain a random draw from this distribution for each simulation.
3. The new residuals are $\breve{\varepsilon}_{t+1} = \varepsilon_{t+1} + \omega_{t+1}$ (B-4)
4. The L_{RD} is calculated using the residuals of the R model minus the distortions: $\varepsilon_{t+1} = \breve{\varepsilon}_{t+1} - \omega_{t+1}$. The likelihood equation is the following:

$$\log L_{RD} = -\frac{1}{T}\sum_{t=0}^{T-1}\left\{\log\sqrt{2\pi} + \frac{1}{2}\left(\breve{\varepsilon}_{t+1} - \omega_{t+1}\right)'\left(\breve{\varepsilon}_{t+1} - \omega_{t+1}\right)\right\} \tag{B-5}$$

The distortions are generated using the feedback rule of ω_{t+1} obtained from the D model.

5. We obtain r_R and p_R for a total of 1,000 simulations for a sample of T = 150.

In order to obtain p_D we follow a similar procedure as in steps 1 to 5. However, in the first step the 150 observations of the state variable are generated using the distorted model D. In the second step the residuals of the distorted model is assumed to be $N(\mathbf{0},\mathbf{I})$. In the fourth step L_{DR} is obtained using $\breve{\varepsilon}_{t+1} = \varepsilon_{t+1} + \omega_{t+1}$ as follows:

$$\log L_{DR} = -\frac{1}{T}\sum_{t=0}^{T-1}\left\{\log\sqrt{2\pi} + \frac{1}{2}\left(\varepsilon_{t+1} + \omega_{t+1}\right)'\left(\varepsilon_{t+1} + \omega_{t+1}\right)\right\} \tag{B-6}$$

The distortions are generated from the sample of the step 1. Once L_{DR} is obtained we compute r_D and p_D for the 1,000 simulations and T = 150.

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