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Explaining Inflation with a Classical Dichotomy Model and Switching Monetary Regimes: Mexico 1932-2013*

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Abstract: This paper applies a novel approach to study the impact of different shocks on the price level. It uses a classical dichotomy model with monetary policy regime shifts at known dates. First, there was a regime dominated by money, afterwards a regime driven by the exchange rate and a third one with inflation targeting. The result is a CVAR with constant long-run parameters but regime-dependent adjustment coefficients. This overcomes the challenge of explaining, within a single theoretical framework, inflation dynamics in Mexico since the country abandoned the gold standard. The model encompasses known results, offers new insights and clarifies decades-old debates on key aspects of the inflationary process such as inertia, the role of money, the exchange rate pass-through and the impact profile of other variables. The model proposed here is very parsimonious, it does not require inflation lags nor dummy variables. It also displays a very good pseudo out-of-sample forecasting performance.

Keywords: Money Velocity, Exchange Rate, Inflation, PPP, Fiscal Deficit, Cointegration, Monetary Regimes, Unbalanced Regressions.

JEL Classification: C32, E41, E42, E52

Resumen: Este documento aplica un enfoque novedoso para estudiar el impacto de diferentes choques sobre el nivel de precios. Utiliza un modelo con dicotomía clásica y cambios sucesivos de régimen de política monetaria en fechas conocidas. Primero hubo un régimen dominado por dinero, después un régimen conducido por el tipo de cambio y un tercero con objetivos de inflación. El resultado es un CVAR con parámetros de largo plazo constantes pero coeficientes de ajuste que dependen del régimen. Esto franquea el reto de explicar, dentro de un mismo marco teórico, la dinámica inflacionaria en México desde que el país abandonó el patrón oro. El modelo abarca resultados conocidos, ofrece nuevos y clarifica algunos debates de décadas sobre aspectos clave del proceso inflacionario tales como la inercia, el papel del dinero, el traspaso del tipo de cambio y el perfil de impacto de otras variables. El modelo aquí propuesto es muy parsimonioso, no requiere rezagos de la inflación ni variables dicótomas. También muestra un desempeño muy bueno en pseudo pronósticos fuera de muestra.

Palabras Clave: Velocidad del Dinero, Tipo de Cambio, Inflación, PPP, Déficit Fiscal, Cointegración, Regímenes Monetarios, Regresiones Desbalanceadas

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1 Introduction

This paper analyzes the dynamics of inflation in Mexico from 1932, when the country abandoned the gold standard, to 2013 and can cover the years ahead as long as the current monetary regime remains unchanged. Its distinct characteristic is that it covers a very long span of time, which is rather unusual. It also proposes the use of a general theoretical and econometric framework that produce parsimony and forecasting efficiency.

It is based on the Lucas (1982) model modified to accommodate a small open economy subject to changing monetary regimes. It departs from the dominant paradigm in that it studies inflation from the angle of the classical dichotomy, i.e., the analytical separation of the real and nominal sectors of the economy. This is unusual in that such kind of models are often deemed inadequate for developing countries (e.g., Vegh, 2013, ch. 5). Classical dichotomy is regarded by New Keynesian theorists as valid only in the long run, but it is considered always true in pure real business cycles (RBC) models.

There are good reasons to look for alternative approaches to the prevalent ones not only for the case of Mexico but many other countries. Macroeconomics textbooks tend to present inflation as a well-understood phenomenon, with precisely known causes and effects. However, available evidence says otherwise and there is a growing skepticism, bluntly summarized by Tootell (2011): “The exact determinants of inflation remain somewhat of a mystery to everyone, including economists.” Coincidentally, Hall (2011) proposes to consider inflation as a nearly-exogenous variable in macroeconomic models for the United States. Uncertainty on the performance of current models has been acknowledged by many economists.

Such pessimism and bewilderment is well justified. For example, there was no consensus on how the unprecedented monetary policy stimulus that followed the world financial crisis would be reflected on inflation. There were four different predictions hotly debated by macroeconomists:\footnote{This debate took place mostly in blogs and newspapers articles although there were some academic papers about it (for example, Williamson (2013).}

1) Inflation would surge (Meltzer, Taylor and others); 2) there would be no inflation (prominently Krugman); 3) deflation would follow (Kocherlakota and Williamson) and; 4) Anything can happen to inflation, i.e., it becomes indeterminate (Yates). Nonetheless, the main Keynesian model for inflation, the Phillips curve (PC, from now on), failed to match the data (Tootell, 2011) and to
produce good forecasts during the Great Recession and long before and after that. There have been some responses to those doubts but not a generally accepted solution has been proposed (Yellen, 2017).

It should must be noticed that most current studies center on trying to solve the forecasting failures of inflation seen in the last years but little else has been said on what has happened to inflation in long periods. One needs a more general framework to study inflation under different conditions, including those when the central bank needs to change its more pressing objectives. This paper proposes one approach where different paradigms can cohabit and complement one another to identify which variables are the drivers of inflation at specific points of time. As asserted before, such factors might change and sometimes they do it sharply.

The main results of this paper are the following: Money was the only source of inflation in Mexico from 1932 to 1981. From 1983 to 2000, the exchange rate played that role and, from 2001 onwards, there has been an inflation target regime. The model here presented identifies the impact of monetary shocks through zero restrictions in the matrices of adjustment coefficients in the cointegrated VARs for both the quantitative equation of money (QEM) and purchasing power parity (PPP). Those restrictions change with the policy regime. These regime changes cannot be studied through well-known alternatives such as the Markov switching models of Hamilton (1989) and smooth transition models (Teräsvirta, 1994). This is because, the main parameters subject to change make the problem intractable.

It is proved that the regime changes did occur at two given dates using unbalanced regressions, which is another novelty of this paper. The model fits the data very well, does not require inflation lags nor dummy variables to account for outliers. It also provides a fresh look about the exchange rate pass-through and the role of money and inflationary inertia. Furthermore, it delivers good out-of-sample forecasts. As should be expected, the impact of nonsystematic factors of inflation onto the price level depend on if the central bank determines a price level, implicit in the first two regimes, or the inflation rate as in the third.

The rest of the paper is organized as follows. Section 2 contains a short literature review on related topics. Section 3 presents the data and analyzes the unit root properties of the series. Section 4 shows the theoretical model. Section 5 describes the three monetary regimes observed during the

\[^{2}\text{1982 was a transition year that can not be made part of any regime.}\]
sample. Section 6 discusses the econometric implications and empirical results. The last section offers the conclusions and final remarks.

2 Historical References and Literature Review

This article touches several branches of economic literature and, therefore, it provides only a few references of each topic: inflation models, models with switching regimes and related econometric issues.

2.1 Models for Inflation in Mexico and with Long Samples

After the use of fiat money became generalized since 1932, the Mexican economy faced several bursts of inflation and currency devaluations. However, from 1956 to 1971 the country kept the fiscal deficit under control. This allowed the central bank to focus in maintaining a fixed exchange rate parity.

Since 1972, inflation became a big problem because public spending was greatly expanded. Much of the increased spending was financed by foreign loans and credit from the central bank. This situation aggravated the current account problems that had began to develop. They led to the depletion of foreign reserves and to a speculative attack on the peso in 1976, that ended a two-decades period of fixed parity. Because of this, a first wave of research on inflation began in the seventies. The main antagonistic views were the monetarist school, which emphasized the role of money, and the so-called Keynesian-structuralist approach, which stressed the social struggle for the distribution of income and the disequilibrium among productive sectors where the prices were determined by costs and market power. For both schools, the effects of currency devaluations on inflation were important but they did not really show it convincingly.

The return to two-digit inflation after the 1995 economic crisis triggered a new wave of models. The main characteristic of these models was precisely the prominence of the exchange rate pass-through. The adoption of inflation targeting at the outset of the new millennium once again caused a new regime change, as implied by the framework of this article. In particular, the declining inflationary impact of the exchange rate after the adoption of these regimes was indicated almost as it was happening in Argentina, Brazil and Mexico by Schmidt-Hebbel and Werner (2002). More
recent work on Mexico (Capistran et al., 2011 and Cortés, 2013) corroborates such result. Those three papers correctly pointed out the change in the monetary regime as the reason for the vanishing exchange rate pass-through. This paper also proves such result and it provides a theoretical framework to explain that and other changes that occur to inflation dynamics when there is a regime change.

A general characteristic of almost all published inflation models for Mexico is the absence of out-of-sample forecasting exercises to validate them. Such exercises seem a good way to discriminate among competing models. As an example, in this article one of the models is able to pass many common statistical tests but it fails forecasting performance tests and, therefore, it must be discarded.

### 2.2 Models with Changing Monetary Policy Regime

Although there is a substantial literature on regime changes in monetary policy, the best known studies are those on the United States, particularly to elucidate the causes of the “Great Moderation.”\(^3\) For example, Sims and Zha (2006) used a Bayesian VAR with Markov switching regimes to find three changes in the monetary policy function at dates that more or less coincide with what “most observers believe monetary policy actually differed.” However, they conclude the estimated shifts are unlikely to explain the changes in US inflation in the 70s and 80s.

A common feature of the literature on changes in the monetary policy regime is the use of a New Keynesian framework. In that approach, the authors look for evidence of a change in the value of the parameters of the policy rule (for example, more weight to inflation deviations in one regime than in another). Another example of that approach is Fernández-Villaverde et al. (2010), where further references on the topic can be found. The case presented here follows a different approach.

### 2.3 Cointegration Models with Regime Switching

The literature on cointegrated VARs (CVARs) with explicit regime changes is scant because its complexity. Kurita and Nielsen (2009) show that if the parameter changes are restricted to those of the lagged terms in differences, the reduced-rank procedure (Johansen) to estimate the cointegra-

\(^3\)An exception is the literature of the declining exchange rate pass-through we discussed before.
tion relationships remains accurate. However, when the changes occur in the adjustment parameters such method is not valid because “[those changes] are reflected in the impact parameter of the common stochastic trends, thereby affecting the asymptotic distributions of cointegration rank tests.” Fortunately, in this paper one does not have to estimate any long-run parameters as they assumed to be known because they belong to two fundamental relationships (QEM and PPP). Nevertheless, these are proved to be valid cointegration relationships.

Massimiliano et al. (2002) suggest a two-step procedure to estimate CVARs with parameters subject to Markov switching, but they have to impose strong restrictions. In the first step, they estimate the long-run parameters. In the second step, they estimate the rest of the parameters through maximum likelihood. However, the complexity involved in the second step imposes limitations on which and how many of those parameters can be allowed to switch.

Although the system here analyzed can be restated as a Markov-switching regime model where no return to the old state is allowed once it is abandoned, its dimensionality and complexity is hard to handle. Thus the study identifies the dates of regime change through historical events. More related to this paper, Barassi et al. (2007) try several procedures to look for a change in the feedback adjustment parameters. However, the results are often inconclusive despite that they limit their analysis to bivariate systems.

3 Data and Descriptive Statistics

The data come mainly from the online sites of Banco de Mexico, Inegi, IMF and the Bureau of Labor Statistics. The variables are in logs. The domestic price level is represented by the time series of the Mexico City Whole Prices Index from 1932 to 2000, when its publication stopped. The rest of the series was completed with the Mexican headline CPI. When both series were available (1970 to 2000), their behavior was similar. The foreign price level \( p^{us} \) is the US CPI. The nominal exchange rate \( e \) is in pesos per dollar. The monetary aggregate \( m \) is currency (the total nominal value of bills and coins held by the public). The measure of economic activity is Mexican GDP.

Table 1 shows the augmented ADF tests for the variables described above and some combinations of them regarded as “equilibrium relationships.” There is also a summary of group unit root tests under the assumptions of both a common and individual trends. This is because the monetary
variables of a country are expected to have common trends. The general conclusion is that the individual variables are $I(1)$ in levels but $I(0)$ in differences. The “equilibrium relationships” are stationary in both levels and differences although there are some problems discussed below.

One of the combinations of variables is the real exchange rate ($rer = e + p^{us} - p$), another is the inverse of the velocity of money ($-v = m - y - p$) and the difference between inflationary money ($m - y$) and foreign prices in local currency ($e + p^{us}$). It should be noticed here that, as these combinations are stationary according to the tests, the variables that form them are cointegrated. So, the real exchange rate $rer$, the velocity of money $v$ and the difference between inflationary money and foreign prices in local currency ($m - y - e - p^{us}$) are to be interpreted as long-run equilibrium conditions.

There are some additional issues to address. First, for the relationships that include money (money velocity and inflationary money deflated by foreign prices) there is a problem at the end of the sample because from 2001 to 2012 there was a persistent process of remonetization. This caused a steady fall in money velocity. Because of this, the stationarity of such relationships from 1932 to 2000 we examined with individual tests and they still reject a unit root process only at 10%.

However, in the most powerful groups tests, the evidence of stationarity despite the remonetization process is clear. Furthermore, as shown later, the long-run behavior of $m - y$ and $e + p^{us}$ is very similar and that leaves little doubt on the strength of their relationship, which has a central role in the analysis carried out below. Second, there is also a test of such relationships for samples that will be identified later as regimes 1 and 2 and their validity is even clearer. For regime 3, the sample is too short to provide any meaningful estimates, especially because the unusual remonetization process that has taken place.

Third, there could be a legitimate concern on the right integration order. For example, the first difference of the Mexican price level can be regarded as stationary only at 5%. Moreover, the US price level cannot reject the hypothesis of a unit root even at 10%. The same happens with money velocity. Because of these results, to examine the robustness of the results the example of Juselius (2006) is followed. She estimates two models. In the first one, she considers that the series are $I(1)$ and in the second, she considers that the series are $I(2)$. When one estimates model for regime 1 considering that the variables are $I(2)$, the conclusions do not change.

6
4 The Classical Monetary Model for a Small Open Economy

The main characteristic of the classical monetary model is the analytical separation of the real and nominal sectors of the economy. A widely maintained view on that model is that, even though it is a necessary benchmark, it is a poor description of an economy except, perhaps, for the long run or a hyperinflation process. However, RBC models take the classical monetary model at face value, considering that “money is a veil.”

The following modified version of the simple Lucas (1982) model contains the necessary theoretical elements for the empirical analysis but any general equilibrium model for a monetary economy with classical dichotomy would work. Every period, a representative-agent consumes a bundle of two freely transportable and perishable goods, one local, $C_t$, and one imported, $C_t^*$. The purchases of goods require the use of local currency in the exact amounts $M_tC_t$ and $M_tC_t^*$. $Y_t$, is provided by fruit trees at a random rate and can be either sold for consumption or exported at the local price $P_t$. The imported good, has a price abroad of $P_t^*$. To buy it, $E_t$ units of local currency have to be traded for each unit of foreign currency at the foreign exchange market so the local price of of the imported good is $P_t^*E_t$.

There is a public sector that demands $G_t$ units of domestic output to pay for the expenses of the central bank, or some other public project. Total product $Y_t$ is therefore equal to $C_t + X_t + G_t$. Government consumption is paid with money issued by the central bank in an amount $\Delta M_t$, an inflation tax that is spent in the same period when it is collected. Here, the central bank sets its monetary policy by determining the growth path of the money supply but there are other possibilities. Each possibility defines a policy regime.

It is worthwhile to stop here to mention that the introduction of a tax does not destroy the classical dichotomy property because output is assumed to be generated exogenously. As Lucas (1982) states, the introduction of production (that is, making output endogenous) in a barter economy entails the same results as long as the one consumer device is kept. However, in a monetary economy the cash-in-advance constraint introduces a wedge between private and social welfare. In that case, if there are choices for leisure or investment, money is not strictly neutral because it will have real effects. However, such effects are known to be tiny and that is why something like a combination of

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4See for example Romer (2011) and Vegh (2013).
5For Mexico, an example of this is Betts and Kehoe (2001).
nominal and real rigidities must be introduced to brush off the classical dichotomy property. Thus, the results of this paper would be maintained in many models with a detailed production technology as long as they do not include nominal rigidities or any other device designed to proscribe classical dichotomy.

Dividends are paid in cash to the consumer/owner at the end of each period so they can be spent only in the next period. This causes that the only asset available to carry out her transactions in period \( t \) is the money stock she carries from period \( t - 1 \). Therefore, her budget constraint is:

\[ M_{t-1} = P_t C_t + E_t P^*_t C^*_t \]  

Money plays a role in this economy through an aggregate cash in advance constraint:

\[ M_{t-1} + \Delta M_t = P_t C_t + E_t P^*_t C^*_t + G_t \]  

If \( G_t \) is transferred to the consumer, then this is equivalent to having no government and the consumer receiving the monetary transfers \( \Delta M_t \), as in the original Lucas (1982) model. The introduction here of a government that keeps the inflation tax is only to provide some story to rationalize sharp rises of the inflation tax, something that has a role in our analysis, particularly for the seventies and eighties.

In the Lucas (1982) model of two currencies solved with a constant relative risk aversion (CRRA) utility function, the real exchange rate depends on the ratio of domestic and foreign outputs (Mark 2001). However, that solution assumes similar sizes of both countries. As the domestic economy we consider is small relative to the foreign one, we simply consider that relative PPP holds, i.e., the real exchange rate \( RER_t \) is a mean-reverting process:

\[ \frac{E_t P^*_t}{P_t} = RER_t \sim I(0) \]  

where \( RER \sim I(0) \) means that the real exchange rate peso/dollar is a stationary or mean reverting process. This might look just as a convenient assumption as for many currencies it is hard to prove that relative PPP holds. Taylor and Sarno (2001) conclude that the validity of such condition in the literature has been subject to shifts and that currently there is the consensus of just “some validity” in the long-run. However, in the Mexican case relative PPP property holds strongly
for the bilateral real exchange rate Mexican peso-U.S. dollar, as was shown in Table 1, where the stationarity of the real exchange rate holds even at the 1 percent level of significance.

In the Lucas model, there are no trade deficits because consumption of the foreign good by local residents is financed by the dividends from the foreign assets they own. A simpler alternative is to assume directly that the value of exports exactly matches the value of imports ($C_t^*$):

$$P_t X_t = E_t P_t^* C_t^*$$  \hspace{1cm} (4)

This is known as “financial autarky” and it is clearly unrealistic, but the trade balance plays no direct role in the dynamics of inflation other than its possible effects on the exchange rate. The consumer faces the dynamic problem of choosing the amounts of domestic and foreign goods that maximize her lifetime CRRA utility:

$$E_t \left[ \sum_{j=0}^{\infty} \phi^j \left( \frac{C_t^n C_t^*(1-\eta)}{1-\gamma} \right)^{1-\gamma} \right]$$  \hspace{1cm} (5)

subject to her budget constraint (1), the cash in advance constraint (2) and the balanced trade condition (4). The parameter $\eta$ is the share of consumption spending in the domestic good, $\phi$ is the discount factor and $\gamma$ is a positive number that, when equal to unity, makes that the utility function takes the logarithmic form.

The solution of this problem is very simple and requires that the consumer spends all of her monetary holdings $M_{t-1}$ in the local and foreign goods (after carrying out foreign exchange transactions). Thus, the demand functions for the goods are:

$$C_t = \eta (Y_t - G_t)$$  \hspace{1cm} \hspace{1cm} (6)

$$C_t^* = \frac{(1-\eta) P_t (Y_t - G_t)}{E_t P_t^*}$$  \hspace{1cm} \hspace{1cm} (7)

So, the consumer’s money holdings, $M_{t-1}$, plus the new money created to pay for government spending, $\Delta M_t$, must be equal to the nominal value of national output:

$$M_{t-1} + \Delta M_t = M_t = P_t Y_t$$  \hspace{1cm} (8)
which is precisely the quantitative equation of money (QEM) with unit velocity, as in Lucas (1982) model. This result is, of course, unrealistic but it can be considered to hold in the long run if the velocity of money is a stationary process. On this, Juselius (2006) states that “The stationarity of money velocity, implying common stochastic movements in money, prices, and income, is then consistent with the conventional monetarist assumption as stated by Friedman (1970) that inflation always and everywhere is a monetary problem.” (p. 29) She then adds that “This case, \((m_t - p_t - y_t^r) \sim I(0)\), has generally found little empirical support”. We are aware of that finding for a number of countries, but we find here that money velocity is indeed a stationary process in Mexico although in the long run the causality among its components has shifted with the monetary regime.

In Lucas (1982), the two national currencies of his model follow arbitrary stochastic processes (as do good endowments). Here, we consider three different assumptions on how the dynamics in the nominal variables is determined by the central bank. The first assumption is similar to that of Lucas’ original work in that monetary policy is carried out by currency injections. The second one considers that the central bank sets a target for the exchange rate. The third one assumes that monetary policy is conducted through an inflation target. Each of these situations entails different dynamic correlations for the inflation rate and the other nominal variables.

In the model we are considering, the only possible reason to increase the price level is to apply an inflation tax and any concerns about real economic activity are ignored. Lucas (1982) states that a richer specification would include “arbitrary correlations” with the endowment process to avoid the neutrality of money, a property he explicitly rejected. However, his most basic model is followed here and possible improvements could be added if serious shortcomings were found.

5 Monetary Policy Regimes

Before developing each case separately and present its implications for the data, a brief summary might be useful. First, when currency is the policy tool the price level is determined within the QEM and the exchange rate only moves to reestablish the PPP condition. Second, when the central bank determines the path of the nominal exchange rate the value of the price level is determined by

\[6\text{This is Juselius’ original notation. The variables } m_t \text{ and } p_t \text{ are the money stock and the price level, as in the notation of this paper, and } y_t^r \text{ is real income, her chosen scale variable, that corresponds to our } y_t.\]
the PPP condition. In such case, currency adjusts passively to reestablish the QEM. Third, when the central bank sets directly an inflation target, then neither money nor the exchange rate have a systematic effect on inflation, which is driven systematically only by the target.

5.1 Money as the Intermediate Policy Target

In Lucas (1982), currency is the driving force for the nominal sector. Here, to better represent the path of money as determined by a central bank, increases in the money supply depend on a moving price level target. This is different from the more modern idea of a fixed price level target. Although there is not a particular reason in estimating the exact form of the monetary rule because there is no trade off between economic activity and inflation in the classical model, as happens in a New Keynesian framework, it might be useful to show an approximation. The monetary rule in this episode could be represented by the following equation:

\[ m_t^s = y_t + p_o^t \]  

(9)

Thus, the supply of currency, \( m_t^s \), depends on GDP, \( y_t \), plus the yearly objective for the price level \( p_o^t \). The sequence of values for the annual price level target depend on the decisions taken by the central bank.

The central bank preferred a fixed exchange rate from 1932 to 1981. However, there were several adjustments to the parity after money growth was used to finance the public deficit. For this reason, the monetary regime was in fact determined by the path of currency despite the preference for a fixed exchange rate.

There is a simple case where the path of the price level target was easy to determine. This happened when there were no devaluations for a long period and the fiscal deficit was kept under control. In these conditions, the policy rule consisted in the creation of enough money (\( m_t^s \)) to maintain the proportionality with output and the foreign price level (in logs):

\[ m_t^s = y_t + p_{t^{us}} \]  

(10)

Public deficit data for the whole sample are not easily available, so one cannot see how accurate this rule is for the complete period. Nonetheless, one can check that during the time span when
the public deficit-to-GDP ratio was low, money growth was essentially determined only by output growth and foreign inflation. Indeed, from 1956 to 1971, currency growth matches very closely the trajectory of output growth plus foreign inflation, especially since 1960 (in 1959 actually there is a wide divergence), as shown in Figure 1. Moreover, for this subperiod the average inflation rate in Mexico 2.6 percent was about the same as that in the United States.

In 1972, fiscal discipline was relaxed and the budget deficit began to surge, financed in part by injections of currency, as Figure 2 shows. Although the period from 1983 to 1993 does not fall into the first regime, the relationship between money growth and the public deficit was still visible. The relationship breaks down in 1994, coinciding with the beginning of legal autonomy for the central bank.

The change in the public budget stance was reflected in the behavior of the public, which became used to expect a higher inflation rate than in the past. This fact was reflected in a shift upwards in the short-run parameter of the inflation model for this period, as shown below.

5.2 The Exchange Rate as the Policy Instrument

The outbreak of the debt crisis in 1982 forced the Mexican government to obtain an emergency loan from the International Monetary Fund. This imposed the country a set of conditions established in an agreement signed in October 1982. Among the most important aspects in it, there were two that changed the behavior of inflation. The first was a commitment by the Mexican government to limit the accumulation of domestic credit at the central bank. The second was the correction of the external deficit through devaluations. These two aspects provoked a change in the monetary policy regime, forcing the central bank to abandon currency as its policy instrument and adopting as such the exchange rate. This change was perceived in the behavior of inflation by several authors, among them Pérez-López (1996) and Garcés Díaz (1999). However, those papers did not elaborate on the issue and did not use it to improve their models, which eventually became obsolete with a new regime change.

The exchange rate became the leading nominal variable and the inflation rate passed to be determined through the PPP condition and not by the money supply in the QEM. Although money was not longer the policy instrument, the QEM was still valid despite the fact that money was not longer useful as a predictor for inflation or the rate of depreciation. Observe that this regime, as the
previous one, implies price level determination (not price level target, which is something different). This is determined by the logarithm of the exchange rate target plus the log of the foreign price level.

It is in this period when the impact of the exchange rate on inflation became a persistent phenomenon in Mexico. The pass-through of exchange rate on prices actually survived even during the initial years of the flexible exchange rate regime. This might look strange at first because the exchange rate was not longer predetermined by the central bank.

The reason for the continuation of the exchange rate pass-through during part of the period when the parity has been determined by the market is not hard to grasp. The central bank was forced to adopt a flexible exchange rate by the 1994 crisis, which depleted its reserves of foreign reserves. The public did not deemed the new regime as permanent in part because the central bank was accumulating foreign reserves and this was interpreted as an initial step to go back to a predetermined exchange rate regime. Because of this, inflation still followed the movements of the exchange rate until 2000.

5.3 Inflation Targeting

After the 1995 devaluation, there was a transition period during which the exchange rate floated and it was observed that the volatility of that variable was much lower than it was projected. This gave room to the adoption of a full-fledged inflation targeting framework since 2001. The public became used to the idea and the new monetary approach was completed with the adoption of a reference interest rate as the policy instrument in 2004. With this, both money and the exchange rate ceased to be systematic causes of inflation.

In the classical monetary model, which is the simplified approach we consider here, the mechanism through which monetary policy works runs through inflation expectations. When the central bank sets the policy interest rate for a long enough period, it is also setting its inflation target by means of the Fisher equation. However, this way to conduct monetary policy is not exempt of the problems of interest rate regimes, specially indeterminacy, as shown by Benhabib et al. (2001). To analyze that issue, they consider a common policy rule where the interest rate policy is set as a response to the inflation rate:
\[ R_t = R_t(\Delta p_t) \] (11)

where \( R_t \) is the policy rate as a function of inflation \( \Delta p_t \). This function, they argue, covers most practical cases regardless if the model has flexible or sticky prices. Furthermore, assuming that the Fisher equation holds:

\[ R_t = R_t^{real} + \Delta p_t \] (12)

where \( R_t^{real} \) is the real interest rate, they show that a steady state with high inflation for the policy rate exists where \( R_t'(\Delta p_t^{sh}) > 1 \) and monetary policy is active. \( \Delta p_t^{sh} \) is the steady state value for high inflation. However, the same framework implies another steady state where the reverse, \( R_t'(\Delta p_t^{sl}) < 1 \), is true. \( \Delta p_t^{sl} \) is the steady state value for low inflation. In the second steady state, inflation is below the intended target and monetary policy is passive. They also find, within a flex price model, that in general the steady state with active monetary policy is unstable while the passive monetary policy state is stable. However, if the analysis is restricted at a small neighborhood around the active policy state, this is the only steady state under perfect foresight.

The key to rule out undesirable equilibria is central bank credibility. To see this, assume that the central bank chooses the optimal path for the price level as the one that minimizes its loss function. This function usually takes a quadratic form in its arguments, typically output gap and the deviations of inflation from the steady state value. The solution for the case of flexible prices is very simple because the output gap term vanishes and the optimal path for the price level is the one that the central bank announces. This occurs because in that case, the central bank does not face the problem of dynamic inconsistence, something that can occur in sticky prices models. In such type of models the central bank can have a reason to deviate. Those models require additional assumptions besides credibility to achieve determinacy as in, for example, Illing and Siemsen (2014).

The achievement of a credible inflation targeting regime brings an interesting consequence for the behavior of inflation, which implications have not been completely appreciated. In this regime, the deviations of inflation from the central bank’s target become hard to predict as inflation typically behaves like noise around a constant. Then, inflation turns into a “near-exogenous” process, in the sense of Hall (2011). He suggests that such situation arises in economies when inflation has been
low for an extended period. However, in the case of Mexico and other countries that property was observed almost immediately after inflation targeting was implemented.

The apparently esoteric issue of indeterminacy became a very practical one after the Federal Reserve introduced unconventional policy measures to face the economic downturn after the 2008 crisis. There was a wide disagreement among economist about how those measures would impact inflation, as discussed in the introduction. Eventually, it was clear that the discussion involved indeterminacy and that the different postures were in fact considerations of which equilibrium the economy was heading to.

Finally, it is interesting to mention that among the proposals to get out of indeterminacy is that the central bank should target the price level instead of the inflation rate.7 In that case, the optimal announced path for prices becomes determined because the central bank is committed ex ante to achieve it. That was the case in regimes 1 and 2 discussed before, where the central bank was committed to a price level determined by either the money supply or the exchange rate. Nonetheless, the empirical relevance of the problem of indeterminacy within an inflation target regime does not appear to be so serious as to cause modern central banks to make the switch to another framework with a price level target.

The model presented here could be an extreme simplification and the exploration of more complete models is needed. Classical dichotomy might be a convenient assumption to facilitate the econometric work but by no means the results that follow automatically disqualify richer models. They could provide deeper insights as long as they pass the out-of-sample forecasting bar set by simpler models.

6 Econometric Implications and Empirical Results

This section analyzes the behavior of inflation in Mexico through conditional single-equation models obtained from a cointegrated VAR (CVAR) with restrictions in the parameters implied by the general equilibrium model with monetary regime changes presented before. This yields interesting and little studied econometric properties validated by Mexican data.

7See, for example, Dittmar and Gavin (2004) or Ambler and Lam (2011).
6.1 A Simple Hypothetical Example and Preliminary Results

It might be useful to start with a hypothetical case. Assume a simple classical monetary economy where the monetary policy from \( t = t_0 \) through \( t = t_1 \) is set through a money supply rule. In this situation, the inflation equation is simply:

\[
\Delta p_t = \beta_{pm} \Delta m_t + \epsilon_{pm}^{m}
\]  

(13)

where \( \epsilon_{pm}^{m} \) is a white noise shock. The index “p” indicates that the respective variable or parameter belongs to the inflation equation and the index “m” that in that regime money is the policy instrument. In a regression, \( \beta_{pm} \) should be close to 1. Now, suppose there is a monetary policy change and from \( t = t_1 + 1 \) through \( t = t_2 \) the money supply is replaced as the policy instrument by the exchange rate. Now, the inflation process is given by the following equation:

\[
\Delta p_t = \beta_{pe} \Delta (e_t + p_t^*) + \epsilon_{pe}^{e}
\]  

(14)

where \( \epsilon_{pe}^{e} \) is another white noise shock. In a typical regression, \( \beta_{pe} \) should be close to 1 as well. The index “e” indicates that in that regime the exchange rate is the policy instrument. Suppose now that a regression is run under the belief that both money and the exchange rate affect the inflation rate from \( t = t_0 \) through \( t = t_2 \). Then, a possible empirical model would be the following:

\[
\Delta p_t = \beta_{pm} \Delta m_t + \beta_{pe} \Delta (e_t + p_t^*) + \epsilon_{pme}^{m}
\]  

(15)

where the double index \( me \) means that both money and the exchange rate enter into the inflation model. It is clear that the estimates for the whole sample would not correspond to the parameters of any model (two pairs are needed instead of just a pair of parameters). A more relevant question is that if the model (15) were applied to each subsample, the estimated parameters would still be unbiased. The answer is negative because there is a problem of endogeneity.

In the first subsample, by assumption, money is exogenous and the exchange rate is endogenous while in the second sample the opposite happens. In a structural VAR, the problem would be reflected in the impulse-response functions. For example, suppose we analyze the data with a VAR and use the Cholesky decomposition as the identification scheme as we do below. For the first regime, the equation for money would go first and for the second regime, the equation for the
exchange rate would take that place. If the order is changed in either regime or the VAR is run for
the whole sample, there would be a violation to the weak exogeneity properties of the system. This
issue was pointed out by Hendry and Mizon (2000), among others.

This issue is crucial in this case, as it shows the analysis of the causality relations among the
price level, money and the exchange rate. These relations are explored in bivariate VARs in the log
levels of those variables applied to each regime.

As these series are nonstationary, the usual Wald test is not valid. Instead, a modification
proposed by Toda and Yamamoto (1995) yields more correct results. This modification simply
adds one lag (in this case, 1 is the maximum order of integration found in the series) to the optimal
number obtained by an information criteria. We use Schwartz’s criteria but others could work as
well as the results are robust to the number of lags. Next, the significance of the excluded variables
has to be tested considering only the optimal number of lags. Table 2 contains the results.

The first point to notice is that in the regime 1 and only then (1932-1981), inflationary money
\((m - y)_t\) causes both the price level \(p_t\) and the exchange rate \(e_t\) and it is not caused by any of them.
In regime 2 and only then, the exchange rate causes both the price level and money and it is not
caused by them. In regime 3, none of the variables causes any of the others.

The analysis of inflation must take into consideration the regime. In impulse-response func-
tions, the identification of shocks must have as the primary innovation that from the leading vari-
able in the regime. Figure 3 shows the point. In them, the impulse responses of bivariate VARs
involving the price level, money and the exchange rate take into consideration the causality tests.
The identification scheme in based on the Cholesky decomposition. In regime 1, money is the first
variable. In regime 2, the first is the exchange rate. In regime 3, it is the price level.

The top pair of graphs of Figure 3 show that money has an effect on the price level and it is not
affected by it. The pair of graphs in the middle, show that the exchange rate affects the price level
but it is not impacted by it. In the pair of graphs at the bottom, the price level provokes a significant
response in the exchange rate but it does not respond to it. This relationship actually depends only
of a few observations when there was a temporary relationship between the exchange rate and the
price level due to the financial crisis, as will be shown later.

In the next sections, there is a more detailed analysis of the behavior of inflation in Mexico
with more detailed techniques. As the data contain unit roots and form cointegrating relationships
(the QEM and PPP), it is better to use a cointegrated VAR (CVAR). The problems with the causal-
ity (weak exogenity) properties remain in the case of the CVAR and give room to little explored
situations when there are regime changes, as discussed below.

6.2 Regime Changes in the Adjustment Parameters in a Cointegrated VAR

A CVAR is a common way to take a theoretical model to the data but this study has several features
that differ from other applications. In particular, the analysis ends up developing error correction
models for each regime instead of a whole system estimated by maximum likelihood. As explained
below, this is dictated by both necessity and convenience.

The beginning is a VAR that represents the dynamics of the variables in the theoretical model.
Due to the annual periodicity of the data, the lag structure of that VAR turns out to be simpler than
that of a model with higher frequency points. Also, because all the variables in the model are ob-
servable, we can restrict ourselves to a VAR(1) formed with I(1) variables that form a cointegration
system, that is, a CVAR.

However, we have considered that the dynamics of the nominal variables can be determined
by the central bank by different means and this makes a regular CVAR inappropriate if we can
study the whole sample. To avoid this, it is common to split the sample and work within only one
regime\(^8\) because Johansen’s reduced rank method fails when the adjustment parameters change, as
discussed in Kurita and Nielsen (2009). The problem with concentrating all attention in just one
regime is to overlook what happened before and, more importantly, if the model is really adequate.
Thus, instead of embedding the variables of interest in a CVAR with constant parameters, we use
one where some, but not all, of the parameters change in a defined way:

\[
\Delta Y_t = \alpha(s_t)\beta'Y_{t-1} + \Phi(s_t)\Delta Y_{t-1} + u_t
\]

where \(Y_t\) is a vector of \(I(1)\) variables from the theoretical model, \(\alpha(s_t)\) is the matrix of adjust-
ment coefficients and \(\beta\) is the matrix of cointegration parameters. Both matrices are of rank \(r\), the
number of cointegrating vectors. The vector \(u_t\), to be later explored, can be interpreted as a linear
function of \(I(0)\) variables and white noise.

The matrix of the feedback coefficients $\alpha(s_t)$ and, possibly, the coefficients for the autoregressive terms $\Phi(s_t)$ depend on the state of nature generated by the monetary policy regime. The matrix for the long-run relationships $\beta$ is assumed to be constant for two reasons. First, if all the coefficients were allowed to change, there would be an identification problem, as discussed by Barassi et al. (2005). Second, in the theoretical model, the parameters of the QEM and PPP are structural, so they should not change.

There is a very important aspect related to the expected changes in the $\alpha(s_t)$ matrix. The representation of the model with changes in the policy instrument becomes messy because the price level, the exchange rate and currency are determined differently in each state. However, the theoretical model by setting the direction of causality in each regime in a unique manner, allows us to separate the VAR for the whole system in two parts. One of them corresponds to the QEM and the another to PPP.

We will use these two subsystems to discuss the expected changes in the feedback coefficients. There is not an established procedure to test such changes so we propose a strategy that makes the analysis tractable. This strategy requires the use of error correction models instead of the whole VAR. The reason is that the availability of critical values for the tests we carry, are not existent within the VAR framework.

In general, the analysis within a CVAR is different than the one carried out through an error correction model except in one case that is the one we face here. As the system here analyzed has only one cointegration relationship and one variable that is not weakly exogenous, then the analysis within a conditional single equation framework does not lose information with respect the one using the full system.

Within the conditional error correction framework all changes that occur in the matrices of adjustment parameters can be tested. As in each case there are a lot of steps, we concentrate in the parameters that involve the equation for inflation. However, it must emphasized that similar equations with the corresponding changes in the parameters, described below, could be obtained

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9“Without loss of generality, [a cointegrated] VAR ... can be factorized into a pair of conditional and marginal models. If the marginal variables are weakly exogenous for the cointegrating vectors $\beta$, then inference about cointegration using the conditional model alone can be made without loss of information relative to inference using the full system (the VAR);”, Ericsson and MacKinnon (2002), p. 288.
for the exchange rate and the monetary aggregate.

The regime changes in the inflation equation do exist in the Mexican data, as the tests reveal. It must be remembered that, because output growth is not affected by monetary policy in a classical monetary model, the policy instrument can be regarded as exogenous. To simplify the exposition, the affirmation that a coefficient “is (statistically) significant” will mean that the corresponding variable is not weakly exogenous.\(^10\)

In the first regime, where money was the policy instrument, the corresponding adjustment parameter for the money equation must be zero while that for the price level must be significant. In the same regime, in the VAR corresponding to PPP, as the price level is determined by the money supply, the exchange rate must be a passive variable. Therefore, in the VAR for PPP, the adjustment parameter for the price level must be zero and the one for the exchange rate must be significant.

In the second regime, the exchange rate is the policy instrument and, therefore, it becomes exogenous. Thus, its adjustment coefficient within the PPP system becomes zero. The price level in that system becomes endogenous, determined by the exchange rate, and its adjustment coefficient becomes significant. Currency, as a consequence of this, becomes an endogenous variable determined by the price level within the QEM (and indirectly by the exchange rate).

In the third regime the central bank chooses an inflation target and adopts a short term interest rate as its policy instrument. In that case, neither the exchange rate nor currency have a systematic effect on the inflation rate. Instead, the most important factor becomes the inflation target itself, as long as the central bank enjoys the public’s credibility. In this situation, the adjustment parameters for the price level both in the QEM and PPP systems become zero. Within the PPP system, if the exchange rate is floating then it cannot be forecast, so its adjustment parameter must be zero as well. As there must be at least one adjustment parameter that is negative for PPP to hold and the foreign price level cannot be the adjusting variable, then the only remaining possibility is the real exchange rate itself.

6.3 Long-run Relationships

As mentioned before, the changes in the feedback parameters make inadequate the reduced-rank method to obtain cointegration coefficients. However, those coefficients are known from theory\(^10\) The sign of the adjustment parameter depends on how the long-run error is written.
so it is only necessary to show those relationships, QEM and (relative) PPP, are indeed long-run equilibrium relationships. Equivalently, money velocity $v_t$ and the real exchange rate $rer_t$ must be both $I(0)$ variables:

$$m_t - y_t - p_t = -v_t \sim I(0)$$  \hspace{1cm} (17)

$$e_t + p_t^{us} - p_t = rer_t \sim I(0)$$  \hspace{1cm} (18)

Relation (17) is the QEM, which takes the price level, $p_t$, currency, $m_t$, and output $y_t$, as determinants of the money velocity $v_t$. Money is currency (bills and coins held by the public) so it can be regarded as a true policy instrument, which is not necessarily the case of broader aggregates.\(^{11}\) Relation (18) is the standard definition of the real exchange rate, $rer_t$, where $e$ is the nominal exchange rate pesos per dollar and $p_t^{us}$ is the US price level.

The residuals $v$ and $rer$ have to be stationary processes for equations (17) and (18) to be regarded as equilibrium conditions and not as tautologies. This was proved in the unit root tests of Table 1. It must be remembered that, from section 6.2, Johansen’s reduced rank method is only valid if the adjustment parameters are constant, but they are not according to the theoretical model. That still could be applied within each regime but this is redundant with the conditional error correction method developed later because there is only one weakly exogenous variable and only one cointegration relationship (see Ericsson and MacKinnon, 2002) and it would not be possible to test for regime changes.

A first approximation of the switching dynamics can be obtained by looking at Figure 4. A movement upwards represents an increase in money velocity and a depreciation of the real exchange rate. The vertical lines delimitate the policy regimes. Figure 4 shows that during the first regime (1932-1982), money velocity leads the real exchange rate by one or two years. In the second regime, the situation turns around and the real exchange rate becomes the leading variable. Figure 5 shows the cross-correlations of money velocity and the real exchange rate observations from $t-3$ to $t+3$, corroborating the observations from the previous graphic.

First, for the whole sample all crossed correlations of money velocity with the shown lags and leads of the real exchange rate are positive and significant. This happens because in the first regime

\(^{11}\)For example, the Fed greatly expanded the monetary base without having much of an impact on M2 or inflation.
the causality runs from money to the exchange rate and the opposite happens in the second regime. This result is true in one sense (one preceded the other at some point) but false in another because it is not considering the regime changes we described before and shown in Figure 4. So, knowing that for the whole sample both money and the exchange rate lead or lag one another might not be very useful, specially if in regime 3 none leads or lags the other.

Now, if the calculations are applied to each regime separately, the results change. The new results are similar to the ones obtained with the causality tests. For the first regime, none of the lags or the contemporary value of the real exchange rate is correlated with money velocity. However, the value of money velocity is positively correlated with the 3 leads of the real exchange rate, indicating that causality runs from money to the exchange rate.

In the second regime, the first lag and the contemporary value of the real exchange rate are positively correlated with money velocity, suggesting that the causality runs from the exchange rate to money.

Notice that in the third regime (2001-2013) both variables are contemporaneously negatively correlated. They diverge from one another because money velocity is steadily falling and the real exchange rate had a sustained depreciation. The negative correlation might lessen or disappear when more years pass, but it is important to comment on the steady decline of money velocity since 2001.

This phenomenon seems to obey to a combination of factors. First, during this third regime, the central bank adopted a full-fledged inflation targeting regime, which eliminated both currency and the exchange rate as systematic causes of inflation. This by itself should allow greater separations of velocity of money and the real exchange rate. The second cause could be related to the increase of the underground economy (informal commerce and extralegal activities), which requires more cash. The third one is the commitment of the central bank to keeping a low inflation rate. When inflation is low, the opportunity cost of holding money is reduced so the public increases its demand for cash. These remonetization process occurred in general, but it has its greater impact in the informal sector, where transactions tend to use lots of cash. This remonetization process cannot

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For the United States, Sprenkle (1993) mentions the underground economy as one of three possible holders of about 84 percent of currency that cannot be explained by regular reasons. The other two suspects being foreigners and children under 18 not included in a Fed survey from which the estimates were made.
be a cause of inflation because it is not imposed on the public, as in the first regime when it was
“helicopter money”, but it responds to an increase of money demand not money supply. If money
velocity keeps falling for several more years, the long-run relationships in which currency is in-
volved will break down completely. However, in the past there have been even larger deviations
that were eventually eliminated (see 4 at the beginning of the first regime).

Equations (17) and (18) hold independently, as shown in Table 1, but it is useful to test their
validity jointly. This result will be very useful to prove that there were regime changes at given
dates. First, eliminate the domestic price level from both relationships. Figure 6 shows the time
path of inflationary money, $m - y$ and foreign prices in local currency $e + p^*$:

The relationship between inflation money and foreign prices in local currency is clear in Figure
6. Thus, from the velocity equation and the real exchange rate by eliminating the price level the
following expression can be obtained:

$$e_t + p^u_t = m_t - y_t + z_{t}^{e1}$$

(19)

where the residual $z_{t}^{e1}$ is stationary if this condition is also a long-run equilibrium. In Table 1,
the unit root test proves this is the case. This equation is one version of the monetary model of
exchange rate determination.

There is another alternative to test such model that can work under the circumstances at hand.
This consists in running a regression for inflation where those terms appear at the same time but
without using the lagged price level. That is the alternative here emphasized because with such pro-
cedure, it will be possible for the inflation equation to prove the occurrence of the regime switching
process described before.

6.4 Testing for Regime Switching in an Unbalanced Regression for Inflation

As a first step, the result that the combination of the variables $(m - y)_t$ (inflationary money) and
$(e + p^u)_t$ (foreign prices in local currency) form a stationary combination is used to formulate the
following model for inflation.

$$\Delta p_t = \beta_m (m - y)_{t-1} + \beta_e (e + p^u)_{t-1} + \phi_m \Delta m_t + \phi_e \Delta (e + p^u)_{t-1}$$

(20)
The left-hand side variable has no trend but those inside the parentheses do, so this is an unbalanced equation where, in general, neither regular statistics nor cointegration statistics can be used to carry out inference.\footnote{It is wrong to think that unbalanced regressions are not used in empirical work. Consider as examples the ADF regressions and the unconstrained conditional error correction model. Both of them, besides the ones of this section, are used in this paper.} However, as there are two I(1) regressors the stationarity of the error term is assured (Pagan and Wickens, 1989 and Baffes, 1997). This occurs in either of the following two cases: 1) When the regression is “incorrect”, in the sense that the nonstationary variables have no equilibrium relationship between themselves, the coefficients for the trending variables are zero and, therefore, the error term has the same stationarity property as the dependent variable; 2) When the equation is correct, in the sense that the nonstationary variables have a long-run equilibrium relationship, the coefficients for the trend variables are different from zero and they cointegrate between themselves.

As known from the unit root tests (the third case in the section of long-run relationships of Table 1), this is the second case, thus the coefficients $\beta$s should be different from zero. Notice that this is not an explicit autoregressive distributed lag model (ARDL), as those used to test for cointegration, because the one-period lagged logarithm of the price level is not on the right hand side. However, it can be turned into one by using the PPP or the QEMs, as done below.

All the models include the lagged levels (i.e. $x_{t-1}$), of currency and foreign prices. However, the contemporary changes of these variables ($\Delta x_t$) are included or not depending on its status as weakly endogenous or exogenous variables. Thus, the model for the whole sample includes the contemporary changes of both variables. For the first regime, only currency growth is included. For the second regime only the change of foreign prices are included. For the third regime neither variation is included but this makes no difference as they are no significant anyway. All estimated coefficients and tests statistics are in Table 3.

First note that in all equations, the coefficients for lagged inflationary money $(m - y)_{t-1}$ and foreign prices $(e + p_{us})_{t-1}$ are nearly identical in absolute value but with the opposite sign. Thus, by assuming they have the same absolute value one can factorize them and obtain (19), which is a confirmation that it is a valid long-run equilibrium relationship.

The second point to note is that for the whole sample, those lagged variables have small coeffi-
cients (0.07 and -0.07) and small t values, suggesting a “wrong” regression because those variables would not be cointegrated but, as known from Table 1, they are. It must be kept in mind that the distribution of these t statistics is not normal because the variables are nonstationary. As the asymptotic distribution for these statistics depends on the variables involved (Pagan and Wickens, 1989), there are no standardized tables to use in evaluating the significance. However, there is a detour to solve the problem.

Notice that from the definition of real exchange rate one can substitute \((e + p^{us})_{t-1}\) for \(p_{t-1} + rer_{t-1}\) in the regression of the first column of Table 3:

\[
\Delta p_t = 1.25 + 0.07(m - y)_{t-1} - 0.07(e + p^{us})_{t-1} + 0.51\Delta m_t + 0.34\Delta e_t \\
+ 0.19\Delta e_{t-1} + \hat{u}_t \\
= 1.25 + 0.07(m - y)_{t-1} - 0.07p_{t-1} + 0.51\Delta m_t + 0.34\Delta e_t \\
+ 0.19\Delta e_{t-1} + (\hat{u}_t + 0.07rer_{t-1})
\]

In the second equality of equation (21), \((e + p^{us})_{t-1}\) was substituted for \(p_{t-1}\). This requires that the term \(0.07rer_{t-1}\) be added to the original estimated error term \(u_t\) to obtain a new error term \((\hat{u}_t + 0.07rer_{t-1})\). Now, with these changes, one obtains an unconstrained error correction model where regular standard statistical inference can be applied.

Although this single-equation method to test for cointegration predates that of Johansen’s reduced rank procedure, it has lost some popularity. As the single-equation method is the main statistical procedure here to test for regime changes, a short explanation on how it works becomes useful.

This method requires there is only one cointegration relationship and only one weakly endogenous variable otherwise it will not work. This is one of the reasons for its decreasing popularity. One has to run a regression for the first difference of the weakly endogenous variable against the lagged levels of the cointegrating variables, i.e., an unbalanced regression. The test consists in comparing the t-statistic of the lagged weakly exogenous against the nonstandard critical values provided by Ericsson and MacKinnon (2002) tables. In that paper, there is a more complete expla-

\[14\] It would be incorrect to substitute \((m - y)_{t-1}\) for \(p_{t-1}\) because the coefficient for the lagged price level must negative in order to have a valid error correction model.
nation, some examples and a comparison with the Johansen and Engle-Granger methods.

Notice that the coefficient for the lagged price level inherits the t-value of $-2.45$. According to the Ericsson-MacKinnon (2002) Table 3 (for five regressors and a constant term), this is well below the critical value of even 10 percent of significance (-3.66). So, this confirms that the lagged nonstationary variables are not significant in this equation despite being cointegrated between themselves.\(^\text{15}\)

This result occurs because the sample includes three different regimes that have different matrices of adjustment parameters, as was discussed before. However, the changes of money and foreign prices are highly significant so money and the exchange rate seem to be causing inflation during the whole sample despite their well-known lack of predictive power since 2001. This is the result of not considering endogeneity problems, as was discussed in section 6.1. In fact such problems can be seen directly by noticing that the new error term, that includes the lagged real exchange rate, is correlated with the contemporary variation of the nominal exchange rate.

The third aspect to note is that for the regimes 1 and 2, the signs for the trend variables are inverted: In regime 1, the coefficient for $\left(m - y\right)_{t-1}$ is positive and that for $\left(e + p^{us}\right)_{t-1}$ is negative. For regime 2 the opposite happens.

This is the result of a regime change. To see this, one can again substitute variables as in equation (21). For the first regime, one can use the PPP condition to substitute the lagged foreign price level $(e + p^{us})_{t-1}$ for its long-run equivalent (up to a stationary deviation), the lagged price level $p_{t-1}$. Thus, the unbalanced regression for the first regime becomes equivalent to an unrestricted error correction model.

Notice that the error term of the transformed regression now would contain a term proportional to the lagged real exchange rate, as in equation (21). However, in this case the new error term is still orthogonal to the regressors because during the first regime the lags of the real exchange rate do not impact contemporary values of money velocity, as shown in the second panel of Figure 5. Observe that the t-statistic of -6.11 is far more negative than the critical value of 1 percent of significance of Table 3 for for three regressors (-4.09) of Ericsson and MacKinnon (2002), confirming the cointegration property for this relationship.

For the second regime, the QEM is used to substitute lagged inflation money $(m - y)_{t-1}$ for

\(^{15}\)In the table, they should have the initials n.s. (nonsignificant) but they are left out to make the point.
the lagged price level \( p_{t-1} \). Thus, one then obtains another conditional error correction inflation model for the second regime. Because in the second regime contemporary and lagged money velocity is uncorrelated with the real exchange rate (third panel of Figure 5), the new error term is still orthogonal to the regressors. One had to add the second lag of the inflation rate in order to eliminate a second order autocorrelation in the regression errors. With this, the t-statistic coefficient can be used for \((mybm - y)_{t-1}\) to assess the validity of the regression. This estimated value is -5.66, which easily exceeds the critical value for the one percent significance in the Ericsson-MacKinnon Table 3 with a constant term and four regressors, -4.36.

For the third regime neither money nor the exchange rate are systematic causes of inflation so they become statistically nonsignificant in the regression, except for the contemporary exchange rate depreciation, which has a small coefficient that is barely significant. Finally, as a final check on the validity of the regressions, the ADF statistic for the residuals of each equation is included. In all cases, the presence of a unit root in the residual is strongly rejected.

The results for the modified unbalanced regressions can be summarized as follows: 1) in the first regime the adjustment coefficient for money velocity is significant and the one for the real exchange rate is zero; 2) for the second regime, the adjustment coefficient for money velocity is zero and the one for the real exchange rate becomes significant; 3) For the third regime, the adjustment coefficients for money velocity and the exchange rate become zero. These changes in the adjustment coefficients are those implied by the theoretical model and we can now estimate a model for inflation for each subsample.

### 6.5 The Model for Inflation in Mexico from 1932 to 2013

The previous section proved the hypothesis of regime changes occurring at some specific dates (1982 and 2001). Now, one can estimate the final inflation equations for each regime. They are an improved version of the ones in Table 3 and have much better statistical properties. The first two regimes have an error correction mechanism but with different explanatory variables, as implied by the theoretical model. As the third one has no systematic causes for inflation, it has a simple structure of a constant plus noise.

In the next section these models prove to work very well in out-of-sample forecasting. On the contrary, other variables often considered as inflation factors, such as the output gap, commodity

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prices and wages have little or no explanatory and forecasting power, as suggested by the theoretical model.

None of the equations contains any lagged values of inflation, meaning that inertial inflation had no role after considering the effect of inflationary money and the separation of the price level from it. The point is interesting because, inflation inertia plays a big role in some models. For example, in Gordon (2013), the estimated Phillips curve for the United States requires the inclusion of autoregressive terms with coefficients that add to one, meaning that the model is in fact for the first differences of inflation. For Mexico, the value of the autoregressive terms falls drastically only when the dominant cause of inflation is included in the model.

6.5.1 Model for Regime 1

In the first regime, currency was the monetary policy variable so the inflation process is an error correction model within the QEM system (equation 22). The results with general-to-specific deletion are shown in the first column of Table 4.

\[ \Delta p_t = c_m + \alpha_{pm} p_{t-1} + \alpha_m (m - y)_{t-1} + \phi_m \Delta m_t + \epsilon_{pm} \]  

The t statistic for the lagged price level coefficient \( \alpha_{pm} \) is negative and highly significant according to the Ericsson-MacKinnon (2002) tables, indicating the QEM equation is a cointegration relationship. The contemporary impact of money growth \( \phi_m \) is very high, with about 0.5, it shows that half the long-run impact of money on prices occurs in the first year. All residual and specification tests are satisfactory.

In an auxiliary regression for money as a function of lagged money velocity and lagged inflation, shown in the first column of Table 5, none of the terms is significant, as expected. This is one way to check that money is a weakly exogenous variable in the first regime within the conditional error correction framework.

\footnote{The disappearance of autoregressive terms is easier for annual data. Although for quarterly or monthly data those terms could still be statistically significant, they would still be of little importance with respect to the main variables. In fact, the presence of autoregressive terms that add less than one in inflation models with monthly or quarterly data is only to describe the intra annual dynamics.}
6.5.2 Model for Regime 2

In the second regime, the exchange rate is the only systematic cause of inflation. Thus, one can get a model from the PPP condition with the price level as the error-correcting variable and both the exchange rate and the foreign price level as weakly exogenous variables:

\[
\Delta p_t = c_e + \alpha_{pm} p_{t-1} + \alpha_m (e + p^{us})_{t-1} + \phi_e \Delta (e + p^{us})_t + u_t^{pe} 
\]  

(23)

The coefficient for the lagged price level \(\alpha_{pm}\) is highly significant, showing the PPP condition is a cointegration relationship with the price level as the adjusting variable. The size of that coefficient \((-0.7)\) plus the contemporary effect of a depreciation on inflation \(\phi_e\) is very high, implying a fast convergence. The very high adjusted \(R^2\) implies that there is almost no room for other explanatory variables. All the statistical tests are satisfactory. In this period, the parameters of the model do not show any signs of instability.

6.5.3 Model for Regime 3

For the third regime, the only systematic cause of inflation is neither money nor the exchange rate, but the inflation expectation itself. As the assumption is that the central bank’s target enjoys credibility, this substitutes the other two drivers of inflation. Because of this, the forecast of inflation based on other variables becomes difficult: there is nothing better than the inflation target itself to forecast annual inflation. Maybe some variables can help at higher frequency but we should not expect a big improvement. It is in this sense that inflation becomes a “near-exogenous” variable (Hall, 2011).

For comparison purposes, the model can be written as another error-correction mechanism. To ensure that the process converges to the target, the deviations from it should be mean-reverting. A way to capture this condition is the following. For the year \(t - 1\), we can write the price level that includes the inflation target \((p^*_t)\) as \(p^*_t - 1 = p_{t-2} + \pi^*\), where \(\pi^*\) is the fixed inflation target.

\[
\Delta p_t = c^* + \alpha^* (p_{t-1} - p^*_t) + u_t^*
\]

(24)

\[
= (c^* - \alpha^* \pi^*) + \alpha^* \pi_{t-1} + u_t^*
\]
In this case, the equation collapses into a simple autoregressive model. The small sample size for this regime might be a problem to estimate the autoregressive coefficient with enough precision. In fact, the estimate for such coefficient is nonsignificant, as can be seen in the third column of Table 4. Because of this, and the fact that the variability of the data for this regime is small, the model for the second regime can be extended to the third without affecting much its statistical properties.

However, that would be incorrect because inflation has not followed the exchange rate movements as in the past. The sharp depreciation of the Mexican peso during the financial crisis of 2008 was not followed by a similar increment in prices (although there was a transitory co-movement). A similar situation has been observed in later episodes.

7 Out-of-sample Forecasting Performance Evaluation

The statistical appraisal of the inflation models of the previous section was good and this section shows that their out-of-sample forecasting performance is equally satisfactory. This type of analysis is absent in most papers on Mexican inflation so there is little to compare with.

The comparison models are: a) the “General” model (equation (20)); b) the models for regimes 1 (“Pure Monetary”) and 2 (“Pure Exchange Rate”); c) a simple AR(1) model (no other lags are significant) and; d) a naive model where the forecasts are a constant (average inflation) so the log of the price level is assumed to follow a random walk with drift. The latter model works very well in regime 3. The exercise consists of the following:

1. For regimes 1 and 2, to estimate each model using only the first half of the sample: 1932-1956 and 1983-1992, respectively.

2. For regimes 1 and 2, to forecast dynamically the second part of the subsample of each regime without re-estimating and conditioning on the explanatory variables.

3. For regime 3, due to its short time span, the estimation period goes from 1983 to 2000, except for the “Naive” model.

4. For regime 3, to forecast dynamically from 2001 to 2013 without re-estimating and conditioning on the explanatory variables, except for the “Naive” model.
5. For regime 3, the average inflation for the “Naive” model is calculated from 2001 to 2007 (it is impossible to do the same with the first three models and the fourth collapses to the “Naive” one). The constant forecast is applied to 2008-2013.

6. The comparisons are carried out with the root mean square error (RMSE).

Table 6 summarizes the results. None of the models performs well in every regime, as expected. “Pure Monetary” does well in regime 1 (where it has the second lowest RMSE), but very poorly in the others. “Pure Exchange Rate” does better than any other only in regime 2.

“General” has the lowest RMSE in regime one but this is misleading. As it contains money as an explanatory variable, it must do well in that period. It does better than the “Pure Monetary” because it includes the devaluation of 1976, which coincided with a change in the short-run parameter of the latter model (all others are stable) and the inclusion of the exchange rate makes up for that instability. The first panel of Figure 7 shows this instability beginning in 1972, when the public deficit began to explode (this was shown in Figure 2). This results in that the model has a deficient forecasting performance in the years 1973 and 1976. The second panel of Figure 7 shows that the model noticeably underpredicts inflation in 1973 and 1976. This means that prices incorporated the inflationary impact of money injections faster than in a low inflation environment.

A way to account for this parameter instability is allowing for the short-run impact of money growth to change in 1972 with an interaction dummy variable. With this, the coefficient until 1971 is 0.26 but it jumps to 0.60 in 1972, when fiscal policy becomes profligate. This regression is shown in the second column of Table 5.

The forecasts of “Pure Exchange Rate” were right on, except in 2000, when the forecast level was much lower than the actual value of inflation (Figure 8). The RMSE (0.06) easily beats those of the others and, in fact, it was quite close to the standard error for the original regression (0.05). The underprediction for 1999 might be a sign that the end of regime 2 occurred in that year and not in 2000.

The good performance of this model ends in regime 3 even though it can still be estimated until 2013 without much change in the parameters. This occurs because PPP still holds at the end of the sample, even though the pass-through from the exchange rate to inflation disappeared. Instead of this effect, the mean reversion property of the real exchange rate was reflected in a revaluation of
the nominal exchange rate nearly enough to erase the effects of the devaluation during the crisis. In Figure 9, the path of actual inflation, actual nominal exchange rate variation and the dynamic forecast of the model estimated for regime 2. Notice that the forecast is off the mark as it follows the movements of nominal exchange rate depreciation and not that of actual inflation. This is the most likely fate of any inflation model that does not account for the regime change in 2000.

In regime 3, the estimate for the autoregressive term is negative but nonsignificant so the model for the log of the price level becomes a random walk. Average inflation is almost equal to the upper limit of the monetary policy band for the inflation target. The situation is similar to that of other economies that have adopted an implicit or explicit inflation target, where the inflation process becomes a flat line around the target with noise around it (i.e., a random walk). This is not surprising, a random walk model can beat Phillips curves in the prediction of U.S. inflation (Atkenson and Ohanian, 2001).

The sample for regime 3 is too short to make a meaningful forecasting exercise but taking the average inflation of half of the subsample should be a decent forecast for the next years and it actually beats easily all the other models in that regime.

8 The Role of Other Variables on the Determination of Inflation

The variables identified as causes of inflation in Mexico can be classified as: 1) systematic causes, when they determine the price level once their impact is fully absorbed; 2) short-run causes, when their effect lasts for some periods but without ultimately changing the price level and; 3) unsystematic but with a permanent impact on the price level.

The first two correspond to regimes 1 and 2, which are equivalent to a regime with price level determination. The third type is a characteristic of regime 3, where the target is the inflation rate. The generic impact profile of each type of shock is provided in Figure 10 (the specific form depends on the parameters of each factor). With annual data the shape is not as smooth because most of the convergence occurs within two years so the periods in Figure 10 could represent quarters or months and not years.

The goodness of fit of the inflation models, specially that for regime 1 where the $R^2$ reaches 0.96
versus the 0.69 of that for regime 2, leaves little room for other explanatory variables. However, those additional variables could become important in some situations. This section explores the role that some popular variables played in each regime, including money and the exchange rate when they were not systematic causes of inflation. These variables are not explicit in the theoretical model and, therefore, fall into the term $u_t$ of the CVAR (16).

### 8.1 Wages

The role of wages and its relationship with the exchange rate in the determination of the price level was studied in Pérez-López (1996) and Garcés Díaz (1999) for samples that begin in 1983 and finish at the end of the nineties. Thus, those studies cover most of regime 2. In the latter paper, the wage variable explains one-third of the long-run changes of the price level while the variable “foreign price level in local currency” $(e + p^{US})_t$ explains two-thirds of it. Because of this property of homogeneity of degree one and that both variables were weakly exogenous with respect to the price level, the inflation model in that paper could have been rewritten with the lagged values of both the real exchange rate and the real wage as the main explanatory variables instead of the error correction term it used $(\frac{1}{3}w + \frac{2}{3}(e + p^{US}))$. Moreover, that model is equivalent to the one here developed for regime 2 because the wage variable $w_t$ was cointegrated with $((e + p^{US})_t$ during regime 2, as were the price level and many other nominal variables. The economic interpretation of those facts is simple: the workers and other price setters had tied their expectations about the price level to the exchange rate and this sped up the pass-through.

The model for regime 2 here obtained is more parsimonious and easier to embed in a unified framework than that of the 1999 paper quoted above and it does not contradict it. During the inflation targeting regime, wage shocks can have only temporary effects on the inflation rate as long as there are not changes in wage policy that can undermine the public’s confidence in the macroeconomic framework. The experience of regime 2 shows indeed that nominal wages can be one of the drivers of the price level.
8.2 The Exchange Rate

This variable was the systematic cause of inflation only in the second regime (1983-2000). There has been a long tradition in Mexico in assigning a big, if not the central, role to the exchange rate in several episodes of inflationary surges, even during regime 1 (1932-1982), when money was the systematic cause. This issue is explored here, in particular the 1976 devaluation.

For this, begin with an error-correction model for inflation similar to that for the second regime but this time applied to the first regime. The results are presented in the second column of Table 7.

It can be seen that although the contemporary value of the nominal exchange rate depreciation is highly significant, the lagged real exchange rate, or error-correction term, is nonsignificant. This eliminates the exchange rate as a systematic cause of inflation during the first period.

Next, augment that regression with the lagged value of money velocity and the contemporary value of money growth. The result is reported in the third column of Table 7. The added terms are highly significant, as expected from previous sections. The contemporary value of nominal depreciation remains highly significant. This could lead one to believe that although the exchange rate was not a systematic cause of inflation in the 1932-1982 period, it had some temporary but strong impact in at least some devaluation episodes.

However, there is a nonobvious mistake with that regression, one that easily could have been committed in other models of Mexican inflation. The problem comes from overlooking that the exchange rate is not a weakly exogenous variable during the first regime. To control for that characteristic, a new regression is run with instrumental variables. The first lag of money growth will work because it Granger-causes the depreciation rate, as shown in Table 2. Furthermore, Granger causality also follows from the known fact that a weakly exogenous variable in a cointegrated system Granger-causes the weakly endogenous variable. The results for inflation with instrumental variables are in the last column of Table 7.

With instrumental variables, the estimated effect of contemporary exchange depreciation loses its statistical significance. So, we can conclude that the exchange rate during the first regime was not an inflationary factor. Even if we were to ignore the results of the regression with instrumental variables, its role would have been modest, temporary and quickly eliminated (two years) and it does not help in out-of-sample forecasts for this period.

In the third regime, the exchange rate again is no longer a systematic cause of inflation, but its
contemporary variation is still significant, as shown in the last column of Table 4. As the strongest exchange rate movements responded to international causes (the world financial crisis), there are no endogeneity issues. The impact is significant but very modest in magnitude: for each one percent of depreciation there is an impact of 0.08 percent, small enough to be hard to spot even if there is a strong depreciation. Furthermore, all of the significance comes from the period 2006-2009 so, the coefficient of this impact could become even smaller when the sample grows.

8.3 Money and Detrended Output (Output Gap)

Other variables such as money in regimes 2 and 3, and detrended output in all regimes were tested as possible explanations of inflation without success. After the first regime, money has not had a distinguishable effect on inflation in Mexico. This can be seen by adding contemporary money growth to the inflation equation for the second and third regimes. Even its contemporary rate of variation is nonsignificant in those regressions, reported in columns two and three of Table 4.

Detrended output, or output gap, is a key variable in models without the classical dichotomy property. It is sometimes replaced by the unemployment rate or some other indicator of idle capacity. Detrended output is significant only in the model for the second regime but it does not help in out-of-sample forecasts. We redid the forecasting exercise for regime 2, described in the previous section. This time we included the log of output detrended with a Hodrick-Prescott filter and estimated the model from 1983 to 1992, as before. We also forecasted dynamically from 1993 to 2000 and the root mean squared error was much larger than in the previous exercise that did not contain detrended output (0.10 vs 0.06).

9 Conclusions and Final Remarks

Although the study is only on Mexico, it can also be applied to other Latin American countries as in Garces-Diaz (2016). Also, the nature of the approach and its results make it useful for more general purposes. Within a standard classical monetary framework (Lucas 1982), we obtained a model with regime changes that explains the behavior of inflation in Mexico during the whole period when fiat money has been widely used (1932-2013).

The theoretical framework assumes classical dichotomy even for the short run. Such approach
has been widely exploited in studies of the business cycle in RBC models, but less so for inflation studies where Keynesian models rule unchallenged despite their frequent troubles. It was shown that there can be viable alternatives, at least for developing countries.

Although the sample used in this paper is shorter than that in Hendry (2001), the model here obtained is far more parsimonious, with at most two explanatory variables in each regime. The model has no lagged values of inflation nor dummy variables to capture outliers or other problems despite the long sample. Although very simple, the model fits the data in a satisfactory manner.

It was shown that if other variables can be added, they contribute very little or nothing to the goodness of fit of the model. These characteristics are reflected in good out-of-sample forecast exercises. It must be said that the list of additional variables here examined is not exhaustive and that some still could be important. For example, Garces Diaz (1999) shows that wages had a role in regime 2 analogous to that of the exchange rate. The connection between that result and the one of this paper comes from the fact that the wage index was also cointegrated with the exchange rate at that time.

This study also showed that the dynamics of inflation, and also that of money and the exchange rate, depends on the variable emphasized by the central bank: 1) When money is the driving variable, the price level is determined within the QEM; 2) when the exchange rate is leading variable, the price level is determined within the PPP condition and money adjusts passively; 3) when the central bank targets inflation, neither money nor the exchange rate can determine the inflation rate and the inflation process becomes noise around a constant.

The results suggest some general implications:

i. The presumption that the classical monetary model with flexible prices requires to work either a very long sample or hyperinflation is not true in general. The results of this paper show that such model can work for low levels of inflation and within short subsamples while also yielding good forecasts.

ii. In regime 3, the best inflation forecast turned out to be a constant related to the inflation target itself. Thus, it cannot be said that the model can do a better job than simple time series alternatives as in regimes 1 and 2. Nonetheless, the model suggests that the inflation property of “near-exogeneity” (i.e., hard to predict based on other variables) in regime 3 is
not necessarily the result of its being low, as Hall (2011) argues for the case of the United States. This is exemplified with the Mexican case during the period 1957 to 1971. During this period, there was low inflation but this was still predictable conditional on other variables. Thus, it seems that the situation of a near-exogenous inflation is reached when a central bank has a credible inflation target (implicit or explicit) and this coincides with the presence of low inflation.

iii. The systematic pass-through of the exchange rate to inflation was a phenomenon that did not exist in Mexico before 1983 and it ended at around 2000.

iv. Several papers have shown a change in inflation inertia when there was a change to inflation targeting regime in Mexico. Inflation inertia here is the result of determinants of inflation within each regime. This is shown by the fact that lags of annual inflation are not even statistically significant if its right causes are used as explanatory variables. Lags of inflation can be different from zero if the frequency is monthly or quarterly because they might be necessary for the intra-annual dynamics.

However, the sum of those coefficients remains far below one if the model is properly specified (see, for example, García Díaz, 1999). This explains why there was a near-unit-root behavior of inflation before inflation targeting was adopted (Chiquiar et al. 2010, Noriega et al. 2013). In regimes 1 and 2, the main variables in the determination of inflation were lagged money velocity and the real exchange rate, respectively. These two are highly autor-regressive variables, that passed on this property to inflation. Since the central bank broke that link in 2001, when it adopted inflation targeting, annual inflation ceased to depend on its past, behaving instead almost as noise around a constant.

v. Not many events seem to affect the parameters of the inflation process. In fact, only changes in the monetary policy framework and the abandonment of fiscal discipline caused a change in the parameters of the model.

vi. It is often stated that inflation has many causes, as Romer (2011), Hendry (2001) and Juselius (2006), but we found that it is hard to show in a regression any significant variables other than the systematic causes (money in regime 1, the exchange rate in regime 2 and the inflation
target in regime three). It is even harder to make use of those additional variables to improve forecasts. As we showed in the model for regime 2, even if a nonsystematic cause survive in regression based on statistical criteria, it might not add anything in terms of predictive power.

References


A Data Sources

The data for this paper come from the following sources:

México:

The GDP ($Y$) for Mexico was constructed with the real series in local currency (base 2008) from the International Monetary Fund’ WEO Database October 2013 and extrapolated backwards from 1979 to 1900 with the rates of change from the historical series of Instituto Nacional de Estadística, Geografía e Informática (INEGI).

The price level $P$, was constructed with the series of National Consumer Price Index from 2001 to 2013 and it was extrapolated backwards from 2000 to 1932 with the implied variation rates of the Wholesale Price Index for Mexico City.

The nominal exchange rate $E$, currency $M$, and $P$ were obtained 1980-2002 Instituto Nacional de Estadística, Geografía e Informática (INEGI) at: http://www.inegi.gob.mx; or from Banco de México at: http://www.banxico.org.mx. For the period 1940-1979, it was taken from the book “Estadísticas Históricas del INEGI.” The data for the ratio of public deficit (borrowing requirements) to GDP ($pd$) was formed with a series taken from Gil-Diaz and Ramos-Tercero (1985) and from Banco de México’s annual reports (also available online) from 1982 onwards.

United States:

CPI ($P^{US}$):Department of Labor Bureau of Labor Statistics, at:
### Table 1: Unit Roots Tests for Variables and Equilibrium Relationships

<table>
<thead>
<tr>
<th>Variables</th>
<th>Level</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Level ($p$)</td>
<td>-0.25</td>
<td>-3.5&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>Exchange Rate ($e$)</td>
<td>0.96</td>
<td>-4.8&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>US Price Level ($p^{us}$)</td>
<td>1.02</td>
<td>-3.2&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>Inflationary Money ($m - y$)</td>
<td>0.67</td>
<td>-7.6&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>Money ($m$)</td>
<td>0.98</td>
<td>-7.6&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>Foreign Prices ($e + p^{us}$)</td>
<td>0.97</td>
<td>-5.2&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

#### Equilibrium Relationships †

<table>
<thead>
<tr>
<th>Equilibrium Relationships †</th>
<th>Level</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Exchange Rate ($e + p^{us} - p$)</td>
<td>-4.11&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-8.20&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>Money Velocity ($p + y - m$)</td>
<td>-3.12c</td>
<td>-9.56&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>Inflationary Money Deflated by Foreign Prices ($m - y - (e - p^{us})$)</td>
<td>-3.12c</td>
<td>-9.70&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

#### Group Tests

<table>
<thead>
<tr>
<th>Method</th>
<th>Level</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Levin, Lin &amp; Chu t (individual unit root)</td>
<td>0.65</td>
<td>-7.10&lt;sup&gt;a&lt;/sup&gt;</td>
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<tr>
<td>Im, Pesaran and Shin W-stat (common unit root)</td>
<td>3.95</td>
<td>-10.01&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>ADF - Fisher Chi-square (common unit root)</td>
<td>1.06</td>
<td>138.17&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

<sup>a, b, c</sup> Unit Root Hypothesis Rejected at 1%, 5%, 10% significance level, respectively.

† The sample for all tests starts in 1932 but it ends in 2000 for money velocity and inflationary money deflated by foreign prices.

The group tests include all the variables at the top of the table.
Table 2: Bivariate Granger Causality Tests For Nonstationary Variables in Each Regime

<table>
<thead>
<tr>
<th></th>
<th>Regime 1</th>
<th>Regime 2</th>
<th>Regime 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent Variable: Price Level ( p_t )</strong></td>
<td></td>
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<td></td>
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<tr>
<td><strong>Excluded Variable</strong></td>
<td>( \chi^2(2) )</td>
<td>p-value</td>
<td>( \chi^2(2) )</td>
</tr>
<tr>
<td>Inflationary Money ((m - y)_t)</td>
<td>9.73</td>
<td>0.01</td>
<td>2.27</td>
</tr>
<tr>
<td>Exchange Rate ( e_{t1} )</td>
<td>2.17</td>
<td>0.34</td>
<td>7.02</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Regime 1</th>
<th>Regime 2</th>
<th>Regime 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent Variable: Inflationary Money ((m - y)_t)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Excluded Variable</strong></td>
<td>( \chi^2(2) )</td>
<td>p-value</td>
<td>( \chi^2(2) )</td>
</tr>
<tr>
<td>Price Level ( p_t )</td>
<td>1.73</td>
<td>0.42</td>
<td>15.89</td>
</tr>
<tr>
<td>Exchange Rate ( e_{t1} )</td>
<td>1.52</td>
<td>0.22</td>
<td>11.80</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Regime 1</th>
<th>Regime 2</th>
<th>Regime 3</th>
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<tr>
<td><strong>Dependent Variable: Exchange Rate ( e_t )</strong></td>
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<tr>
<td><strong>Excluded Variable</strong></td>
<td>( \chi^2(2) )</td>
<td>p-value</td>
<td>( \chi^2(2) )</td>
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<tr>
<td>Price Level ( p_t )</td>
<td>2.23</td>
<td>0.33</td>
<td>1.56</td>
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<tr>
<td>Inflationary money ((m - y)_t)</td>
<td>7.57</td>
<td>0.01</td>
<td>0.26</td>
</tr>
</tbody>
</table>

*The test was modified following Toda and Yamamoto (1995).*

*The Wald test statistic is distributed as a \( \chi^2(2) \).*
Table 3: Unbalanced Regressions for the Inflation Rate ($\Delta p_t$) in Each Regime

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>1.25 (2.39)</td>
<td>3.46 (6.65)</td>
<td>-7.46 (-5.27)</td>
<td>n.s.</td>
</tr>
<tr>
<td>$(m - y)_{t-1}$</td>
<td>0.07 (2.43)</td>
<td>0.2 (6.81)</td>
<td>-0.46 (-5.66)</td>
<td>n.s.</td>
</tr>
<tr>
<td>$(e + p^{as})_{t-1}$</td>
<td>-0.07 (-2.45)</td>
<td>-0.18 (-6.11)</td>
<td>0.40 (5.20)</td>
<td>n.s.</td>
</tr>
<tr>
<td>$\Delta m_{t-1}$</td>
<td>0.51 (7.33)</td>
<td>0.5 (8.06)</td>
<td>n.i.</td>
<td>n.s.</td>
</tr>
<tr>
<td>$\Delta e_{t-1}$</td>
<td>0.34 (7.92)</td>
<td>n.i.</td>
<td>0.53 (7.33)</td>
<td>0.08 (1.95)</td>
</tr>
<tr>
<td>$\Delta p_{t-1}$</td>
<td>n.s.</td>
<td>n.s.</td>
<td>n.s.</td>
<td>n.s.</td>
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<td>$\Delta p_{t-2}$</td>
<td>n.s.</td>
<td>n.s.</td>
<td>-0.24 (-2.73)</td>
<td>n.s.</td>
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<tr>
<td>$\Delta m_{t-1}$</td>
<td>0.12 (2.71)</td>
<td>n.s.</td>
<td>n.s.</td>
<td>n.s.</td>
</tr>
<tr>
<td>$\Delta e_{t-1}$</td>
<td>0.19 (4.18)</td>
<td>n.s.</td>
<td>n.s.</td>
<td>n.s.</td>
</tr>
<tr>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ADF statistic</td>
<td>81 $a$</td>
<td>49 $a$</td>
<td>18 $a$</td>
<td>13 $a$</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.87</td>
<td>0.77</td>
<td>0.94</td>
<td>0.42</td>
</tr>
<tr>
<td>SE</td>
<td>0.06</td>
<td>0.06</td>
<td>0.07</td>
<td>0.01</td>
</tr>
<tr>
<td>Jarque-B</td>
<td>0.42</td>
<td>0.59</td>
<td>0.76</td>
<td>0.63</td>
</tr>
<tr>
<td>LM(1) autocor</td>
<td>0.14</td>
<td>0.13</td>
<td>0.06</td>
<td>0.71</td>
</tr>
</tbody>
</table>

$t$ statistics are between parentheses.

n.s. means excluded for being nonsignificant and n.i. means it was not included.

$a$, $b$, $c$ represent 1%, 5%, 10% significance level, respectively.

ADF statistic to test the stationarity of the residuals of an unbalanced regression.

The Ericsson-MacKinnon critical values of 1% significance for each of the first three regressions are -4.3, -4.09 and -4.3, respectively.

For Jarque-B and the LM(2) Autocor statistics the $p$ values are provided.
Table 4: Inflation ($\Delta p_t$) Model in Each Regime

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>3.64 (5.77)</td>
<td>-1.73 (-5.41)</td>
<td>0.04</td>
</tr>
<tr>
<td>$p_{t-1}$</td>
<td>-0.21 (-5.31)</td>
<td>-0.76 (-7.72)</td>
<td>n.i.</td>
</tr>
<tr>
<td>$(m - y)_{t-1}$</td>
<td>0.24 (5.84)</td>
<td>n.i.</td>
<td>n.i.</td>
</tr>
<tr>
<td>$(e + \mu^x)_{t-1}$</td>
<td>n.i.</td>
<td>0.73 (7.37)</td>
<td>n.s.</td>
</tr>
<tr>
<td>$\Delta m_t$</td>
<td>0.45 (6.99)</td>
<td>n.i.</td>
<td>n.s.</td>
</tr>
<tr>
<td>$\Delta e_t$</td>
<td>n.i.</td>
<td>0.54 (9.79)</td>
<td>0.08 (3.2)</td>
</tr>
<tr>
<td>$\Delta m_{t-1}$</td>
<td>n.s.</td>
<td>n.s.</td>
<td>n.s.</td>
</tr>
<tr>
<td>$\Delta e_{t-1}$</td>
<td>n.s.</td>
<td>n.s.</td>
<td>n.s.</td>
</tr>
<tr>
<td>T</td>
<td>49</td>
<td>18</td>
<td>13</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.68</td>
<td>0.96</td>
<td>0.42</td>
</tr>
<tr>
<td>SE</td>
<td>0.05</td>
<td>0.05</td>
<td>0.01</td>
</tr>
<tr>
<td>Jarque-B</td>
<td>0.16</td>
<td>0.94</td>
<td>0.63</td>
</tr>
<tr>
<td>LM(2) autocor</td>
<td>0.63</td>
<td>0.36</td>
<td>0.71</td>
</tr>
<tr>
<td>LM(1) arch</td>
<td>0.91</td>
<td>0.75</td>
<td>0.93</td>
</tr>
<tr>
<td>$CUSUM$</td>
<td>pass</td>
<td>pass</td>
<td>pass</td>
</tr>
<tr>
<td>$CUSUM^2$</td>
<td>pass</td>
<td>pass</td>
<td>pass</td>
</tr>
<tr>
<td>N-step proj.</td>
<td>fail</td>
<td>pass</td>
<td>pass</td>
</tr>
</tbody>
</table>

$t$ statistics are between parentheses.

n.s. means excluded for being nonsignificant.
n.i. means it was not included.

The Ericsson-MacKinnon critical values of 1% significance for the first two regressions is -4.09.

For Jarque-B, LM(2) autocor LM(2) arch the p values are provided.
Table 5: Complementary Regressions for Regime 1 (1932-1981)

<table>
<thead>
<tr>
<th>Regressors</th>
<th>Dependent Variable</th>
<th>Dependent Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta m_t$</td>
<td>$\Delta p_{t-1}$</td>
</tr>
<tr>
<td>constant</td>
<td>-0.17</td>
<td>3.85</td>
</tr>
<tr>
<td></td>
<td>(-0.17)</td>
<td>(6.37)</td>
</tr>
<tr>
<td>$p_{t-1}$</td>
<td>0.03</td>
<td>-0.25</td>
</tr>
<tr>
<td></td>
<td>(-0.15)</td>
<td>(-6.21)</td>
</tr>
<tr>
<td>$(m - y)_{t-1}$</td>
<td>-0.02</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>(-0.16)</td>
<td>(6.29)</td>
</tr>
<tr>
<td>$\Delta p_{t-1}$</td>
<td>0.29</td>
<td>n.s.</td>
</tr>
<tr>
<td></td>
<td>(1.27)</td>
<td></td>
</tr>
<tr>
<td>$\Delta m_t$</td>
<td>n.i.</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.83)</td>
</tr>
<tr>
<td>$dum_{72-81}\Delta m_t$</td>
<td>n.i.</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.74)</td>
</tr>
<tr>
<td>T</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.09</td>
<td>0.74</td>
</tr>
<tr>
<td>SE</td>
<td>0.11</td>
<td>0.05</td>
</tr>
<tr>
<td>Jarque-B</td>
<td>0.05</td>
<td>0.79</td>
</tr>
<tr>
<td>LM(2) autocor</td>
<td>0.52</td>
<td>0.92</td>
</tr>
<tr>
<td>LM(1) arch</td>
<td>0.94</td>
<td>0.81</td>
</tr>
<tr>
<td>CUSUM</td>
<td>fail</td>
<td>pass</td>
</tr>
<tr>
<td>CUSUM$^2$</td>
<td>fail</td>
<td>pass</td>
</tr>
<tr>
<td>N-step proj.</td>
<td>fail</td>
<td>pass</td>
</tr>
</tbody>
</table>

$dum_{72-81}$ is a dummy variable equal to 1 from 1975 to 1981.

$ t $ statistics are between parentheses.
n.s. means excluded for being nonsignificant.
n.i. means it was not included.
The Ericsson-MacKinnon critical values of 1% significance is -4.09
For Jarque-B, LM(2) autocor LM(2) arch the p values are provided.
Table 6: Out-of-sample forecasting comparisons (RMSE)

<table>
<thead>
<tr>
<th>Model</th>
<th>Regime 1</th>
<th>Regime 2</th>
<th>Regime 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>“General”</td>
<td>0.04</td>
<td>0.12</td>
<td>0.05</td>
</tr>
<tr>
<td>“Pure Monetary”</td>
<td>0.07</td>
<td>0.13</td>
<td>0.42</td>
</tr>
<tr>
<td>“Pure Exchange Rate”</td>
<td>0.08</td>
<td>0.06</td>
<td>0.04</td>
</tr>
<tr>
<td>“AR(1)”</td>
<td>0.10</td>
<td>0.17</td>
<td>0.10</td>
</tr>
<tr>
<td>“Naive”</td>
<td>0.10</td>
<td>0.29</td>
<td>0.01()</td>
</tr>
</tbody>
</table>

The numbers in the cells are the root mean square errors (RMSE).

E. and F. mean the estimation and forecasting period, respectively.

“General” is \(\Delta p_t = \beta_m (m - y)_{t-1} + \beta_e (e + p^u)_{t-1} + \phi_m \Delta m_t + \phi_e \Delta e_{t-1}\)

“Pure Monetary” is \(\Delta p_t = \beta_m (m - y)_{t-1} + \beta_p p_{t-1} + \phi_m \Delta m_t\)

“Pure Exchange Rate” is \(\Delta p_t = \beta_e (e + p^u)_{t-1} + \beta_p p_{t-1} + \phi_e \Delta e_t\)

“AR(1)” is \(\Delta p_t = \beta_p \Delta p_{t-1}\)

“Naive” is the average of inflation of half the sample except for Regime 3.

Table 7: The Impact of Other Variables on Inflation ($\Delta p_t$)

<table>
<thead>
<tr>
<th>Regressors</th>
<th>Regime 1 (1932-1981)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>OLS</td>
<td>IV ($\Delta m_{t-1}$)</td>
<td></td>
</tr>
<tr>
<td>constant</td>
<td>-0.15</td>
<td>3.35</td>
<td>3.54</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.56)</td>
<td>(5.99)</td>
<td>(5.71)</td>
<td></td>
</tr>
<tr>
<td>$p_{t-1}$</td>
<td>n.s.</td>
<td>-0.20</td>
<td>-0.21</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-5.63)</td>
<td>(-5.40)</td>
<td></td>
</tr>
<tr>
<td>$(m - y)_{t-1}$</td>
<td>n.s.</td>
<td>0.23</td>
<td>0.24</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(6.07)</td>
<td>(5.80)</td>
<td></td>
</tr>
<tr>
<td>$(e + p^{us})_{t-1}$</td>
<td>n.s.</td>
<td>n.s.</td>
<td>n.s.</td>
<td></td>
</tr>
<tr>
<td>$\Delta m_t$</td>
<td>-0.45</td>
<td>0.43</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7.36)</td>
<td>(6.96)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta e_t$</td>
<td>0.45</td>
<td>0.27</td>
<td>n.s.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.11)</td>
<td>(3.82)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta m_{t-1}$</td>
<td>n.s.</td>
<td>n.s.</td>
<td>n.s.</td>
<td></td>
</tr>
<tr>
<td>$\Delta e_{t-1}$</td>
<td>n.s.</td>
<td>n.s.</td>
<td>n.s.</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>49</td>
<td>49</td>
<td>49</td>
<td></td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.26</td>
<td>0.76</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>SE</td>
<td>0.08</td>
<td>0.04</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>Jarque-B</td>
<td>0.04</td>
<td>0.51</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td>LM(2) autocor</td>
<td>0.01</td>
<td>0.51</td>
<td>0.59</td>
<td></td>
</tr>
<tr>
<td>LM(1) arch</td>
<td>0.34</td>
<td>0.97</td>
<td>0.79</td>
<td></td>
</tr>
<tr>
<td>CUSUM</td>
<td>pass</td>
<td>pass</td>
<td>n.a.</td>
<td></td>
</tr>
<tr>
<td>CUSUM$^2$</td>
<td>pass</td>
<td>pass</td>
<td>n.a.</td>
<td></td>
</tr>
<tr>
<td>N-step proj.</td>
<td>fail</td>
<td>pass</td>
<td>n.a.</td>
<td></td>
</tr>
<tr>
<td>Diff. in J-stat</td>
<td>n.a.</td>
<td>n.a.</td>
<td>0.33</td>
<td></td>
</tr>
</tbody>
</table>

1. Statistics are between parentheses.

n.s. mean nonsignificant and not included, respectively.

For Jarque-B, LM(2), autocor LM(2) arch and
Diff. in J-stat the p values are provided.
Figure 1: Monetary Rule for 1956-1971

- Currency growth
- Mexican GDP growth plus US inflation
Figure 2: Inflation and Public Deficit (FRPS).
Figure 3: Impulse Responses of Bivariate VARs in Each Regime.
Figure 4: Money Velocity and the Real Exchange Rate Across Regimes.
Figure 5: Dynamic Correlations for Money Velocity and the Real Exchange Rate Across Regimes.
Figure 6: Inflation Money and Foreign Prices in Local Currency.
Figure 7: Instability of the Short-run Effect of Money Because a Change in Fiscal Policy.
Figure 8: Forecasting Performance of the Model in Regime 2.
Figure 9: Inflation, the Depreciation Rate and a Dynamic Forecast in the Inflation Targeting Regime.
Figure 10: Conditional Impulse Responses of Inflationary Factors in Each Regime.