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Abstract

We consider a decision maker who enjoys choosing from a varied set of alternatives. Building on behavioral evidence, we propose testable axioms which characterize preference for variety, and provide a representation theorem. We go on to illustrate the potential effects of preference for variety in a model of retailing. Consumer welfare may be decreasing in the competitiveness of the retailing sector as competition eliminates the scope for retailers to offer variety. Mainstream consumers with a preference for variety and consumers with eccentric tastes enjoy a symbiotic relationship. Competition over mainstream consumers makes retailers offer more exotic goods, while eccentric consumers subsidize their carrying costs.

Keywords: Preferences, variety, representation theorem, retail, competition.
JEL Classification: D0, D03, D4.

Resumen

Consideramos a un agente que disfruta elegir de un conjunto variado de alternativas. Basándonos en evidencia acerca del comportamiento de los consumidores, proponemos axiomas verificables que caracterizan preferencias por variedad y presentamos un teorema de representación. Además, ilustramos los efectos potenciales de las preferencias por variedad en un modelo de ventas al menudeo. El bienestar de los consumidores puede disminuir al aumentar la competitividad del sector, ya que la competencia elimina las economías de escala necesarias para que los minoristas ofrezcan variedad. Los consumidores mayoritarios que tienen preferencias por variedad y gustos comunes mantienen una relación simbiótica con consumidores que tienen preferencias excéntricas. La competencia por los consumidores mayoritarios hace que los establecimientos ofrezcan bienes exóticos, y los consumidores excéntricos subsidian los costos de ofrecer dichos productos.

Palabras Clave: Preferencias, variedad, teorema de representación, menudeo, competencia.

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1 Introduction

The enjoyment of shopping is an experience shared by most people around the world. Whether it is a trip to an outlet mall for a new pair of jeans or going to the local fruit stand for bananas, pleasure is often derived not only from the ultimate consumption of what is bought, but also from the contemplation of what is available. This offers a compelling explanation of why retailers offering similar goods compete on variety as well as price. In this paper, we propose a utility representation which incorporates these motives, provide axiomatic foundations for this representation, and study some of its consequences for the industrial organization of retailers.

To make the phenomenon of interest concrete, suppose a decision maker (henceforth referred to as DM) lives in a town with two movie theaters where three different movies (a,b,c) are showing. Movies a and b are blockbusters, and close substitutes, while c is a documentary so that DM prefers a to b to c: $\{a\} \succ \{b\} \succ \{c\}$. Theater 1 offers only one movie (a), while Theater 2 offers movies b and c. Traditional models predict that DM would choose to see movie a at Theater 1, and that the fact that Theater 2 offers two choices is irrelevant ($\{a\} \succ \{b,c\}$). However, if DM enjoys choosing from a varied billboard, and the difference in consumption value between a and b is small, it may be that she will choose to see movie b in Theater 2 ($\{b,c\} \succ \{a\}$).[1] We refer to such a DM as having a *preference for variety.*[2]

We derive a utility representation for a decision maker for whom the available alternatives are relevant even if they are not chosen. While we have in mind a two stage problem in which DM chooses a menu and subsequently a good from that menu, we explicitly model only the first stage. Second stage behavior is left as part of the interpretation of first stage choices. The utility provided by a menu A is the sum of the utility derived from variety and the utility of final consumption:[3]

[1] Note that DM is not uncertain about her future preferences so there is no chance that she will choose to watch movie c. Because of this, the example is not consistent with preference for flexibility (Kreps, 1979).

[2] The term "preference for variety" is used in the literature on monopolistic competition (e.g. Anderson et al., 1995) to refer to a representative consumer's penchant for consuming different products. This is meant to model the variety of preferences in the population rather than an individual's enjoyment of variety. Therefore, the analysis in this paper is of a different nature than that previously stressed in the industrial organization literature.

[3] We consider preferences over lotteries when deriving our representation theorem. For clarity, we postpone discussion of lotteries to Section 3.

$$U(A) = (1-\beta)\sum_{a\in A} U(\{a\}) + \beta \max_{b\in A} U(\{b\})$$

where U is a utility function defined over menus, $\{a\}$ is the menu containing only good a, and $\beta \in (0,1)$. Adding a good to a menu adds utility proportional to that good's consumption value. The parameter β measures the relative importance of variety and consumption for DM. U can take negative values for unpleasant goods, so that adding such a good to a menu can make it *less* appealing.

We propose two testable axioms which, along with standard axioms on choice over sets, guarantee that a preference relation has a *preference for variety representation*. We define the variety of a menu A as the vector of ones and zeroes[4] describing which goods in the feasible set Z are offered in A. Axiom 4, which we call *Preference for Quality*, states that DM values options which enhance her consumption value, even if they do not enhance variety. Axiom 5, *Preference for Variety*, states that DM values variety, and that the variety added by good a is evaluated according to DM's preference relation over menus containing only one good. With this assumption we are asserting that adding the same amount of variety in two different dimensions may increase the utility of a set by different amounts; that is, DM is concerned about the quality of the added variety and not merely the quantity.

While our representation theorem builds upon subjective state-dependent preferences (Dekel et al., 2001), in the spirit of Gul and Pesendorfer (2001), we emphasize that it may be that the different utility functions in a subjective state-space representation do not reflect uncertainty about the decision maker's future preferences, but instead different ways in which alternatives in a menu affect the experience of choosing from it. To rule out the subjective state-space interpretation we need to observe both choices over menus and consumption choices; if there are goods which influence an agent's choice of menu even though they are never consumed, then only our interpretation can provide a consistent explanation. While conclusive evidence is hard to come by, the research cited in Section 2 points in the direction of preference for variety.

As our moviegoer example suggests, taking consumers' preference for variety into account can lead to predictions which clash with those of traditional models. In Section 4 we provide two examples by analyzing a simple model of retailing with consumers who have a preference for variety. Retailing is a natural application of this theory: when consumers choose which retailer to go to, they are

[4] We will generalize this to include lotteries in Section 3.

choosing the set of goods that they will be able to contemplate and purchase from. The following predictions of our model of retailing are of particular interest.

In a spatial model of retailing, we find that reducing barriers to entry reduces consumer welfare. Thus, contrary to the predictions of traditional models, competition is bad for consumers. This is because the resulting influx of retailers into the market reduces revenues per store so that retailers find it harder to pay the fixed costs associated with offering additional goods. This, in turn, reduces the variety component of consumers' utility.

Section 4.2 illustrates another interesting implication of the model. Mainstream consumers with a preference for variety benefit from the presence of consumers with eccentric tastes, and vice-versa. This is because retailers offer a wide variety of goods in an attempt to please profitable mainstream consumers, even though these consumers will not buy these eccentric goods. This is good for consumers with eccentric tastes who are too few in number for retailers to serve for their own sake. On the other hand, because consumers with eccentric tastes buy at least some of the goods that are offered, the retailers' costs of carrying these goods are lower than they would be without these consumers. Consequently, retailers offer a wider variety of goods, which mainstream consumers enjoy even though they are not interested in purchasing them. In contrast to other models that use heterogenous tastes in the population to explain the provision of variety (Anderson et al., 1995), our results are driven by the preference for variety of one dominant group of consumers, and we are able to draw conclusions about the impact that one group's presence has on another's welfare.

The paper is organized as follows. Section 2 surveys relevant empirical and experimental evidence. Section 3 presents formal definitions of variety, our axioms on preferences, and our representation theorem. In Section 4, we set up our application of preference for variety to retailing. Section 4.1 presents a spatial model of retailing. Section 4.2 contains an application to consumers with heterogeneous tastes. Section 5 concludes.

2 Evidence

According to Shiv and Huber (2000), when shopping "consumers assess the likely satisfaction with each item before making the final choice" (p.202). When using this method of evaluating goods, called anticipating satisfaction or preconsumption mental imagery (McInnis and Price, 1987), consumers mentally reproduce the experience of consuming all goods under consideration. We believe this is the psychological source of preference for variety.

That hedonic motivations are important in consumers' shopping behavior is well documented (Arnold and Reynolds, 2003). While we do not pretend to capture the full complexity of shoppers' motivations, our paper fits well with the observation that a subset of shoppers enjoy contemplating the goods on offer. This is a distinguishing feature of a category of shoppers referred to as *browsers* (Jarboe and McDaniel, 1987).

There is a long line of papers which study preferences over assortments or choice sets, and the effects of assortment size on satisfaction. Greifeneder et al. (2009) provide a brief survey and try to identify choice conditions which lead to preference for variety. Findings indicate that consumers enjoy variety (i.e. choosing from larger sets) when they are familiar with the type of good being considered and have clearly defined preferences. In an article aptly named "The Lure of Choice," Bown et al. (2003) conduct a series of experiments which study the effect of clearly inferior alternatives (lures) on choice behavior. Larger menus, those with a lure, were chosen significantly more often than those without, while the lure itself was almost never the final choice.

With respect to preferences over choice set size, in an influential paper, Iyengar and Lepper (2000) describe an experiment in which subjects are presented with assortments of jams and chocolates of different sizes. Subjects were more likely to stop at the display when the jam assortment was large, but were more likely to make a purchase when the assortment was small. The former behavior suggests that subjects were lured by variety; variety increased the attractiveness of the display. In an experimental setting in which subjects who suffer from information overload are asked to choose the size of their choice set, Kaiser (2011) shows that very large sets are seldom selected. However, as subjects become more experienced they choose bigger sets. With respect to variety and satisfaction, Botti and Iyengar (2004) find that subjects who must make an unpleasant choice find variety detrimental to their experience. Subjects were offered an entree of fried scorpion, stewed snake meat, fried ants, or boiled spider egg. Those who were offered a larger set of alternatives anticipated enjoying the dish less than those who were not given a choice.

Marketing studies show that placing a *decoy* product, which is not meant to be sold, alongside a featured product can lead to increased demand. For instance, the retailer Williams Sonoma once offered a bread-maker for $275. When they added a similar, but larger, model to their product line priced 50% higher than the original, sales of the old bread-maker almost doubled while sales of the new model were close to zero (Ariely, 2008, p. 14-15). While the mainstream explanation for this in the Psychology literature is that the decoy highlights the virtues of the featured product for the consumer (the "attraction effect," Huber et al., 1982), a competing interpretation is that adding an irrelevant product to your offerings increases the attractiveness to consumers of coming to your store – thereby enabling an increase in sales.

Richards and Hamilton (2006) and Trindade (2010) point out that increasing variety is a strategy used by supermarkets to attract consumers and gain market share. They find that variety is positively related to price, a result that fits well with our results on market structure in Section 4. It is important to note that, consistent with our Axiom 4, the consumer behavior literature has found that it is not only more variety that matters, but the kind and quality of products offered. Oppewal and Koelemeijer (2005) find that, even when the most preferred product is always available, evaluations of assortment increase with its size. This indicates that the utility of a set is not only the utility of its best element, as standard Economic Theory would predict. Furthermore, they find that assortment evaluation is more responsive to increases in the variety of more important attributes. This suggests that variety is valued in a way consistent with the way products are valued for consumption. Boatwright and Nunes (2001) find that cutting items in stores has no negative consequences on profits if and only if the items that are eliminated from it are "repeated" or very similar to others in their characteristics. Thus, variety is valuable to consumers while sheer quantity is not.

Before moving on, let us pause to consider how the evidence mentioned above is reflected in our utility representation:

$$U(A) = (1-\beta)\sum_{a\in A} U(\{a\}) + \beta \max_{b\in A} U(\{b\})$$

As suggested by Bown et al. (2003), Huber et al. (2000), and Oppewal and Koelemeijer (2005), a menu may be more attractive when inferior alternatives are added to it. That is, adding an item that will not be chosen affects the first term in U, but not the second. As Oppewal and Koelemeijer (2005) suggest, adding items which are more attractive for consumption makes a menu more attractive than adding options DM would not care to consume – the value to DM of adding a good to a menu is proportional to his consumption utility. Indeed, if these alternatives are unpleasant, their consumption utility may be negative and the menu becomes less attractive (Botti and Iyengar, 2004). Adding more copies of a good already available does not enhance the menu (Boatwright and Nunes, 2001); each good in A shows up only once in U. Note that uncertainty about DM's enjoyment of a good z plays no role in our representation. Thus, we are focusing on conditions under which people tend to prefer choosing from larger sets as identified in Greifeneder et al. (2009).

The evidence cited above can be explained piecemeal by a combination of preference for flexibility, changing or heterogeneous tastes, the attraction or asymmetric dominance effect, and perhaps other psychological or behavioral tendencies. One of the great strengths of our representation is that it can account for all of these irregularities by means of a single mechanism. The ultimate validity of our explanation awaits experimental testing of our Axioms 4 and 5.

3 Representation

Let Z be a finite set of distinct prizes or *goods* with $|Z| \geq 3$. $\Delta(Z)$ is the set of all probability distributions, or *lotteries*, on Z. $\mathscr{A}$ denotes the collection of all closed subsets of $\Delta(Z)$ and its elements, $A \in \mathscr{A}$**,** are referred to as *menus***.** Let $\succcurlyeq$ be a preference relation on $\mathscr{A}$. We endow $\mathscr{A}$ with the topology generated by the Hausdorff metric.[5]

Throughout the paper, elements of Z are denoted a, b, or c. Elements of $\Delta(Z)$ are typically denoted x, y, or z. With a slight abuse of notation, we will write a both when referring to the good a and to the lottery which awards a with probability one. Note that each lottery is a $|Z|$-dimensional vector of probabilities. For a given lottery $x \in \Delta(Z)$, let x_a denote the probability with which x awards good a. Elements of $\mathscr{A}$ are A, B, or C. $\{x, y\}$ is a menu containing only lotteries x and y.

We have in mind an agent facing a two-period decision problem. The agent chooses a menu in period 0 and subsequently selects a lottery from that menu in period 1. However, we do not explicitly model the agent's period 1 choice, leaving it as part of the interpretation of the agent's period 0 preference.

The following definition clarifies our notion of variety. The *variety* of a menu A is the vector of maximal probabilities with which some lottery in A awards each good in Z.

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[5]Defined for any pair of non-empty sets, $A, B \in \mathscr{A}$, by:

$$dh(A;B) := max \left[\max_{a \in A} \min_{b \in B} d(a,b), \max_{b \in B} \min_{a \in A} d(a,b) \right]$$

where $d : \mathbb{R}^2 \to \mathbb{R}$ is the standard Euclidean distance.

Definition 1 *The variety of a menu A,* $V(A) \in \mathbb{R}^{|Z|}$*, is:*

$$V(A) = \left\{ \max_{x \in A} \{x_a\} \right\}_{a \in Z}$$

and $V_i(A)$ *denotes the i'th component of* $V(A)$*.*

Thus, the greatest variety a menu can offer is the $|Z|$-dimensional vector of ones. Furthermore, the sets Z and $\Delta(Z)$ provide the same variety. Note that our definition of variety takes the set of possible variations as fixed and exogenous. Thus, all possible variations are in Z, and variety itself does not depend on preferences. However, as will become clear below, agents may value different types of variety in different ways.

The menu $A \cup B$ may or may not have higher variety than A. Whenever, for every $a \in Z$ and $x \in B$, there is $y \in A$ such that $y_a \geq x_a$, we will have that $V(A \cup B) = V(A)$. On the other hand, if b is a degenerate lottery awarding a good not offered by any lottery in A, we will have $V(A \cup \{b\}) - V(A) = b$.

When comparing menus, DM will evaluate B's contribution to the variety in $A \cup B$: $V(A \cup B) - V(A) \geq 0$. However, $V(A \cup B) - V(A)$ typically will not be a lottery. Because the preference relation $\succcurlyeq$ is defined over sets of lotteries ($\mathscr{A}$), preference-based comparisons of the variety added by different menus to A cannot be made directly. We facilitate this evaluation in two ways. First, we add only one lottery at a time so that we can transform the added variety into a lottery[6] and make DM's preferences over singleton menus our point of reference. This leaves no room for ambiguity and ensures that there is an interpretable correspondence between how variety is valued and how goods are valued for consumption. Second, we construct a lottery by augmenting $V(A \cup \{x\}) - V(A)$ with outcomes in an arbitrary good $c \in Z$.

Definition 2 *The c-variety added by x to A is the lottery* $v^c(x,A) \in \Delta(Z)$ *such that:*

$$v_i^c(x,A) = \begin{cases} V_i(A \cup \{x\}) - V_i(A) & \text{for } i \neq c \\ 1 - \sum_{i \neq c} (V_i(A \cup \{x\}) - V_i(A)) & \text{for } i = c \end{cases}$$

Note that the preceding definitions are independent of any utility representation and serve only to clarify concepts and notation. We impose six axioms on preferences. Axioms 1-3 are standard in the setting of preferences over menus (Dekel et al., 2001).

[6]The variety added by a menu can include entries whose sum is greater than one.

Axiom 1 (Weak Order) $\succeq$ *is a complete and transitive binary relation.*

Axiom 2 (Countinuity) *The sets* $\{B : B \succ A\}$ *and* $\{B : A \succ B\}$ *are open.*

Axiom 3 (Independence) $A \succ B$ *and* $\alpha \in (0,1)$ *implies* $\alpha A + (1-\alpha)C \succ \alpha B + (1-\alpha)C$.

Axiom 4 states, holding variety constant, DM prefers menus which contain lotteries which yield higher consumption utilities, as measured by DM's preferences over singleton menus (Figure 3.2).

Axiom 4 (Preference for Quality) *If* $V(A) = V(B)$ *and for every* $x \in B$ *there is a* $y \in A$ *such that* $\{y\} \succ \{x\}$, *then* $A \succ B$.

In the standard model of choice over menus, a version of Preference for Quality without the restriction to sets of equal variety is imposed. Indeed, in the standard model, *the only way* in which the value of a menu can be affected is by adding a most preferred good. Here, we leave open the possibility that a menu's variety matters for DM's assessment of it.

The following axiom plays a dual role. First, it says that variety is utility-enhancing (utility-reducing if added variety is composed of undesirable goods). Second, it says that the value of additional variety is independent of the elements of the menu A, and preserves the order which $\succcurlyeq$ imposes on singleton menus (Figure 3.3).

Axiom 5 (Preference for Variety) *There is a good* $n \in Z$ *such that, for all menus* A *and lotteries* x, y *satisfying* $\{z\} \succeq \{x\}$ *and* $\{z\} \succeq \{y\}$ *for some* $z \in A$, $A \cup \{x\} \succ A \cup \{y\}$ *if and only if* $\{v^n(x,A)\} \succ \{v^n(y,A)\}$.

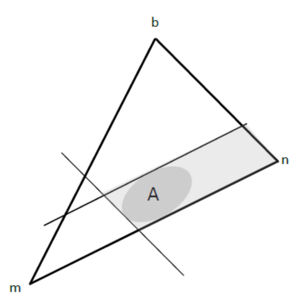

Figure 3.1: The shaded area includes all the lotteries that do not add variety to set A. For any x that belongs to the shaded area $V(A) = V(A \cup \{x\})$.

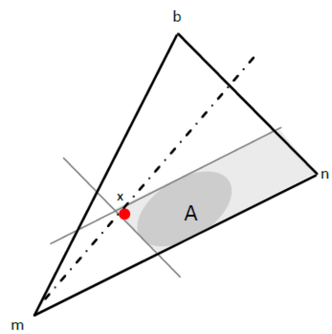

Figure 3.2: Axiom 4. Dotted lines indicate indifference curves for DM's preferences over singleton sets. The lottery x does not add variety to set A. However, lottery x adds consumption value so that $A \cup \{x\} \succ A$.

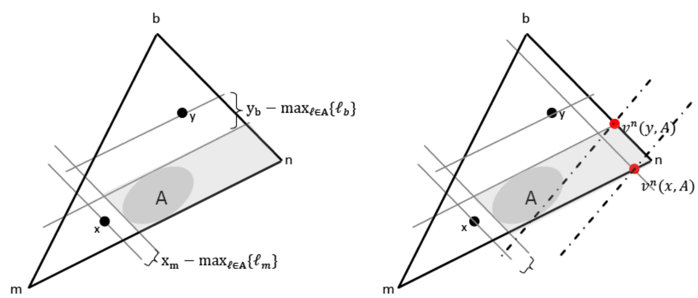

Figure 3.3: Axiom 5 - In order to evaluate whether adding lottery y to the set A is preferred to adding lottery x, DM compares the n-varieties added by each lottery according to DM's preferences over singleton menus.

Note that, for any menu A containing at least one element x which is preferred to n, $v^n(n,A) = v^n(x,A)$ and therefore $A \cup \{n\} \sim A$. Thus, Axiom 5 implies that the set of prizes Z includes an element which DM neither likes nor dislikes. It is natural to think of the good n as the option of choosing nothing. If we follow this interpretation, the implication of the axiom is that, as long as there is something desirable in a menu, adding the ability to choose nothing does not change the experience of choosing from it. The good n provides us with a natural candidate for zero utility, as well as playing an important role in DM's comparisons across choice sets.

Remark 1 *Axiom 5 has the following implications:*
i) ***Neutral Element:*** *There is a good $n \in Z$ such that, for all A where $\{z\} \succeq \{n\}$ for some $z \in A$, $A \cup \{n\} \sim A$.*

ii) ***Positive Preference for Variety:*** *For any x such that $\{v^n(x,A)\} \succ \{n\}$, $A \cup \{x\} \succ A$.*
iii) ***Negative Preference for Variety:*** *For any x such that $\{z\} \succeq \{x\}$ for some $z \in A$ and $\{n\} \succ \{v^n(x,A)\}$, $A \succ A \cup \{x\}$.*

It is instructive to note that Axioms 4 and 5 are distinct from other prominent axioms in the literature. In particular, they do not imply Monotonicity (Kreps, 1979): $A \cup \{x\} \succeq A$. Instead, adding a good to a menu which is less preferred than the neutral good n will make the menu less attractive. Similarly, our axioms do not imply Set Betweenness (Gul and Pesendorfer, 2001): $A \preceq A \cup B \preceq B$. If the menus A and B contain only goods which are strictly preferred to n, and $A \cap B = \varnothing$, then $A \prec A \cup B$ and $B \prec A \cup B$. In fact, Set Betweenness is satisfied only for cases in which $A \preceq \{n\} \preceq B$.

Finally, there is an additional requirement that we must make of preferences.

Axiom 6 (Non-Triviality) *There are goods $a, b, c \in Z$ such that $\{a\} \succeq \{b\} \succ \{c\}$.*

Axiom 6 differs from other non-triviality axioms in the literature (see Dekel et al., 2001) by requiring that *two* goods be strictly preferred to a third, rather than just one good being strictly preferred to another. This is a very weak requirement of preferences, but it is critical for our analysis because it makes it possible for a menu to have higher quality than another without having different variety. That is, it makes Axiom 4 meaningful. Consider the menu $A = \left\{\frac{1}{2}a + \frac{1}{2}c, \frac{1}{2}b + \frac{1}{2}c\right\}$. If we add the lottery $\frac{1}{2}a + \frac{1}{2}b$, we have that $V(A) = V(A \cup \{\frac{1}{2}a + \frac{1}{2}b\})$ but, using Independence (Axiom 3), $\{\frac{1}{2}a + \frac{1}{2}b\} \succ z$ for $z \in A$. We use this construction repeatedly in the proof of our representation theorem.

We are now ready to state our main result. Our representation is the following:

Definition 3 *A Preference for Variety representation is a utility function $U : \mathscr{A} \to \mathbb{R}$ and a constant $\beta \in (0,1)$ such that $U(\{n\}) = 0$ for some $n \in Z$ and:*

$$U(A) = (1-\beta)\sum_Z U(\{a\}) \max_{x \in A} x_a + \beta \max_{y \in A} \sum_Z y_b U(\{b\})$$

represents $\succcurlyeq$.

The representation captures our notion of preference for variety quite directly. The first component in the utility function echoes Axiom 5; DM enjoys each good in proportion to its consumption value and the highest probability with which it is available in the choice set A. The second component reflects a preference for quality as extra weight is given to the best available item in A. Our interpretation is that, once DM has chosen a menu, she will use the restriction of U to singletons to make her consumption decision. The parameter β measures the relative importance of the two components of utility. We allow the utility function U to take negative values for undesirable goods (i.e. bads).

The recursive formulation of Preference for Variety emphasizes the fact that choice sets are evaluated according to the utility that their components would provide if offered individually. Equivalently, the representation can be written as:

$$U(A) = (1-\beta)\sum_{Z} u(a)\max_{x\in A} x_a + \beta \max_{y\in A} u(y)$$

where u is an expected utility function defined on $\Delta(Z)$ and $u(y) = U(\{y\})$. This formulation makes it clear that Preference for Variety is a special case of a finite additive expected utility representation (Dekel et al., 2009). It is only in the interpretation, and in our (here unmodeled) predictions about choice from the set that our representation differs. Furthermore, Preference for Variety is consistent with Preference for Flexibility (Kreps, 1979) if and only if $U(\{a\}) \geq U(\{n\}) \geq 0$ for every $a \in Z$. It is the possibility that some goods are undesirable ($U(\{a\}) < 0$) that distinguishes our representation.

Theorem 1 *The binary relation $\succcurlyeq$ has a Preference for Variety representation if and only if it satisfies Weak Order, Continuity, Independence, Preference for Quality, Preference for Variety, and Non-Triviality (Axioms 1- 6).*
If $\succeq$ has a Preference for Variety representation (U,β), it is also represented by (U',β') if and only if $U' = \alpha U$ for some $\alpha > 0$ and $\beta' = \beta$.

Proof. In Appendix A. ■

We prove Theorem 1 in two steps. First, we show that Axiom 5 implies that every menu has a finite subset which is indifferent to it. As Dekel et al. (2009) have shown, this together with Axioms 1-3 is sufficient for a finite additive expected utility representation. The problem is then to determine the nature of the expected utility functions which make up this representation. Axiom 5 implies that there must be components which are sensitive only to changes in the variety of each good in Z. Having established this, we show that Axioms 4 and 6 imply that a component reflecting preferences over singleton menus must be included, and that no other component is consistent with our axioms.

4 Two Applications to Retailing

In this section, we propose a simple model of retailing and present two examples in which endowing consumers with a preference for variety leads to novel insights. If consumers exhibit a preference for variety, retailers may try to attract customers by offering a wide assortment of products rather than by charging low prices. In turn, the price-variety combination offered will change along with the number of retailers in the market, the cost of stocking goods, the number and types of consumers, and other factors.

A store is a profit maximizing agent that chooses an assortment A and a vector of prices p, and who incurs costs of providing assortment A, $c(A)$. We assume that costs are linear in the size of the assortment $c(A) = \kappa |A|$ and the set of available goods (Z) is large enough for stores never to be constrained in their ability to offer variety.

The profits of the store are:

$$\Pi(A, p) = \sum_{a \in A} p_a Q(a|A) - c(A) \tag{1}$$

where p_a is the price of product a and $Q(a|A)$ is the demand for a given that set A is offered.

There is a set of identical consumers of measure one who are interested in purchasing only one good. For each consumer who goes to the retailer in question, demand will be one if the expected utility of consuming the good minus its price p_a is greater than this difference for any other product in the store. If some good is dominated**,** the demand for that product will be zero. Note also that we must take a stand here about whether prices matter for consumers' evaluation of choice sets. In the spirit of the Williams Sonoma episode we discuss in Section 2, we take the position that they do.[7] Thus, we are assuming that a "good" in this setting is a good-price pair[8] and $U(\{a, p_a\}) = U(\{a, 0\}) - p_a = U(\{a\}) - p_a$.

The utility of going to a store that offers set A to the consumer is:

$$U(A) = (1-\beta) \sum_{a \in A} (U(\{a\}) - p_a) + \beta \max_{b \in A} [U(\{b\}) - p_b] \tag{2}$$

[7] This is also makes our model more tractable. We see no reason why taking the opposite stance would qualitatively change our results.

[8] Formally, the space of goods is now $Z \times \mathbb{R}$. While our results only hold for a finite space of goods, as would be generated if we considered bounded prices denominated in cents, we take the liberty of using a continuous approximation.

Retailers will never stock undesirable goods since they are costly and do not give the store any competitive advantage.

This specification allows us to make several simplifications. First, given that the carrying cost c is the same for all goods, retailers will always offer the best feasible good: $b \equiv \arg\max_{a \in Z} U(\{a\})$. When a retailer adds a product to its lineup in order to increase its variety it does not intend to sell it. Therefore, because consumers' enjoyment is proportional to how much utility the consumption of good a would provide to the consumer, the retailer will price the lure so that $U(\{a\}) - p_a = U(\{b\}) - p_b$. We write $u \equiv U(\{b\})$ and $p \equiv p_b$.

With this price scheme, the consumer will still consume the best option available b, and each lure added to a retailer's stock will have the same effect upon consumer utility. This insight allows us to simplify our analysis in two ways. First, it is without loss of generality that we assume that goods are added to a retailer's inventory in decreasing order of their value to consumers. Thus, taking z to be the last unit of variety added, $U(z)$ is decreasing.[9] Second, we summarize variety as a single decision variable $x \in \mathbb{R}_+$. One may interpret small changes in variety as increases in the probability that a new good is offered. We subsume the term $(1-\beta)$ into x and adjust the carrying cost κ accordingly. Taking all of this into account, we write the consumer's utility from going to a retailer offering an assortment with variety x and price p as:

$$U(x,p) = (x+\beta)(u-p) \tag{3}$$

Before moving on, we should distinguish the provision of variety via lures in our model from two related yet distinct retailing strategies. Our lures are similar to loss leaders (Lal and Matutes, 1994) in that they do not add to profit through sales; instead, they are used to get consumers in the door. However, loss leaders are meant to be sold, and their attractiveness to consumers stems from their low price. Lures, on the other hand, are stocked to be contemplated and to enhance the shopping experience, but are not meant to be sold.[10] Attention grabbers (Eliaz and Spiegler, 2011) are not meant to be consumed; instead they get consumers in the door by attracting their attention. The mechanism which we focus on is clearly different from this, as consumers in our model consider all available menus. Furthermore, our model predicts that stocking items which have high consumption

[9]We abuse notation in two ways here: we write U as a function of the continuous variable z, and we omit the brackets used to specify a menu.

[10]In Section 4.2 we present an example in which a small volume of lures is sold to consumers with eccentric tastes.

value for shoppers, net of prices, will be more effective at attracting them to a store than stocking items with low consumption value. The effectiveness of attention grabbers, on the other hand, need not have any relation to their consumption value.

4.1 A Spatial Model of Retailing

A unit mass of consumers with a preference for variety are uniformly distributed over a circle of circumference one. We refer to this circle as *the economy*. Consumers' preferences are quasilinear in money and transportation costs, which are t per unit of length (l) traveled. Their utility is:

$$U(x,p,l) = (x+\beta)(u-p) - tl \tag{4}$$

Retailers have a fixed cost $f > \beta\kappa > 0$ of entry. We assume that $u^2 > 4\kappa t$ and focus on symmetric equilibria in which retailers are evenly spaced on the economy and all offer the same variety-price combination.

Consider an economy in which there are $N-1$ retailers who offer consumers a shopping experience worth $\bar{U}$ net of transportation costs. If the $N'th$ retailer offers consumers utility U net of transportation costs, the number of consumer who shop there is:

$$d(u) = \frac{U-\bar{U}}{t} + \frac{1}{N} \tag{5}$$

In order to provide utility level U, the retailer solves:

$$\max_{p,x} dp - x\kappa - f \tag{6}$$

$$\text{such that } (x+\beta)(u-p) = U \tag{7}$$

The equilibrium values are:

$$\begin{aligned}
U &= \frac{1}{2N\kappa}\left(u\sqrt{-4\kappa t + u^2} - 2\kappa t + u^2\right); \\
p &= u - \frac{1}{2}\sqrt{2}\sqrt{u\sqrt{u^2-4\kappa t} - 2\kappa t + u^2}; \\
x &= \frac{1}{2\sqrt{\kappa}}\left(\sqrt{2}\sqrt{\frac{1}{N^2\kappa}\left(u\sqrt{u^2-4\kappa t} - 2\kappa t + u^2\right)} - 2\sqrt{\kappa}\beta\right).
\end{aligned}$$

Note that the expression for x is decreasing in N. Therefore, parameters which lead to retailers having large market share (low N) also lead to retailers offering high variety (high x). This provides a preference-based explanation of the positive correlation between product line length and market share as noted in Draganska and Jain (2005), Kekre and Srinivasan (1990), and Bayus and Putsis (1999).

We define consumer welfare to be the average of the net benefit of shopping for our consumers:

$$W = U - \frac{t}{4N} \tag{8}$$

We summarize the comparative statics of the model in the following Proposition:

Proposition 1 *In the symmetric equilibrium of the spatial retailing model:*

1. *Greater preference for variety (smaller β) implies:*
   (a) *fewer retailers (N),*
   (b) *no change in price (p),*
   (c) *more variety (x),*
   (d) *higher consumer welfare (W).*
2. *An increase in transportation costs (t) leads to*
   (a) *fewer retailers (N),*
   (b) *higher price (p),*
   (c) *and higher variety provided (x).*
3. ***Lower fixed costs (f) lead to:***
   (a) *more retailers (N),*
   (b) ***lower consumer welfare (W).***
4. *There exists an $\bar{f} > 0$ such that for all $f < \bar{f}$ no variety is provided ($x = 0$).*

Proof. In Appendix B.1. ■

The first result is intuitive: if consumers have a stronger preference for variety, the variety provided by retailers will be greater. Providing more variety increases costs for retailers, and therefore there will be fewer retailers in the market. In order to analyze the welfare effects of a decrease in β, we first note that transportation costs will increase. However, the presence of fewer retailers in the market means that increases in variety will attract more customers than they would have at a higher β and thus retailers will provide more of it. The net effect is that consumer welfare increases.

When transportation costs are high there will be fewer retailers because, even though they can charge a higher price, they also have to offer more variety in order to compensate consumers. In equilibrium the increase in costs due to more variety is greater than the increase in revenue due to the price increase. The effects of an increase in transportation costs can be understood in light of recent parallel trends in retailing and suburbanization. If we think of people's move to the suburbs as an exogenous positive shock to transportation costs, our model correctly predicts that fewer retailers, each offering a wider selection of products, will serve the market.

The third result is the most intriguing. Small fixed costs facilitate the entry of retailers leading to an increase in the number of retailers and a decrease in transportation costs. However, when N is large, the competition generated by more retailers harms consumers by decreasing the variety provided. The net effect is a decrease in consumer welfare. The result is reminiscent of (but distinct from) the "excessive entry" result in Mankiw and Whinston (1986) where firms entering the market lower social welfare by lowering output per firm making net profits lower. In contrast, our result implies that consumer (not necessarily social) welfare is decreasing with firm entry.

4.2 Differing Tastes

Thus far, our analysis has simplified reality by assuming that consumers agree on their ranking of different goods. In this section, we look at the implications of having consumers whose preferences differ from the bulk of the population. We posit a stylized model in which there is a large group of consumers with *mainstream* tastes, and a small group with heterogeneous or *eccentric* preferences.

Mainstream consumers have a preference for variety, so they will evaluate assortments according to the utility function 3. On the other hand, eccentric consumers are interested only in one particular good, which may be different from mainstream consumers' most preferred good, and have no preference for variety.

We assume that, for every good in Z, there is a group of eccentric consumers that value it the most. Alternatively, one may interpret our model as one in which every good in Z will be matched to a group of eccentric consumers with a certain probability.

The central insight provided by this extension is that there is a symbiotic relationship between consumers with mainstream tastes and a preference for variety and consumers with eccentric tastes. Because carrying costs may outstrip expected revenues from carrying eccentric goods given small market demand, retailers may not carry them. However, if mainstream consumers have a preference for variety, offering eccentric goods at a good price becomes part of stores' optimal marketing strategy. Therefore, eccentric consumers who would not have had the opportunity to buy their preferred good can now find it at a mainstream retailer. On the other hand, eccentric consumers lower the carrying costs of eccentric goods by purchasing them, whereas mainstream consumers would leave them on the shelves. Thus, retailers will be willing to carry a greater variety of goods than they would if there were no eccentric consumers, increasing the variety component of mainstream consumers' utility.

We adjust our model as follows. Let there be two retailers, a set of mainstream consumers of measure m, and a set of eccentric consumers Υ of size $e \geq 0$. In Υ, there is a group of consumers of density[11] e who enjoy only good $g \in \mathbb{R}_+$. These consumers, who we index by their most preferred good, have utility functions:

$$U^g(A) = \begin{cases} u^e - p_g & \text{if } g \in A. \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

Thus, for each good offered by the retailer (each unit of variety x), there is a group of eccentric consumers of density e who will buy it. Note that the baseline model presented at the beginning of this section is a special case of this section's model in which $m = 1$ and $e = 0$.

We make two assumptions on the number of eccentric consumers. First, $e < \frac{\kappa}{u^e}$ ensures that it is not profitable for a retailer to serve only eccentric consumers. Second, $m \geq e|Z|$ states that mainstream consumers outnumber eccentric consumers. We also assume that eccentric consumers value the eccentric good at least as much as mainstream consumers do $u^e \geq u$, so that the inequality above implies $u < \frac{\kappa}{e}$. Therefore, when a retailer offers good g, it pays the corresponding inventory cost κ but will also collect revenue $p_g e$, where p_g is the price at which

[11] Equivalently, if we consider $e \in [0,1]$, we may interpret e as the probability that a lure will sell.

good g is sold. We also make the following assumption on the quality of available lures: $u - U(\{z\}) < \frac{\kappa}{m}x$ for every $z \leq x$. This condition guarantees that lures will be offered at positive prices. In this model, the carrying costs of stocking a good are independent of the number of customers who will buy it. The key condition for our results is that inventory costs per customer served are decreasing in the number of customers served.

Because a retailer cannot be profitable without attracting mainstream consumers, pricing and inventory decisions will be driven by the need to attract them.

Lemma 1 *i) Retailers will offer the price-variety combination which most effectively attracts mainstream consumers.*

ii) Lures will be priced so as to make mainstream consumers indifferent between all goods.

Proof. Proof is in Appendix B.2. ■

The following results make the idea of a symbiotic relationship between mainstream and eccentric consumers concrete. We begin with a result on the benefits of eccentric consumers for their mainstream counterparts. The intuition behind the result is simple: eccentric consumers subsidize the carrying costs of goods that the mainstream consumers like to have in their choice set but will not buy. Therefore, the variety offered by retailers is strictly increasing in the number of eccentric consumers. It is important to note that different from other models where the variety provided is determined by the taste distribution and the production function of firms (Anderson et al., 1995), in our setting the decision of firms to provide variety depends only on the preferences of the mainstream group while the decision of how much variety to provide takes into account the distribution of tastes of all the groups.

Proposition 2 *The utility of mainstream consumers is increasing in the number of eccentric consumers (e).*

Proof. Proof is in Appendix B.2. ■

Before proceeding to the next result, it is useful to define a notion of welfare for eccentric consumers:

$$W^e = \int_{\Upsilon} U^g dg \tag{10}$$

Clearly, W^e is strictly increasing in the variety offered, as U^g is strictly positive when g is offered by the retailer and zero otherwise. Increasing the number of mainstream consumers increases the retailer's revenues without affecting carrying costs for eccentric goods. Competition makes retailers spend this extra revenue on increased variety.

Proposition 3 *The welfare of eccentric consumers is increasing in the number of mainstream consumers (m) and in their preference for variety (decreasing in β).*

Proof. Proof is in Appendix B.2. ■

5 Conclusion

Introspection suggests that we derive pleasure from contemplating the consumption options available to us. When we go to the mall, we spend hours browsing before buying that one pair of jeans that motivated us to go in the first place. When standing at the deli counter at a supermarket, we linger and silently mouth the names of exotic imported cheeses before buying the block of cheddar we need for dinner. This pleasurable contemplation of options plays a part in our decision-making: sometimes we choose to go to the mall rather than the standalone store because we correctly anticipate that we will enjoy window-shopping. It has been the goal of this paper to formalize a notion of variety, provide a utility representation which incorporates its enjoyment, and highlight the importance of taking preference for variety into account in Economic models.

While one cannot distinguish preference for variety from uncertainty over future preferences (Dekel et al. 2001) by observing choices *over* menus, the two models make different predictions of choices *from* menus. That is, either interpretation can rationalize the same consumer's choice of retailer. Yet, preference for variety implies that the consumer's purchases at the retailer are not ex-ante uncertain. Only our model can explain the presence of lures – goods which are stocked but seldom or never purchased. Our interpretation of the preference for variety representation asserts that it may be that the different utility functions in a subjective state-space representation do not, in fact, reflect uncertainty about the decision maker's future preferences, but instead different ways in which alternatives in a menu affect the experience of choosing from it.

Our definition of variety emphasizes that it is new goods or product attributes, rather than repetitive versions of the same product, which are important to decision makers. Following evidence from a wide range of sources, we axiomatize the notion that goods which are meant for contemplation are evaluated in a manner consistent with preferences over consumption. We then show that any preference relation which satisfies our axioms has a preference for variety representation.

In an application of our utility representation to a model of retailing, we provide two examples of how taking preference for variety into account can lead to surprising conclusions. First, in a spatial model of retailing, we show that making the retailing market more competitive (decreasing the fixed costs of entry) can be welfare-reducing. Second, in a version of the model with consumers with heterogeneous tastes we find that as long as there is a group of consumers with preference for variety all groups will benefit from each other's presences.

Moving forward, we must keep in mind that preference for variety is only part of the picture, applicable more in some circumstances than others. Standing in counterpoint is evidence that processing information is costly; an abundance of options can generate an amount of information that a decision maker cannot process, causing her to make worse decisions. Future research should attempt to provide a unified framework in which preference for variety and sensory overload coexist.

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A Appendix: Proof of Representation Theorem

We begin with some preliminary definitions and results which lead to the intermediate representation result from Dekel et al. (2009). We then go on to build on this to reach our preference for variety representation by appealing to Axioms 5 and 4.

Definition 4 *$A' \subseteq conv(A)$ is critical for A if for all B with $A' \subseteq conv(B) \subseteq conv(A)$, we have $B \sim A$.*

Axiom 7 (Finiteness) *Every menu A has a finite critical subset.*

Together, these axioms lead to the following result:

Theorem 2 *The preference relation $\succ$ has a finite additive expected utility representation if and only if it satisfies Weak Order, Continuity, Independence, and Finiteness.*

The previous result guarantees that preferences can be represented by a utility function of the form:

$$U(A) = \sum_{S_1} \max_{x \in A} u(x,s) - \sum_{S_2} \max_{y \in A} u(y,s) \tag{11}$$

Where $S \equiv S_1 \cup S_2$ are subjective states of the world, each associated with an expected utility function $u(x,s) = \sum x_a u(a,s)$. Note that this implies that is an expected utility function over singleton menus: $U(\{x\}) = \sum_Z x_a \left(\sum_{S_1} u(a,s) - \sum_{S_2} u(a,s) \right)$. States in S_2 are "negative states" in which DM suffers rather than enjoys his choices. Our preference for variety representation adds structure by replacing the Finiteness axiom with two axioms which serve to pin down the ‘state-dependent utility functions’ in a way which is consistent with the evidence cited in Section 2.

Lemma 2 *Axioms 1, 3, and 5 imply Finiteness.*

Proof. Step 1: Axioms 3 and 5 imply $A \sim conv(A)$:

Let $w \in conv(A) \backslash A$. Then, $V(A \cup \{w\}) = V(A)$. By Axiom 5, the only way that $A \nsim A \cup \{w\}$ is if $\{w\} \succ \{z\}$ for all $z \in A$. We rule this out as follows:

$w=\sum_{i=1}^{n}\alpha_i x_i$ for $x_i\in A$, $\alpha_i\geq 0$ and $\sum_{i=1}^{n}\alpha_i=1$. By Carathéodory's theorem, we may take $n\leq |Z|+1$.

Let $w_1=x_1$ and $w_{j+1}=\frac{\sum_{i=1}^{j}\alpha_i}{\sum_{i=1}^{j+1}\alpha_i}w_j+\frac{\alpha_{j+1}}{\sum_{i=1}^{j+1}\alpha_i}x_{j+1}=\sum_{i=1}^{j+1}\frac{\alpha_i}{\sum_{i=1}^{j+1}\alpha_i}x_i$. Note that $w_n=w$.

By Axiom 1, either $\{w_j\}\succeq\{x_{j+1}\}$ or $\{w_j\}\preceq\{x_{j+1}\}$. Consider the following algorithm:

1. $\hat{w}_1=x_1$
2. If $w_j\succeq x_{j+1}$, then $\hat{w}_{j+1}=\hat{w}_j$.
3. If $w_j\preceq x_{j+1}$, then $\hat{w}_{j+1}=x_{j+1}$.

For every j, $\hat{w}_j\in A$. By Axiom 3, $\{w_j\}\succeq\{x_{j+1}\}$ implies $\{w_j\}\succeq\{w_{j+1}\}$ and $\{x_{j+1}\}\succeq\{w_j\}$ implies $\{x_{j+1}\}\succeq\{w_{j+1}\}$. By transitivity, $\{\hat{w}_{j+1}\}\succeq\{w_{j+1}\}$. Therefore, for any lottery $w\in conv(A)$, there is a lottery in $x\in A$ such that $\{x\}\succeq\{w\}$. We conclude that, $conv(A)=A\cup(conv(A)\backslash A)\sim A$.

Step 2: There is a finite set $\mathscr{F}_A\subseteq A$ such that $\mathscr{F}_A\sim A$.

For $a\in Z$ and $A\in\mathscr{A}$, define the set $\mathscr{L}_a(A)=\{x\in A\ s.t.\ x\in\arg\max_{x\in A}\{x_a\}\}$. $\max_{x\in A}\{x_a\}$ is well-defined because all menus are defined to be a closed sets. Let $\mathscr{L}_q(A)=\{x\in A\ s.t\ \{x\}\succeq\{y\}$ for every $y\in A\}$. For each $i\in Z\cup q$, choose one lottery $\hat{x}^i\in\mathscr{L}_i(A)$. Define $\mathscr{F}_A=\cup_{i\in Z\cup q}\hat{x}^i$. $\mathscr{F}_A\subseteq A\subseteq conv(A)$ is a finite set.

We have constructed $\mathscr{F}_A$ in such a way as to guarantee that $V(A\cup\{w\})=V(A)$ for all $w\in A\backslash\mathscr{F}_A$, and $\{\hat{x}^q\}\succeq\{y\}$ for every $y\in conv(A)$. By Axiom 5, $\mathscr{F}_A\sim A$.

Step 3: $\mathscr{F}_A$ is critical for A.

Consider a set B such that $\mathscr{F}_A\subseteq conv(B)\subseteq conv(A)$. Because $conv(B)\subseteq conv(A)$, we can take $\mathscr{F}_{conv(B)}=\mathscr{F}_A$. Thus, $\mathscr{F}_A\sim conv(B)\sim conv(A)\sim A$. By Step 1, $conv(B)\sim B$. Using transitivity again, $B\sim A$. We conclude that $\mathscr{F}_A$ is critical for A. ■

Before moving forward, note how the way we constructed the finite critical subset in the above proof shows that Axiom 5 restricts the number of ways DM's utility can change. We use the same logic in the proof of our representation theorem to identify all relevant subjective states.

It is straight-forward to verify that a utility function with a preference for variety representation satisfies Axioms 1-5. In what follows, we verify that any preference relation satisfying these axioms has a preference for variety representation.

Step 1: Finite Additive Expected Utility

By Theorem 2 and Lemma 2, there exists a finite additive EU representation:

$$U(A) = \sum_{S_1} \max_{x \in A} u(x,s) - \sum_{S_2} \max_{x \in A} u(x,s)$$

In what follows, we work with a given finite expected utility representation. Note that we need only consider expected utility functions for which $u(a,s) > u(b,s)$ for some $a, b \in Z$. Otherwise, the function is a constant and may be eliminated by a normalization of U.

Step 2:

As we argue in the text, by Axiom 5, there is a $n \in Z$ such that $\{n\} \cup A \sim A$ for all A containing at least one lottery z such that $\{z\} \succeq \{n\}$. Let $N \subset Z$ denote the set of all such neutral goods.

Lemma 3 *For every $i \in Z \backslash N$ there is an $s \in S$ such that $u(i,s) > u(a,s) = u(b,s)$ for all $a, b \neq i$.*

Proof. Let $i \in Z \backslash N$ be a good such that there is no $s \in S$ for which

$$u(a,s) = \begin{cases} > k & \text{if } a = i \\ = k & \text{otherwise} \end{cases}$$

For each s, choose $b^s \in \arg\max_{a \in Z \backslash i} u(a,s)$ and $c^s \in \arg\min_{a \in Z \backslash i} u(a,s)$. Then, for small $\lambda > 0$, define $z^s = \lambda i + (1-\lambda) c^s$.

Consider the set $A = \{b^s, c^s\}_{s \in S}$. Note that $V_i(A) = 0$. Choose one z^s and consider $U(A \cup \{z^s\})$. For any $s' \in S$, we have that

$$u\left(x^{s'}, s'\right) - u(z^s, s') = (1-\lambda)\left(u\left(b^{s'}, s'\right) - u(c^s, s')\right) - \lambda u(i, s').$$

By construction, $u\left(b^{s'}, s'\right) - u(c^s, s') > 0$. Therefore, for small enough λ, $u\left(b^{s'}, s'\right) > u(z^s, s')$, implying that $U(A) = U(A \cup \{z^s\})$.

However, z^s adds variety to A: $\{v^n(z^s, A)\} \nsim \{n\}$. By Axiom 5, $A \cup \{z^s\} \nsim A$. This contradicts the premise that U represents the preference relation in question. ■

Given this lemma, we know that there are at least $|Z \backslash N|$ subjective states. The following lemma verifies that this is the maximum number of states in which $u(a,s)$ takes on a higher value for a single good, except in special circumstances.

Step 3:

Lemma 4 *There are at least $|Z \backslash N| + 1$ subjective states, $|S| \geq |Z \backslash N| + 1$.*

Proof. By Axiom 6, there are three goods such that $\{a\} \succeq \{b\} \succ \{c\}$. Suppose that $|S| = |Z \backslash N|$ so that each u is responsive to only one good. Consider the menu $A = \{\frac{1}{2}a + \frac{1}{2}c, \frac{1}{2}b + \frac{1}{2}c\}$ and the lottery $x = \frac{1}{2}a + \frac{1}{2}b$. Then, $\max_{y \in A \cup \{x\}} u(y,s) = \max_{y \in A} u(y,s)$ for all $s \in S$. However, by Axiom 3, $\{\frac{1}{2}a + \frac{1}{2}b\} \succ \{\frac{1}{2}a + \frac{1}{2}c\} \succeq \{\frac{1}{2}b + \frac{1}{2}c\}$. Furthermore, Axiom 4 implies $U(A \cup \{x\}) > U(A)$. Therefore, there must be at least one additional subjective state. ■

Step 4:

Lemma 5 *If* $u(a,s) \neq \begin{cases} > k & \text{if } a = i \\ = k & \text{otherwise} \end{cases}$ *for some* $i \in Z \backslash N$*, then* $u(\cdot,s)$ *is an affine transformation of* U.

Proof. There are two cases we must deal with. First, consider a state in which we can find three goods such that $u(a,s) \geq u(b,s) > u(c,s)$. If $u(\cdot,s)$ is not an affine transformation of U, there are lotteries $y = \lambda a + (1-\lambda)b$ and $x \in \Delta Z$ such that $U(\{x\}) = U(\{y\})$ but $u(y,s) > u(x,s)$. Consider the menu $A = \{x, \lambda a + (1-\lambda)c, \lambda c + (1-\lambda)b\}$. Note that $\{x\} \succeq \{y\}$ and $V(A) = V(A \cup \{y\})$. By Axiom 5, $A \sim A \cup \{y\}$. However, $u(y,s) > u(z,s)$ for all $z \in A$ so that $U(A) \neq U(A \cup \{y\})$, contradicting our hypothesis that U represents the preference relation in question.

The second case is a state in which $u(a,s) = \begin{cases} > k & \text{if } a = n \\ = k & \text{otherwise} \end{cases}$ for some $n \in N$ and $k \in \mathbb{R}$. By Axiom 6, there is a good $\{a\} \succeq \{n\}$. By Axiom 5, $\{a\} \cup \{n\} \sim \{a\}$. However, $u(n,s) > u(a,s)$ so $U(A) \neq U(A \cup \{n\})$. Again, this contradicts our hypothesis that U represents the preference relation in question. ■

Step 5: Pinning down U

The preceding lemmas lead us to the conclusion that preferences satisfying Axioms 1-5 are represented by a utility function of the form:

$$U(A) = \max_{x \in A} \{\alpha + \beta U(\{x\})\} + \sum_{i \in Z} \left(k_i \max_{y \in A} y_i + k_i'(1 - \max_{y \in A} y_i) \right)$$

where $\alpha, \beta > 0$ and k_i and k_i' are constants of either sign. Denoting the constant $\gamma = \alpha + \sum_{i \in Z} k_i'$:

$$U(A) = \gamma + \beta \max_{x \in A} U(\{x\}) + \sum_{i \in Z} \left(k_i - k_i'\right) \max_{y \in A} y_i$$

Adding or subtracting a constant does not change the ordering of menus, so that we may choose $\gamma = 0$. Restricting attention to singleton menus, for each $a \in Z \backslash N$:

$$k_a - k'_a = (1-\beta)U(\{a\})$$

Furthermore, for $n \in N$:

$$U(\{n\}) = \beta U(\{n\})$$

from which we conclude that $U(\{n\}) = 0$. Note that $\beta \neq 1$ because that would imply $U(A) = \max_{x \in A} U(\{x\})$ which is ruled out by Axiom 5. Having $U(\{n\}) = 0$ allows us to include neutral goods in the summation above.

In fact, $\beta \in (0,1)$. We know that $\beta > 0$ because $\alpha + \beta U$ is an affine transformation of U. To see that $\beta < 1$, consider $U(\{a\} \cup \{b\}) - U(\{a\}) = (1-\beta)U(\{b\})$. By Axiom 5, $\{a\} \cup \{b\} \succ \{a\}$ if $\{b\} \succ \{n\}$, or equivalently $U(\{b\}) > 0$. Thus, $\beta < 1$.

β is **uniquely determined**. Take three goods such that $\{a, \lambda c + (1-\lambda)a\} \sim b$ for $\lambda \in (0,1]$ and $U(\{a\}) \geq U(\{c\}) \neq 0$. Then,

$$U(\{a\}) + (1-\beta)\lambda U(\{c\}) = U(\{b\})$$

implying that:

$$\beta = \frac{U(\{a\}) + U(\{c\}) - U(\{b\})}{\lambda U(\{c\})}$$

We conclude that any preference relation satisfying Axioms 1-5 has a Preference for Variety representation:

$$U(A) = (1-\beta)\sum_{Z} U(\{a\}) \max_{y \in A} y_a + \beta \max_{x \in A} U(\{x\})$$

B Appendix: Proofs Related to Applications to Retailing

B.1 A Spatial Model of Retailing

The retailers' interim maximization problem is:

$$\begin{aligned} & \max_{p,x} dp - \kappa x - f \\ s.t.\, U &= (x+\beta)(u-p) \end{aligned}$$

or substituting the constraint into the maximand:

$$\max_x d\left(u - \frac{U}{x+\beta}\right) - \kappa x - f$$

The first order condition of this problem is:

$$\begin{aligned}\frac{dU}{(x+\beta)^2} - \kappa &= 0 \\ &\Leftrightarrow \\ x &= \sqrt{\frac{dU}{\kappa}} - \beta\end{aligned}$$

The second order condition is clearly satisfied, so that this is indeed a unique global maximum. Substituting the optimal value of x into the expression for p we have:

$$p = u - \sqrt{\frac{U\kappa}{d}}$$

We may now write the retailers' problem in terms of U:

$$\max_U \Pi = \max_U d(U)\left(u - \frac{\sqrt{U\kappa}}{\sqrt{d(U)}}\right) - \kappa\left(\frac{\sqrt{Ud(U)}}{\sqrt{\kappa}} - \beta\right) - f$$

where $d(U) = \frac{U-\bar{U}}{t} + \frac{1}{N}$. We can rewrite this problem as:

$$\max_U d(U)u - 2\sqrt{d(U)U\kappa} + \kappa\beta - f$$

The problem's first order condition is:

$$d'(U)u - d'(U)\frac{\sqrt{U\kappa}}{\sqrt{d(U)}} - \frac{\sqrt{d(U)\kappa}}{\sqrt{U}} = 0$$

Note that $d'(U) = \frac{1}{t}$ and, when we use the equilibrium condition $U = \bar{U}$, $d(U) = \frac{1}{N}$. Therefore, the FOC simplifies to:

$$u\sqrt{U} - \sqrt{\kappa N}U - t\frac{\sqrt{\kappa}}{\sqrt{N}} = 0$$

Using the quadratic formula, we derive:

$$\sqrt{U} = \frac{u \pm \sqrt{u^2 - 4\kappa t}}{2\sqrt{\kappa N}}$$

By assumption, the term in the numerator's root is positive. We take the larger value for $\sqrt{u}$, which when squared yields:

$$U = \frac{u^2 - 2\kappa t + u\sqrt{u^2 - 4\kappa t}}{2\kappa N}$$

Substituting U in the price and variety, we get:

$p = u - \frac{1}{2}\sqrt{2}\sqrt{u\sqrt{u^2 - 4\kappa t} - 2\kappa t + u^2}$

$x = \frac{1}{2\sqrt{\kappa}}\left(\sqrt{2}\sqrt{\frac{1}{N^2\kappa}\left(u\sqrt{u^2 - 4\kappa t} - 2\kappa t + u^2\right)} - 2\sqrt{\kappa}\beta\right)$

The condition of zero profits (no entry) gives:

$N^* = \frac{1}{-\kappa\beta + f}\left(u - \sqrt{2}\sqrt{u\sqrt{-4\kappa t + u^2} - 2\kappa t + u^2}\right)$

Proof of Proposition 3

- Part 1

Greater preference for variety (smaller β) implies fewer retailers (N)

$N^* = \frac{1}{f - \kappa\beta}\left(u - \sqrt{2}\sqrt{u\sqrt{-4\kappa t + u^2} - 2\kappa t + u^2}\right)$

Variety is decreasing in β :

$x^* = \frac{1}{2\sqrt{\kappa}}\left(\sqrt{2}\sqrt{\frac{1}{N^2\kappa}\left(u\sqrt{u^2 - 4\kappa t} - 2\kappa t + u^2\right)} - 2\sqrt{\kappa}\beta\right)$

Price is independent of β and N :

$p^* = u - \frac{1}{2}\sqrt{2}\sqrt{u\sqrt{u^2 - 4\kappa t} - 2\kappa t + u^2}$

Welfare increases when β increases:

$W = U - \frac{t}{4N}$

We know that $U \geq \frac{t}{N}$ (this happens due to the fact that $u^2 \geq 4\kappa t$ and U is increasing in u^2). Therefore $\frac{\partial W}{\partial N} > 0$.

- Part 2

By inspection of the expressions for the equilibrium values of p, N, and x, we see that price and the number of firms are decreasing in costs of transportation. Variety is increasing in transportation costs.

- Part 3

$W = \frac{1}{N}(\frac{1}{2\kappa}u\sqrt{u^2-4\kappa t}-2\kappa t+u^2-\frac{t}{4})$.

Fixed costs (f) enters this expression only through N, which is decreasing in f. Because the term in parenthesis is positive, this implies that consumer welfare is increasing in f.

We now prove that there is an $\bar{f}>0$ such that for all $f<\bar{f}$ no variety is provided ($x=0$).

Note that:

$x^* = \max\{\frac{1}{2\sqrt{\kappa}}\left(\sqrt{2}\sqrt{\frac{1}{N^2\kappa}\left(u\sqrt{u^2-4\kappa t}-2\kappa t+u^2\right)}-2\sqrt{\kappa}\beta\right),0\}$

N grows unboundedly as f decreases to $2\kappa\beta$ and therefore the positive term goes to zero.

B.2 Differing Tastes

Lemma 6 *i) Retailers will offer the price-variety combination which most effectively attracts mainstream consumers.*

ii) Lures will be priced so as to make mainstream consumers indifferent between all goods.

Proof. The retailer's problem must now incorporate the effect of demand for eccentric goods into its objective function. Therefore, the retailer must consider the trade-offs between offering cheap lures and increasing revenues and variety by charging more for lures. For any given price of the mainstream consumption good p, and level of mainstream consumer utility $\bar{U}$, the retailer minimizes costs solving:

$$\begin{array}{rll} \min\limits_{x,p(z)} \kappa x - e\int_0^x p(z)dz & & \\ s.t & & \\ \int_0^x U(z)-p(z)dz & =\bar{U} & (1) \\ U(z)-(u-p) & \leq p(z) \leq U(z) & (2) \\ x & \geq 0 & (3) \end{array}$$

The first inequality in the second constraint guarantees that mainstream consumers prefer the mainstream good to the lure. The second inequality in the second constraint ensures that offering the lure adds to mainstream consumers' utility. Using the first constraint, we can rewrite the problem as:

$$\begin{aligned} \min_x \kappa x - e\left(\int_0^x U(z)dz - \bar{U}\right) & \\ s.t. & \\ U(z) - (u-p) &\leq p(z) \leq U(z) \\ x &\geq 0 \end{aligned}$$

Given that $U(z)$ is decreasing, the objective function is convex and first order conditions will characterize an interior solution where the constraint is not binding: $\kappa - eU(x) = 0 \Rightarrow x = U^{-1}\left(\frac{\kappa}{e}\right)$ with prices for each good z left indeterminate, subject only to the aggregate condition $\int_0^x p(z)dz = \int_0^x U(z)dz - \bar{U}$.

However, by assumption we have that $U(x) < \frac{\kappa}{e}$ so that the interior solution is not feasible. Rather, since U is decreasing, the optimal variety x is the lowest amount of variety which may be provided while satisfying the constraints on consumer utility and the lower bound on prices. In other words, the constraint $U(z) - (u-p) \leq p(z)$ is binding. Intuitively, the retailer loses money by offering more variety so that the optimal amount of variety for it to offer is the lowest amount at which the constraints are satisfied.

We can now incorporate the result above to the retailers' problem. A deviation by one retailer from the solution of the problem below will lead to losses since it must either involve lowering prices, which leads to losses given the zero-profit condition, or to losing mainstream consumers, which leads to losses since eccentric consumers are not profitable by assumption. Also, a retailer cannot attract all eccentric consumers and use them to expand variety, thus attracting mainstream consumers because lures are already priced as low as is feasible without making mainstream consumers switch to buying lures.

$$\begin{aligned} \max_x (x+\beta)(u-p) & \\ \kappa x - e\int_0^x p(z)dz &= mp \\ U(z) - (u-p) &= p(z) \end{aligned}$$

The first constraint is the zero profit condition implied by Bertrand competition. Substituting the second constraint into the first:

$mp = \kappa x - e\int_0^x U(z) - u + pdz = \kappa x - e\int_0^x U(z)dz + ex(u-p)$

$\Rightarrow p = \frac{\kappa x - e\int_0^x U(z)dz + exu}{m+ex}$

Substituting both constraints into the objective function:

$$\max_x (x+\beta)\left(u - \frac{\kappa x + e\left(xu - \int_0^x U(z)dz\right)}{m+ex}\right)$$

■

Proposition 4 *The utility of mainstream consumers is increasing in the number of eccentric consumers (e).*

Proof. Let $U(x(e);e)$ denote mainstream consumers' utility at the optimum level of variety. An application of the envelope theorem tells us that

$$\frac{dU(x(e);e)}{de} = \frac{\partial U(x(e);e)}{\partial x}x'(e) + \frac{\partial U(x(e);e)}{\partial e} = \frac{\partial U(x(e);e)}{\partial e}$$

Calculating this derivative explicitly:

$$\frac{\partial U(x(e);e)}{\partial e} = (x+\beta)\,\frac{\kappa x^2+\kappa x-mx\left(u-\frac{1}{x}\int_0^x U(z)dz\right)}{(m+ex)^2}$$

which is positive if:

$$\kappa x > m\left(u-\frac{1}{x}\int_0^x U(z)dz\right).$$

This inequality holds by our assumption on the quality of lures: $u-U(x) < \frac{\kappa}{m}x$.

The condition above can be given further interpretation through a bit of manipulation. The zero profit condition states that $\kappa x - e\int_0^x p(z)dz = mp$. We can use this expression to write a necessary condition for the inequality above:

$$\frac{1}{x}\int_0^x U(z)dz > u-p$$

This condition states that the average value to the mainstream consumer of the variety goods provided is greater than the value of purchasing the consumption good. This is a necessary condition for lures to be offered at positive prices. ■

Proposition 5 *If there are more mainstream than eccentric consumers in the market (m < ex), the welfare of eccentric consumers is increasing in the number of mainstream consumers (m) and in their preference for variety (decreasing in β).*

Proof. The total utility of eccentric consumers is strictly increasing in the number of eccentric consumers which are able to purchase their most preferred good. Thus, an increase in x corresponds to an increase in the utility of eccentric consumers. Let $x^*(m,\beta)$ denote the argmax of the retailer's problem above as a function of our parameter of interest. To show that $x^*(m,\beta)$ is increasing in m and decreasing in β, we apply monotone comparative statics (Milgrom and Shannon, 1994): if $\frac{\partial U(x;m)}{\partial m\partial x} > 0$ then $\frac{\partial x^*(m,\beta)}{\partial m} > 0$.

$$\frac{\partial U(x;m)}{\partial m} = (x+\beta)\,\frac{\kappa x+e(xu-\int_0^x U(z)dz)}{(m+ex)^2}.$$

$$\frac{\partial U(x;m)}{\partial m\partial x} = \frac{\kappa x+e(xu-\int_0^x U(z)dz)}{(m+ex)^2} + (x+\beta)\left(\frac{\kappa(m-ex)+e(m+ex)(u-U(x))-2e^2x\left(u-\frac{1}{x}\int_0^x U(z)dz\right)}{(m+ex)^3}\right) > 0.$$

The first term is clearly positive, while the second is positive as long as $m \geq ex$, which is true by assumption since $x \leq |Z|$.

Similarly,

$\frac{\partial U(x;\beta)}{\partial \beta} = u - \frac{\kappa x + e(xu - \int_0^x U(z)dz)}{m+ex}$.

$\frac{\partial U(x;\beta)}{\partial \beta \partial x} = \frac{-e^2 x\left(\frac{1}{x}\int_0^x U(z)dz - U(x)\right) - m(\kappa + e(u - U(x)))}{(m+ex)^2} < 0.$ ■