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An Empirical Study of the Mexican Banking System's Network and its Implications for Systemic Risk*

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Abstract: With the purpose of measuring and monitoring systemic risk, some topological properties of the interbank exposures and the payments system networks are studied. We propose non-topological measures which are useful to describe the individual behavior of banks in both networks. The evolution of such networks is also studied and some important conclusions from the systemic risks perspective are drawn. A unified measure of interconnectedness is also created. The main findings of this study are: the payments system network is strongly connected in contrast to the interbank exposures network; the type of exposures and payment size reveal different roles played by banks; behavior of banks in the exposures network changed considerably after Lehmans failure; interconnectedness of a bank, estimated by the unified measure, is not necessarily related with its assets size.
Keywords: Systemic risk, financial networks, payment systems.
JEL Classification: C01, C02, C44, C63, G21.

Resumen: Con el propósito de medir y monitorear el riesgo sistémico, se estudian algunas propiedades topológicas de las redes de exposiciones interbancarias y del sistema de pagos. Proponemos medidas no topológicas que son útiles para describir el comportamiento individual de los bancos en ambas redes. La evolución de dichas redes también es estudiada y se extraen algunas conclusiones importantes desde la perspectiva del riesgo sistémico. También se crea una medida unificada de interconectividad. Los resultados principales de este estudio son: la red del sistema de pagos está fuertemente conectada en contraste con la red de exposiciones interbancarias; el tipo de exposiciones y el tamaño de los pagos revelan diferentes roles desempeñados por los bancos; el comportamiento de los bancos en la red de exposiciones cambió considerablemente después de la caída de Lehman; la interconectividad de un banco, estimada con la medida unificada, no necesariamente está relacionada con el tamaño de sus activos.
Palabras Clave: Riesgo sistémico, redes financieras, sistemas de pagos.

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Introduction

Despite the importance of systemic risk in the international financial regulatory arena, authorities have failed to reach an agreement on a common widely accepted definition for this concept. The lack of a representative definition has been the main motivation behind the wide range of proposals to study and measure systemic risk. In the search for a structural framework to evaluate it, financial authorities and researchers have considered important aspects, so that they have created a spectrum of possibilities to define systemic risk, which has allowed them to dimension the complex phenomena we are dealing with. Nevertheless, the clear disadvantage of having different approaches to study systemic risk is the impossibility of comparisons among the results different methods provide. In this sense, a common definition will give a structural framework to evaluate systemic risk in a standard way and will allow us to manage it to a certain extent.

Among many methodological proposals, graph models (network theory) occupy a preponderant place and this is reflected by numerous papers recently being published. Moreover, most of the financial stability reports include a section on network models, contagion, interconnectedness, etc. The concept of *"too interconnected to fail"* is widely used by domestic and international financial authorities around the globe. This concept is associated with the centrality (relevance) of a node in a network.

Network theory and models have been employed in economics and finance before their recent popularity in systemic risk. Nowadays, there is an important body of literature on network models in finance and economics and it would be impossible to give a full account of all papers, but we can mention only the ones which we consider to be either relevant or close to the purpose of this paper. In Nagurney (2003), the author introduces some of the applications and lines of research, whereas in Allen and Babus (2008) the authors discuss several aspects in finance, which benefit from the network treatment. Finally, Goyal (2007), is the most comprehensive text on network economics.

Despite all the good work that has been done on financial networks, there is room for improvement, in particular regarding empirical evidence of real exposures and payment systems' networks. In this work, some relevant measures for systemic risk measurement and monitoring are proposed, beyond the well known topological metrics. Moreover, all this empirical evidence is put into the systemic risk context, something which has not been done comprehensively in the past, to the best of our knowledge.

Furthermore, there are some important underlying questions on the use of this approach to study systemic risk such as: Which network or networks should be used? Is the network of interbank exposures the relevant network to determine systemic relevance? Is the payment system flows network the one that should be used? How do we incorporate the time dimension to study systemic relevance in financial networks?

Nevertheless, in order to be able to study the interconnections between the financial institutions, relevant data is a crucial element. For instance, the data obtained to build the payment system network is relatively easier to access by the authorities and regulators, as generally the central banks are in charge of the management of Large Value Payment Systems. However, this is not the case for the interbank exposures network. There are only a few central banks which could have an opportunity to estimate the interbank exposures network; but a lot has been written on the topology of the interbank exposures network by using simulation or by making some strong assumptions.

The dynamics of financial networks is a fundamental aspect on the determination of systemic relevance by means of network models, which has not received adequate attention until now. A static understanding of the relevance of an institution in a network dominates the ideas behind the new regulation, which tries to incorporate interconnectedness for the determination of Global Systemically Important Banks (G-SIBs). In contrast, some relevant findings are reported in this work on dynamic aspects of two well recognized networks for the Mexican banking system: the payment system flows and the interbank exposures networks. This study provides empirical evidence on the dynamics of the different roles played by Mexican banks in such networks.

There are many relevant aspects that a network can reveal: the number of connections between the nodes, the degree of connectivity among them, the relative relevance of a node, the persistence of some of the links, etc. Nevertheless, the interpretation of results vary according to the context. Depending on the network that is used to study the role played by a node, different conclusions might be drawn. For example, in this work we found that the interbank exposures network posses different connectivity to the payments system flows network, with the latter being more intensively connected.

Another relevant finding in this work is that some nodes (banks) which play an important role in the interbank exposures network (as lender or borrower), play less important roles in the payments system network. On the other hand, there are some nodes which would not be considered important by their size or their roles in the interbank exposures network but they are important players in the payments system network. Moreover, we provide empirical evidence on how the relevance of banks in the studied networks changes with time as a consequence of an important external shock; namely, the bankruptcy of Lehman Brothers. Furthermore, the proposed measure of interconnectedness captures the dynamics of the connectivity relevance of a node in terms of low/high value payments or as lender/borrower in the payments system or the interbank exposures network respectively.

Although, network theory has proved to be effective in the study of payment systems, it is necessary to understand its limitations in the context of systemic risk. It is not possible to determine the systemic relevance or the contribution to the systemic risk of an institution by means of network theory alone, as some important components might be ignored. Some relevant aspects, among many others, which allow us to identify its systemic relevance are: the risk profile of the individual financial institutions, the assets and liabilities structure of banks, and their funding behavior.

Moreover, most studies on systemic risk and networks use only the direct links between institutions and frequently indirect links are not considered. Although, we acknowledge that it is not an easy task to include indirect links into the study of systemic risk, more efforts must be made to incorporate such links as they represent relevant sources of common shocks and contagion.

The rest of the paper is organized as follows: Section 1 introduces the literature of network models in the context of systemic risk. Section 2 describes the measures that will be used to describe the properties of the studied networks. Section 3 describes the data used to build the payments system and the interbank exposures networks. Section 4 presents some relevant metrics for the interbank exposures and the payments networks; and, Section 5 presents a centrality study for both networks. Finally, Section 6 concludes and proposes future lines of research.

1. Network models and systemic risk

Since the seminal paper by Allen and Gale (2000), network related models have been present in the context of financial contagion and systemic risk. However, the topology of real interbank exposures networks is far from the two topologies suggested in Allen and Gale (2000). For example, Wells (2002) studies the UK interbank exposures network in the context of systemic risk and Boss et al. (2004b) as well as Boss et al. (2004a) provide some empirical data on the interbank exposures network of the Austrian banking system; in Iori et al. (2005), the authors provide some empirical evidence on the topological properties of the Italian Money Market. In Blavarg and Nimander (2002), the Danish market flows are studied by analyzing two networks: the money market and the customer transactions network.

The above-mentioned studies have analyzed the interbank exposures network with the more general purpose of studying contagion and systemic risk. Nevertheless, as complete information on the network of exposures is not easily available, most of the previously mentioned works had to rely on a common assumption: the maximum entropy principle. Unfortunately, this principle might be the cause of the underestimation of contagion, and as a consequence, of systemic risk as has been clearly demonstrated in Mistrulli (2007).

Simulation of financial networks has been also applied to study contagion, as in Nier et al. (2007) and Gai and Kapadia (2010). In both papers the authors use randomly generated networks to study financial contagion. Random formation models use some scale-free properties, which apparently interbank exposures networks exhibit, to generate simulated networks.

There is branch of research in central banks which includes "network models" within a wider simulation framework, as in Boss et al. (2006), Aikman et al. (2009), Alessandri et al. (2009), Márquez-Diez-Canedo and Martínez-Jaramillo (2009), Martínez-Jaramillo et al. (2010b), Martínez-Jaramillo et al. (2010a), Lopez-Castañon et al. (2012), Gauthier et al. (2010b), Gauthier et al. (2010a).

From an international perspective, in Espinosa-Vega and Solé (2010) the authors use network theory to simulate credit and funding shocks to different financial systems. In Minoiu and Reyes (2011) the authors study the network of global banking from 1978 to 2009. On a different approach, in Hoggarth et al. (2010) the authors study the propagation of the consequences of the previous international crisis on the international financial system, and in Garratt et al. (2011) the authors study contagion in the international financial network.

1.1. Payment systems

Network models in payment systems have been rather successful in describing relevant aspects of the payment systems flows. The literature is vast, and it would be easy to miss some relevant research[2]. However, as it is not our intention to provide a full review of the literature, we will only give some references to the work we know until now. Among such studies, one can find research that describes various payment systems around the

[2]We make our network analysis considering the size of the payment orders for small and large value payments. A similar approach is used in Alexandrova-Kabadjova and Solis (2012), in which the authors analyze the difference in the intraday liquidity management for large and small value payments by studying different sets of payment transactions, divided according to their value.

world: Soramäki et al. (2006), Bech and Atalay (2008), Becher et al. (2008), Rordam and Bech (2008), Pröpper et al. (2008), Wetherilt et al. (2010).

1.2. Network formation models

As a consequence of the difficulties of finding the relevant data to determine the network of exposures, financial network theory has been forced to use either strong assumptions (like maximum entropy) or to resort to different theories and tools, like random graphs (see Erdös and Rényi (1959)) or scale free networks (see Barabási and Albert (1999)). Random graphs are important because most network formation models are based on variations of random graphs. On the other hand, scale-free networks can be described in a simplistic way as networks in which the distribution of the nodes' degree follows a power law distribution[3]. Scale-free networks have become a popular subject of study as many of the well-known networks possess this property (for example, the WWW, the Wikipedia and the citation networks, among many others).

In the context of systemic risk, the financial network has been described as a robust-yet-fragile structure because of this scale-free property. In the occurrence of random shocks, this property of the high connectivity of only a few nodes and the low connectivity of most would result in the situation that a random shock would affect low-connectivity nodes with high probability; therefore, the structure is robust. However, if the random shock affects one or few highly connected nodes, the network would suffer major disruption; therefore, the structure is fragile.

Despite the usefulness of the network-formation models, we believe that modeling the interbank network of exposures by means of random graphs is not appropriate because relationships in such networks are created for strategic reasons and are also determined by other relevant factors like the abundance or scarcity of liquidity. There are many relevant aspects in the context of systemic risk which might not be captured by such generative models as will be shown later.

2. Notation and measures

A graph is an ordered pair $G = (V, E)$ where V is the set of vertices and E is the set of edges which is a set of unordered pairs of V. Sometimes, in order to be explicit, this object can be called a simple graph or undirected graph. On the other hand, a directed graph or digraph is a graph $D = (N, A)$ where N is the set of nodes and A is the set of arcs in which A is a set of ordered pairs of nodes. In digraphs, the order of nodes in the definition of an arc is important and usually represents a relationship in which direction is relevant.

A weighted graph can be described as a graph with weights, $w(e)$, assigned to each edge, e. A weighted digraph can be defined as a directed graph with weights, $w(a)$, assigned to each one of the arcs, a.

[3]This means that the fraction $p(d)$ of vertices with degree d satisfies:

$$p(d) = \frac{d^{-\eta}}{\zeta(\eta, d_{min})}, \quad d \geq d_{min} \quad \eta > 0 \tag{1}$$

in which $\zeta(\eta, d_{min}) = \sum_{d=d_{min}}^{\infty} d^{-\eta}$ is known as the generalized Riemann Zeta Function.

The terms "weighted graph" and "weighted digraph" have been used as synonyms with the term "network", although this is not fully accurate.In the past, networks have been used for optimization problems like the maximum flow problem, the traveling salesman problem, etc. In particular, in finance there has been widespread use of the term "network" to refer to weighted graphs or weighted digraphs, and we follow the same convention in this paper.

In addition to the representation of graphs useful for exploratory purposes, there exists a matrix representation which is often used for the storage of such structures and for the implementation of algorithms. Such representations are the adjacency matrix and the weighted matrix. The adjacency matrix, A, represents the existence or absence of an edge or arc in a graph or digraph. The entries of such a matrix, a_{ij}, represent the existence of a relationship between nodes i and j. If the modeled relationship does not exist between nodes i and j, then $a_{ij} = 0$. Otherwise, $a_{ij} = 1$.

The weighted adjacency lending matrix, W^L, entries w^L_{ij} represent the weight (cost, flow, exposure) from bank i to bank j. In a similar fashion, we define the weighted adjacency borrowing matrix W^B. By definition, the following equality holds: $W^L = (W^B)^T$. We also define the weighted adjacency matrix, W, as $W^L + W^B$, its entries, w_{ij}, represent the weight of the total existing relationship between nodes i and j.

There are two important concepts which will be used recurrently in the rest of the paper: walks and paths. For a non-directed graph ,G, a *walk* is a sequence of vertex and edges $v_0, e_1, ..., v_{n-1}, e_k, v_k$, in which $v_0 = v_k$. A *path* is a walk in which no vertex and no edge is repeated. For directed graphs, the concepts are similar but the edges have direction (arcs).

The following sections will describe some of the metrics which are widely used to describe several aspects of different types of graphs and digraphs.

2.1. Topological measures

The topological measures of a network describe its structural properties. The study of network topology is very important in real life physical networks like the electric grid, circuits, computer networks, etc. Two of the most simple, yet important, metrics in graph and network models are the number of vertex (nodes) $n = |V|$ and the number of edges (arcs) $m = |E|$. These quantities give us a clear idea of the size of the network and the density of the connections. Such simple quantities will be used to express many other metrics and to calculate the computational complexity of the algorithms used to solve particular problems.

2.1.1. Degree

The degree of a node in a network is a simple measure but a very useful one. This measure captures the number of nodes that a node is connected to. The degree, $d(i)$, of a vertex, i, in a graph is defined as:

$$d_i = \sum_{j \in N(i)} a_{ij} \tag{2}$$

where $N(i)$ is the set of neighbors of vertex i; that is, the set of vertices which have an edge with vertex i.

The inner degree, d_i^-, and the outer degree, d_i^+, of a node, i, in a digraph are defined as:

$$d_i^- = \sum_{j \in N^-(i)} a_{ij} \;\; and \;\; d_i^+ = \sum_{j \in N^+(i)} a_{ij} \tag{3}$$

where $N^-(i)$ is the set of inner neighbors of vertex i, which is the set of nodes having an arc ending in node i. $N^+(i)$ is the set of outer neighbors of vertex i, the set of nodes which have an arc starting in node i.

2.1.2. Clustering coefficient

The clustering coefficient, c_i, is a measure of the density of the connections around a vertex i and is defined as:

$$c_i = \frac{2}{d_i(d_i - 1)} \sum_{j,h} a_{ij} a_{ih} a_{jh}. \tag{4}$$

The clustering coefficient indicates that if two vertices, which have a connection with a third vertex, have a connection between them; that is, it indicates if they form a triangle. The average clustering coefficient measures the density of triangles in the graph.

2.1.3. Reciprocity

The *reciprocity* in a directed graph, G, is the fraction of arcs in any direction for which there exists an arc in the opposite direction. It is important to note that in a directed graph, in general, $a_{ij} \neq a_{ji}$, and the reciprocity is defined as:

$$r = \frac{\sum_{i \in V} \sum_{j \in N(i)} a_{ij} \mathbf{1}_{\Omega}^{(i,j)}}{\sum_{i \in V} \sum_{j \in N(i)} a_{ij}} \tag{5}$$

in which $\Omega = \{(i,j) \in V \times V : a_{ij} = a_{ji}\}$ and

$$\mathbf{1}_{\Omega}^{(i,j)} = \begin{cases} 1 & \text{if } (i,j) \in \Omega, \\ 0 & \text{if } (i,j) \notin \Omega \end{cases}$$

is the indicator function over the Ω set.

2.1.4. Affinity

Affinity is a measure, which on the basis of the degree of a node, describes the type of nodes to which such a node tends to have a link.

$$a_i = \frac{1}{d_i} \sum_{j \in \mathcal{N}(i)} d_j \tag{6}$$

If a_i is increasing with d_i, then nodes with high-degree tend to have relationships with nodes which possess similar degree. If a_i decreases with d_i, then the majority of the neighbors of high-degree nodes have lower degree. Conversely, nodes with low degree tend to have relationships with high degree nodes.

This measure describes if the nodes in a network tend to have relationships with nodes of similar degree or nodes with a different degree.

2.1.5. Completeness index

The completeness index of a graph is a measure of how close a graph is to the complete graph. The complete graph has an index of 1, whereas the graph with no edges has an index of 0. The closer the index is to 1, the closer the graph is to being fully connected. The completeness index, $C(G)$, of an undirected graph, G, is defined in the following way:

$$C(G) = \frac{\sum_i \sum_j a_{ij}}{n(n-1)}. \tag{7}$$

The completeness index for a directed graph is:

$$C(G) = \frac{\sum_i \sum_j a_{ij}}{2n(n-1)}. \tag{8}$$

2.1.6. Components of a network

The sort of networks which can be observed in real life are very large, making it very difficult to visualize. In this case, it is possible to partition the network into components. Such a partition is going to be given by the connectivity properties of the nodes belonging to each partition. In Dorogovstev et al. (2001) the author proposes to partition V in the following way:

- The *disconnected components* (DC). These are the zero degree vertices or weakly connected small components.
- The *giant weakly connected component* (GWCC) is the largest component in which every pair of vertices is connected by a path .

The giant weakly connected component can also be partitioned in the following way:

- The *giant strongly connected component* (GSCC) or *core* is the largest component in which, for each pair of vertices i y j, there exists a *path* from i to j and a *path* from j to i.
- The *giant out-component* (GOUT) consists of the vertices which can be reached from the GSCC by a *path*.
- The *giant in-component* (GIN) consists of the vertices from which the GSCC can be reached by a *path*.
- The *tendrils* are the vertices which cannot reach the core and are unreachable from it.

Given that, in the Mexican case, the networks representing the banking system are pretty small, we will only make a distinction between the nodes which belong to the core and the ones which do not. In this way we can distinguish from the banks which are easily reachable and the ones which are not.

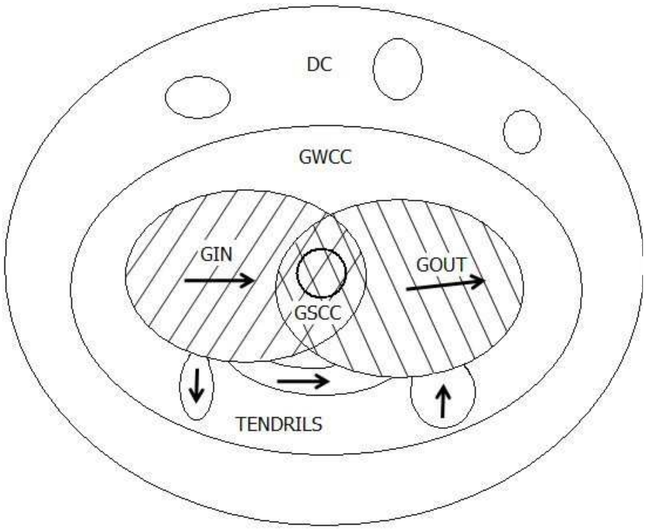


Figure 1: *The components of a network*

2.2. Other measures

In addition to the well-known topological measures, other relevant measures in the context of systemic risk will be described in this section. This set of measures is based on the weights associated with the arcs. Their interpretation is straightforward; for example, the strength of a node in the network can be interpreted as the intensity of its interactions with all its counterparts. Centrality measures will be described in detail in further sections, due to their relevance in the context of this paper.

2.2.1. Strength

The total strength of a node is a very simple measure but a very important one, and can be interpreted as an intensity-of-interaction measure. This measure is even used as a criteria to determine centrality in a network as will be explained later in the paper. The strength, s_i, of a vertex in the network is defined as:

$$s_i = \sum_{j \in N(i)} w_{ij} \tag{9}$$

In addition to total strength, inner and outer strength are relevant measures because they could be useful to determine if a bank plays a more important role as a lender or a borrower, in the case of the interbank exposures network. The inner strength, s_i^B, and the outer strength, s_i^L, of a node in a weighted digraph are defined as:

$$s_i^B = \sum_{j \in N(i)} w_{ij}^B \ \ and \ \ s_i^L = \sum_{j \in N(i)} w_{ij}^L \tag{10}$$

2.2.2. Flow

The flow of a node is a very relevant measure in the context of exposures networks; such a measure can be used to characterize a node as a net lender or net borrower in the network. This characterization in turn, can be used to take some actions in order to manage systemic risk depending on the importance of the node in the network.

In the context of the interbank exposures network, the elements w_{ij}^B (w_{ij}^L) of the borrowing and lending matrices express the exposures between bank i and bank j. The flow between the banks i and j is defined as: $f_{ij} = w_{ij}^L - w_{ij}^B$, which is the net exposure of bank i to bank j.

The total flow of a node is defined as follows:

$$f_i = \sum_{j \in N(i)} f_{ij}. \tag{11}$$

In the context of exposures networks, if $f_i > 0$ then the bank i is a net lender, whereas if $f_i < 0$ bank i can be considered as a net borrower.

2.2.3. Herfindahl-Hirschman Index (HHI)

The Herfindahl-Hirschman Index is commonly associated with competition and market power. In our context, such an index will be used to describe the concentration of lending or borrowing by individual nodes in the network. As we will see in some concrete examples later, this index is relevant for measuring contagion transmission. For example, two banks, A and B, are exposed to the same three counterparts each. Bank A has its lending highly concentrated in one of its three counterparts, let's say C. On the other hand, bank B is lending in a less concentrated fashion. In this specific example, bank A will suffer larger losses than bank B if bank C fails, something which might have an impact on the transmission of contagion if the exposure of bank A to bank C is large enough to cause its failure.

The Lending HHI is computed in the following way:

$$HHI^L(i) = \sum_{j \in \mathcal{N}(i)} \left(\frac{w_{ij}^L}{\sum_j w_{ij}^L} \right)^2. \tag{12}$$

The Borrowing HHI can be calculated as follows:

$$HHI^B(i) = \sum_{j \in \mathcal{N}(i)} \left(\frac{w_{ij}^B}{\sum_j w_{ij}^B} \right)^2. \tag{13}$$

2.2.4. Preference index

The interbank market is a very important vehicle for liquidity transmission and it is very important to study the way in which banks lend to each other. The preference index is proposed in Cocco et al. (2009) to measure the intensity of the interaction between each pair of banks. Banks lend to each other for many different reasons but they prefer to do so with some banks rather than others. Such a preference is important in particular in times of stress, when liquidity dries up. Having a strong relationship with a particular bank would mean that such a provision of funding would be available in times of stress, depending on the severity of the liquidity shock.

- *Lending Preference Index (LPI).* For each lender, L, and each borrower, B, the LPI is computed as the ratio between the amount which L has lent to bank B and the total amount which L has lent to all its counterparts, in the last k days[4].

$$LPI_{L,B,t} = \left(\sum_{s=1}^{k} F_{t-s}^{L \longrightarrow B} \right) / \left(\sum_{s=1}^{k} F_{t-s}^{L \longrightarrow system} \right) \tag{14}$$

  where $F^{L \longrightarrow B}$ is the total amount which L has lent to B and $F^{L \longrightarrow system}$ is the total amount which L has lent to all its counterparts.

- *Borrowing Preference Index (BPI).* Analogously the BPI can be defined as:

$$BPI_{L,B,t} = \left(\sum_{s=1}^{k} F_{t-s}^{L \longrightarrow B} \right) / \left(\sum_{s=1}^{k} F_{t-s}^{system \longrightarrow B} \right) \tag{15}$$

If L is an important lender (borrower) for B the ratio should be close to 1.

2.3. Centrality measures

Centrality is a concept commonly used in social networks and it has been extensively studied for several decades, as it has several relevant interpretations like power, influence, independence, control, etc. One of the first attempts to characterize centrality measures is Sabidussi (1966). In this paper the author establishes some criteria that a centrality measure should hold. Additionally, in Freeman (1979) the author revises three different types of node centrality (degree, betweenness and closeness centrality) from the perspective of social networks. More recently, Borgatti and Everett (2006) rightfully state that the measures of centrality asses the involvement of the nodes on the walk structure of the network and propose four classes of network centrality measures.

It is very important to put centrality measures into the right context. To apply network theory concepts and measures blindly could result in wrong interpretations. This is particularly important from the perspective of policy making. In the context that we are interested in, there are numerous works which compute centrality indexes in payment systems and interbank exposures networks.

Centrality measures offer the possibility of ranking the nodes to assign them a measure of relevance in a network. The larger the centrality measure the greater importance such a node has in a network. This is closely related to the determination of systemic importance[5] as one of the components to determine it is interconnectedness, which can be translated into centrality. However, systemic importance and centrality should not be used as synonyms, as systemic importance is a concept which involves more aspects than interconnectedness alone.

In the context of financial and banking systems, borrowing from Henggeler-Muller (2006), it is possible to say that a financial institution is important in a financial network if it has the following characteristics:

[4] The authors in Cocco et al. (2009) propose to use the last 30 days.

[5] See BCBS (2011) for a description of the methodology

- Possesses many linkages to other members of the network (degree)
- The total amount of its assets, liabilities or flow in the network is very large (strength)
- Its failure could transmit contagion in a few steps (closeness)
- Its counterparts are considered also as relevant (eigenvector and PageRank)
- There are many paths which pass through it (betweenness)

We consider that each centrality measure provides an important element to determine the relevance of a node in the network and, for that reason, we will combine them instead of choosing one as the centrality measure for our networks.

Some of the most relevant research on centrality in financial networks is the following: in Nacaskul (2010), the author proposes Entropic Eigenvector Centrality as a measure of systemic relevance and tests the measure over a range of stylized network topologies. In Saltoglu and Yenilmez (2010), the authors propose a modified version of the PageRank algorithm for assigning systemic relevance to banks in Turkey and analyze it in different periods of time before a financial crash.

2.3.1. Strength centrality

The strength of a node, v, as was seen previously in 2.2.1, is simply the sum of its interbank assets and liabilities in the case of total strength:

$$C_S(v) = s_v. \tag{16}$$

In the case of inner strength, it is the sum of its interbank total assets, and, in the case of outer strength, it is the sum of its total interbank liabilities. To differentiate between inner and outer strength is important, as we are interested in determining which bank is lending (borrowing) the most in the network. Such differentiation is important in the context of systemic risk because a bank can play a specific role in the network; that is, a bank can be very important as a lender or as a borrower in the interbank exposures network and it is important from the regulators' perspective as the failure of such institutions would have different repercussions depending on the role played in the network. This measure is very easy to compute from the algorithmic point of view but fails to consider the relevance of its counterparts and the number of possible affected neighbors in the case that an institution fails.

2.3.2. Degree centrality

Degree centrality, as defined in (17), is one of the most simple measures of network centrality. A node is more important in a network if it is connected to many other nodes, as its failure would have an impact in many other participants. In addition to degree centrality, it is possible to define out-degree centrality and in-degree centrality. One of the most attractive characteristics of these centrality measures is that they are very cheap to compute. One of the main criticisms of these types of measures is that they do not consider the importance of the neighbors and the weight size. Clearly both factors are important in determining the relevance of a bank in a network. For the purposes of this study, degree centrality of vertex, v is defined as:

$$C_D(v) = d_v. \tag{17}$$

Although in other studies this measure is standardized, dividing it by the maximum number of counterparts a bank may have $(n-1)$.

2.3.3. Betweenness centrality

Betweenness centrality, in social networks, is associated with being strategically located on the communication paths of many other nodes in the network. A node with high betweenness centrality would have an important influence on other nodes as it can stop or distort the information that passes through it. This measure of centrality is particularly important in the payment system network. As in the case of the previous two measures, it is possible to compute inner-betweenness centrality and outer-betweenness centrality.

Let $\sigma_{ij} = \sigma_{ji}$ denote the total number of shortest paths between i and j. And let $\sigma_{ij}(v)$ be the total number of shortest paths between i and j that pass through vertex v, then:

$$C_B(v) = \sum_{i \neq v \neq j \in V} \frac{\sigma_{ij}(v)}{\sigma_{ij}}. \tag{18}$$

The algorithm used to calculate this measure was proposed by Brandes (2001).

2.3.4. Closeness centrality

Closeness centrality has an interpretation of independence in social networks in terms of communication control. A node with high closeness centrality would depend less on other intermediary nodes to receive messages. In the context of systemic risk and financial contagion, this measure can be associated with the capacity of a node to spread contagion, as such a node is close to the rest of the network. It is defined as:

$$C_C(v) = \sum_{j \in V \setminus \{v\}} \frac{1}{d_G(v,j)}. \tag{19}$$

In which $d_G(v,j)$ denotes the length of the shortest path that joins v and j. It is also possible to compute inner-closeness centrality and outer-closeness centrality.

2.3.5. Entropic Eigenvector Centrality (EEC)

Eigenvector Centrality was first proposed in Bonacich (1972) with further development into Entropic Eigenvector Centrality by Nacaskul (2010). In Bonacich (1972), the author proposed the eigenvector, e, of the adjacency matrix, A, associated to the largest eigenvalue as the vector whose entry, i, is the centrality measure of node i.

$$\lambda \mathbf{e} = A\mathbf{e} \tag{20}$$

This measure has the advantage that takes into consideration the centrality of the neighbors to compute the centrality of a node. Additionally, the proposed Eigenvector centrality can also be seen as a weighted sum of not only direct connections but indirect connections of any length. Thus, it takes into account the entire pattern in the network.

Nacaskul (2010) proposes to use the eigenvector (EEC) associated with the largest eigenvalue of a matrix referred as P instead of the adjacency matrix. This matrix is the weighted adjacency matrix, W, additionally weighted with the value of the entropy of exposures of the corresponding vertex (see Appendix A). Entries p_{ij} of matrix P are defined as:

$$p_{ij} = \left(1 + \frac{\tau_i}{\tau^{max}}\right) \cdot w_{ij} \tag{21}$$

in which τ_i is the entropy calculated over the rows of a normalized version of the weighted adjacency matrix whose entries are $w_{ij}^{norm} = \frac{w_{ij}}{\sum_{j=1}^{n} w_{ij}}$:

$$\tau_i = -\sum_{j=1}^{n} w_{ij}^{norm} \ln(w_{ij}^{norm}) \quad \forall i \in \{1, ..., n\} \tag{22}$$

and τ^{max} is the maximum value possible for the entropy:

$$\tau^{max} = -\sum_{j=1}^{n} \frac{1}{n} \ln\left(\frac{1}{n}\right) = \ln(n). \tag{23}$$

In order to obtain strictly positive entries for the eigenvector, Nacaskul (2010) also proposes to change each zero entry in W for some non-zero low value in equation (22).

2.3.6. PageRank (PR) centrality

PageRank centrality is based on Google's algorithm proposed in Page et al. (1999), which considers the World Wide Web (WWW) as a digraph. The importance of this measure lies in the fact that unlike other centrality measures (degree, closeness, betweenness, etc.) it considers the relevance of neighbors to determine the relevance of a node in the network, as in the case of the entropic eigenvector centrality. PageRank is defined in the following way:

$$PR(i) = \frac{(1-d)}{N} + d \sum_{j \in \mathcal{N}^-(i)} \frac{PR(j)}{L(j)} \tag{24}$$

where $i \in V = \{1, ..., n\}$ is the set of nodes which represents the Internet sites and $L(u)$ is the number of links which depart from u (that is, its outer degree) and d is a factor which the authors in Page (1997) recommend to set at 0.85.

Vector $\mathbf{PR}$ of $n \times 1$ with entries $PR(i)$ is the one which solves the equation

$$\mathbf{PR} = \frac{1-d}{N}\mathbf{1} + d\mathcal{M}\mathbf{PR} \tag{25}$$

where $\mathbf{1}$ is a column vector with dimension n with the entries all equal to one and $\mathcal{M}$ is the matrix of $n \times n$ given by

$$\mathcal{M}_{ij} = \begin{cases} 1/L(j) & \text{if there exists an arc from } j \text{ to } i \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

If $\mathbf{E}$ is defined as the $n \times n$ matrix with all entries equal to one and $\sum_{i \in V} PR(i) = 1$ we have:

$$\mathbf{PR} = \left(\frac{1-d}{N}\mathbf{E} + d\mathcal{M}\right)\mathbf{PR} =: \widehat{\mathcal{M}}\mathbf{PR}. \tag{27}$$

Therefore, $\mathbf{PR}$ is the eigenvector of the matrix $\widehat{\mathcal{M}}$ associated to the first eigenvalue.

Nevertheless, arcs on the WWW have no weight associated with them but in the interbank exposures network and the payment system flows networks, the weights associated with the arcs provide useful information. Therefore, weights should be incorporated in the centrality algorithms in order to avoid losing information. In Yenilmez and Saltoglu (2011) the authors suggest that the PageRank of each node should be multiplied by the dominant weight. For example, if i has links with j, then the largest weight between w_{ij} and w_{ji} should be used by the algorithm. In this way the centrality of a bank will increase in the direction of its dominant weights.

$$PR(i) = \begin{cases} (1-d) + d\displaystyle\sum_{j\in\mathcal{N}(i)}\left(PR(j)\frac{w_{ji}^L}{\displaystyle\sum_{z\in\mathcal{N}(j)} w_{jz}^L}\right) & \text{si } w_{ji}^L > w_{ji}^B\ , \\ (1-d) + d\displaystyle\sum_{j\in\mathcal{N}(i)}\left(PR(j)\frac{w_{ji}^B}{\displaystyle\sum_{z\in\mathcal{N}(j)} w_{jz}^B}\right) & \text{si } w_{ji}^B > w_{ji}^L\ , \\ (1-d) + d\displaystyle\sum_{j\in\mathcal{N}(i)}\left(PR(j)\left(\frac{\frac{w_{ji}^L}{\sum_{z\in\mathcal{N}(j)} w_{jz}^L} + \frac{w_{ji}^B}{\sum_{z\in\mathcal{N}(j)} w_{jz}^B}}{2}\right)\right) & \text{si } w_{ji}^L = w_{ji}^B. \end{cases} \tag{28}$$

2.3.7. A principal components unified measure of centrality

Given that we have observed that different centrality measures provide us with different aspects of the relevance of a node in the network, it is important to preserve all the information provided by such measures. Nevertheless, from the policy-making perspective, it is important to have only one measure of importance (centrality) in the network. As a consequence, we propose a statistical technique called Principal Component Analysis (see Appendix B for a brief introduction) that provides us with a unique index of centrality, incorporating the information of several centrality measures.

In the context of systemic risk and financial contagion $X_{i,t}^j$ defines the centrality measure j of bank i at time t with $i(t) \in \{27, 28, \ldots, 41\}$, $j \in \{C_S, C_D, C_B, C_C, EEC, PR\}$ and $t \in \{1, 2, \ldots, 1500\}$.

In order to obtain a unique set of coefficients and forecasting ability we decided to run principal components globally. The limitations of this approach are mentioned in Elman (1990). On the other hand, to enhance precision, we ran principal components daily. Given that some centrality measures are highly correlated and to avoid numerical instability, we ran principal components with six centrality measures and with three measures (betweenness, closeness and strength). The decision to use these measures was based on three criteria. The first was to use the least correlated variables. Then, it was decided to take into account those measures that were more important in the context of financial contagion. Lastly, it was decided to use those measures whose computation was less computationally intensive. An alternative, possibly more robust, approach is detailed in Voegtlin (2004).

3. Data

This section describes the data used to construct the daily networks of the interbank exposures and the payment system flows. Even though, the data available for one of the networks might be larger, so we decided to use the same time period for comparison purposes.

3.1. The Mexican interbank exposures market

The Mexican central bank has daily data which can be used to calculate the matrix of interbank exposures in the Mexican financial system from January 2005 onwards. The period of time contemplated in this study goes from the 3rd of January 2005 to the 31st of December 2010. Although currently there are 42 banks in the system, all the metrics and studies reported will consider the evolution of the banking system; that is, we start the study with 27 banks and finish it with 40, which is the number of active banks at the end of 2010. The interbank exposures considered comprise all the possible deposits, credits and loans, including credit lines as part of the interbank market. As is pointed out in Graf et al. (2005), the assumption of maximum entropy in the distribution of the interbank exposures is not realistic, at least not in the Mexican banking system.

Given the detail of the information from the interbank exposures network, it is possible to separate the data and to study the impact that different types of exposures have on the properties of the total exposures network. The base case will be the network described in the previous paragraph with a minor modification: FX exposures will be eliminated for the banks which use the services provided by the Continuous Linked Settlement (CLS) Bank[6]. The point at which the Mexican peso is included as a currency settled by the CLS changes some of the properties of the network. In particular, it changes some of the individual measures of the banks which participate in the CLS. This change in the structure of the network provides an interesting case on how a central counterpart changes the structural properties of a network.

The second case studied is the network which includes all of the FX exposures. This case is useful in order to understand the effect that the CLS had on the properties of the network. We also analyzed the properties of the network without the FX exposures in order to evaluate if the network changes only as a consequence of the Lehman Brothers failure. This case is also important because it would otherwise be impossible to evaluate the impact that Lehman had on the interbank exposures network as the Mexican peso entered the CLS approximately one month before the Lehman failure.

[6]The settlement of FX operations by means of the CLS eliminates completely the FX settlement risk.

3.2. The Mexican Large Value Payments System

Payment systems have evolved over time as modern economies are becoming more dependent on the services they provide. The technology and the need for international cooperation have been the main factors driving the changes observed in many aspects of the payments service industry. Ongoing innovation is likely to diversify payment types competing on consumer service level, whereas efficiency and cost reduction could be the main reasons for integration of payments processing and settlement. It is possible that in the near future real-time high-value payments, low-value electronic payments, and even card payments will be settled together. To achieve this, settlement engines need to incorporate a Liquidity Saving Mechanism, which allows settlement of a large number of low-value payments with a minimum pressure on intraday liquidity consumption.

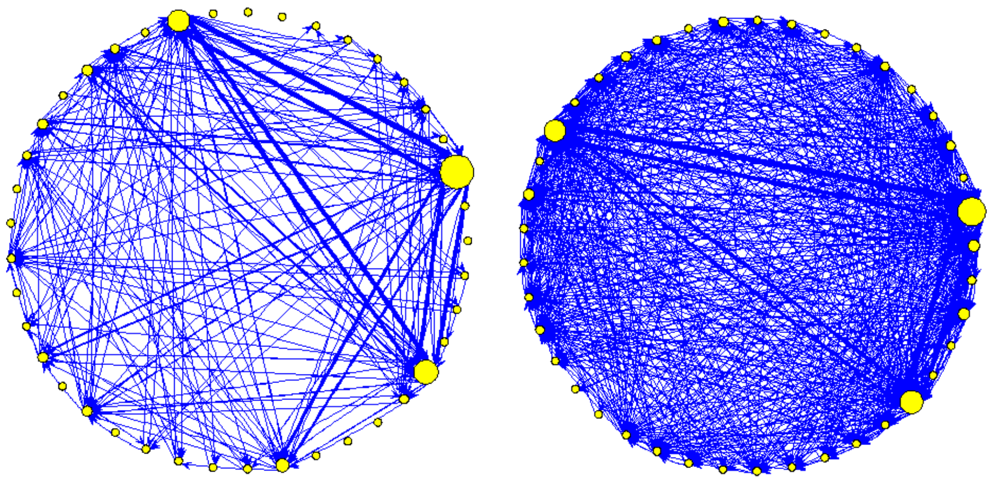
In Mexico, the Payments System that processes and settles large-value payments in real time is named SPEI and is operated by the Mexican central bank. It started to operate on the 13th of August 2004 and it has integrated a Liquidity Saving Mechanism (LSM), which allows continuous netting of payments during the day. SPEI settles payment orders in real time, charging less than a 0.05 USD per payment. It processes, on average, around 300,000 operations daily. More than 80% of the transactions are payments with a value lower than 10,000 USD and only 1.3% of the transactions are above 1,000,000 USD.

In this paper, we use daily information from all the payments which are settled by SPEI. We build the network by accumulating all the payments in one day between each pair of banks in both directions. Entry w_{ij} of such a matrix is the accumulated sum of all the payments which took place from bank i to bank j in a particular day. It is important to note that, in general, $w_{ij} \neq w_{ji}$. In the case of the payments network we will have a daily network for each day in which SPEI operated as is the case for the interbank exposures network. We will cover the same period, from 2005 to 2010, in order to be able to compare some of the characteristics of both networks.

In the case of SPEI, it is possible to separate the payments by value in order to construct different types of networks. This means that it is possible for us to study the SPEI network by using the total value during the day, the total value only considering payments with value above a certain threshold, and the total value considering payments with value below this threshold. This separation is important in order to compare the properties of the network of total payments along with the network of large-value and low-value payments. This separation of payments would reveal the role and the importance of a bank depending on the type of payment (low-value vs. large-value). The threshold value that we use to separate low-value and large-value payments is 10,000,000 MXN.

4. Global and individual measures

In this section, we will report some of the measures developed in this study. In order to report them in an orderly fashion we will split them first by the level in which they are computed: global vs. individual. The next level of organization is by the type of network: payments vs. interbank exposures. The global measures describe the network as a whole, whereas the individual measures describe banks individually.



(a) January 3, 2005 (b) July 27, 2010

Figure 2: The payment system flows network on two different dates.

4.1. Global measures

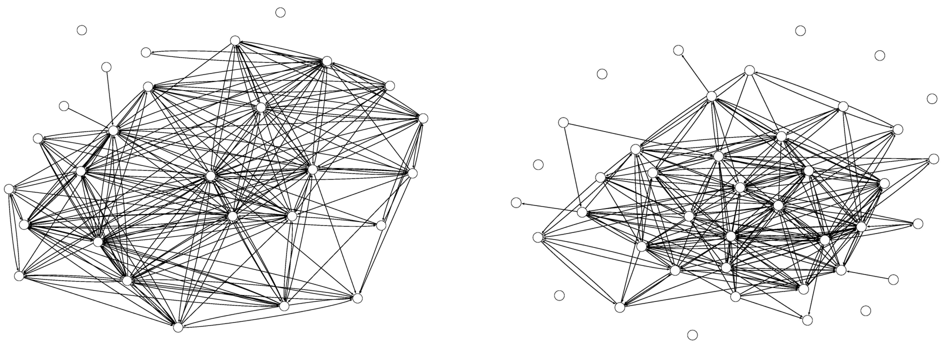
Here we report some of the global measures of the two objects of study: the SPEI and the interbank exposures networks. Among such global measures we will find the volume, the total number of arcs, the average degree and the completeness index. Figure 2 shows the SPEI network on two different days: January 3, 2005 and July 27, 2010. From this figure we can see that the connectivity of the network changed dramatically.

Figure 3 shows the base case of the interbank exposures network[7] on two different days: January 3, 2005 and December 31, 2010. From this figure we can see that despite the incorporation of new banks in the banking systems, some of the newcomers did not participate in this market, at least not every day.

4.1.1. Payments system

In Figures 4 and 5 we can see the evolution of the global measures for the SPEI's base case. In the figures we can see that the system's usage experienced a big jump at the beginning of 2005 and this is reflected in all measures. At the beginning of the period of study, some of the series show big upward movements which are related to the government's payroll transfers. Such sudden changes start to disappear due to the incorporation of more banks in the system. The black vertical line marks the date of the failure of Lehman Bothers[8], which apparently did not change some of this network's global measures. The daily number of arcs has an increasing trend which means that

[7]The figures here were created by using the Pajek software.
[8]This line will appear in several further graphs.



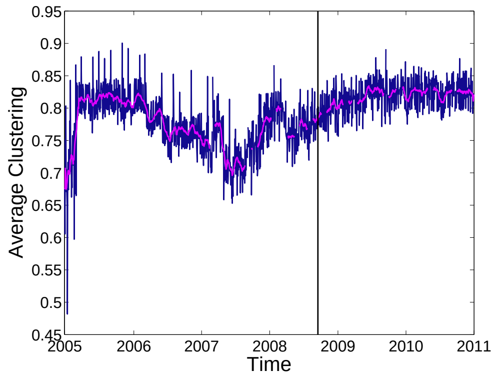
(a) January 3, 2005 (b) December 31, 2010

Figure 3: The interbank exposures network on two different dates.

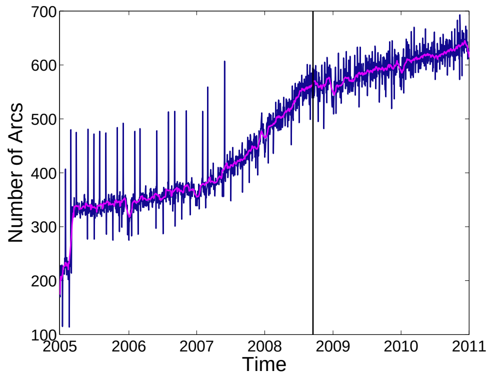
there are more lines of transfers between banks. This trend is also related to the incorporation of more banks in the system. The average degree suffers a decline as does the completeness index. This is natural as the number of banks increases. The completeness index is a more sensitive measure because the number of possible arcs increases dramatically with the incorporation of new banks. The average clustering coefficient is large, meaning that there is a tendency that a bank A is sending/receiving payments to/from a bank B, and sending/receiving payments to/from another bank C, which is also sending/receiving payments to/from bank B. This can be interpreted as a high degree of circulation of payments in the system.

Table 1 shows some of the structural properties of three different SPEI networks: large-value payments, low-value payments and the network which includes all payments regardless of their value. From the table we can observe that the core of the large-value network is smaller, the network is less complete, and the number-of-arcs measure is also smaller than the other two networks, but the total volume of the network is much larger than for the low-value network. Reciprocity in the three networks is the same, indicating that there is a large reciprocity in the network. Summarizing, structurally speaking, the low-value and large-value networks are different. We have a similar table considering the number of operations instead of the value, but the results are quite similar to the ones summarized in this table.

Figure 6 shows two graphs: (a) the evolution of the size of the core and (b) the evolution of the relative clustering coefficient. From Figure 6(a), it is possible to observe that, at the end of the period, almost every node in the network belongs to the core. This fact means that the SPEI network is strongly connected in contrast to the interbank exposures network. The relative clustering coefficient (Figure 6(b)) is obtained by dividing the average clustering coefficient of the SPEI network by the average clustering coefficient of the random graph with the same number of nodes and with the same aver-

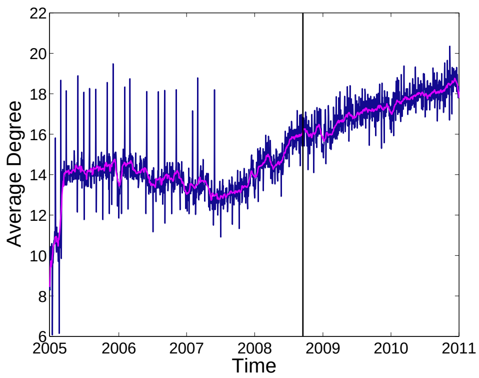


(a) Daily average clustering coefficient

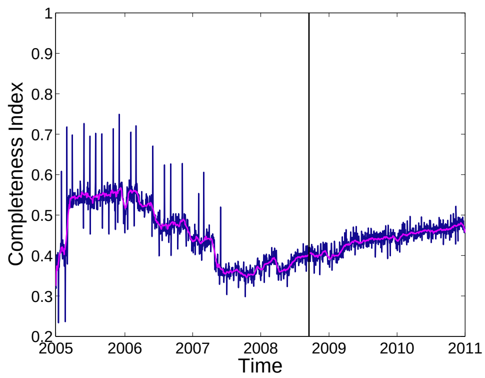


(b) Daily total number of arcs

Figure 4: Evolution of the payment system network.



(a) Daily Average degree

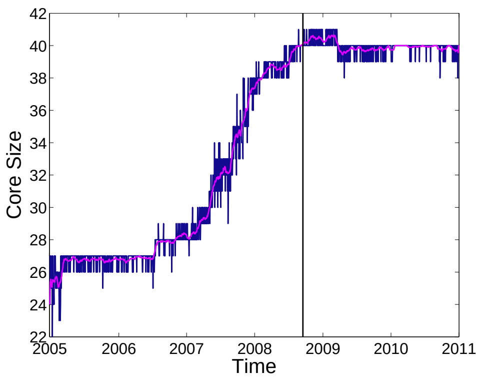


(b) Daily completeness index

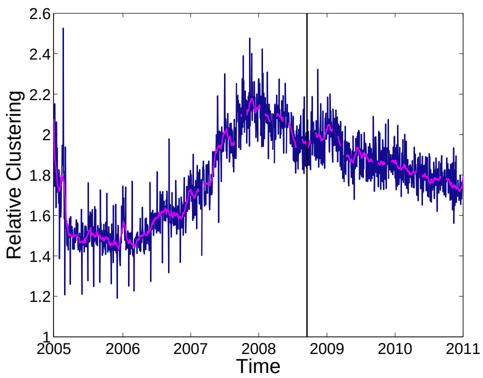
Figure 5: Evolution of the payment system network.

Mean	Large Value	Low Value	Total
Core Size	30.8	33.0	34.2
Completeness Index	0.3	0.4	0.4
Average Degree	10.1	13.3	15.2
Reciprocity	0.8	0.8	0.8
Average Distance	1.7	1.6	1.5
Total Arcs	290.2	405.9	470.9
Average Strength*	24.0	1.24	25.2
Total Volume*	415.8	22.41	438.7

Table 1: *Statistics of the SPEI network (quantities marked with * are expressed in millions).*



(a) Core size



(b) Relative clustering

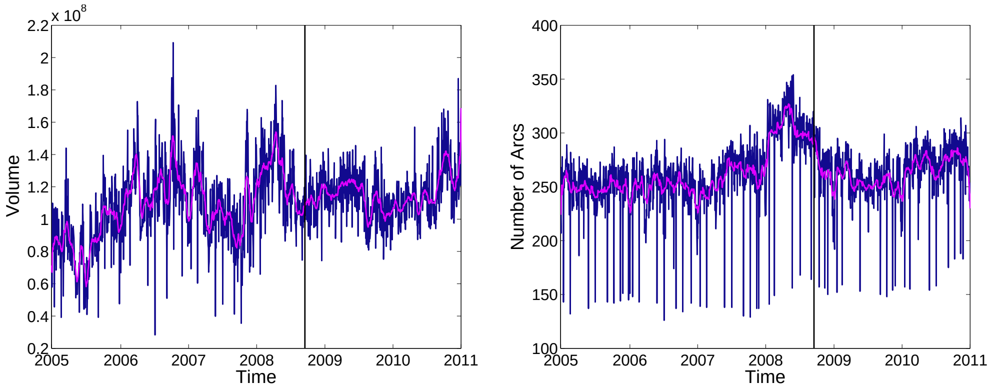
Figure 6: Evolution of the payment system network.

age degree[9]. We can see that the SPEI network deviates from a random graph and that it possesses more triangles than a random network.

4.1.2. Interbank exposures

Figures 7 and 8 show the evolution of the global measures for the interbank exposures base case. In such figures we can see that the interbank exposures experienced an increasing trend from the beginning of 2005 until the end of 2006. From this peak we observe a decline in the volume of exposures until the end of 2007. Afterward, an upward trend started until the incorporation of the Mexican peso to the CLS (June 2, 2008). From this day forward we observe a decrease, as more FX operations were settled by using the CLS. Shortly afterward, we observe the black line which marks the Lehman failure, which apparently did not change this network's global measure. The

[9]The average clustering coefficient of a random graph is equal to the connection probability p: $c = p = \frac{\bar{d}}{n-1}$. In which $\bar{d}$ is the average degree of the graph.



(a) Daily total volume on the interbank market

(b) Daily total number of arcs

Figure 7: Evolution of the base case of the interbank exposures network.

daily number of arcs has an increasing trend until some of the Mexican banks started to settle FX operations by the CLS[10]. There is a further decline after the bankruptcy of Lehman. This shows that the Mexican interbank market was sensitive to such an event on the number of links but not on the total volume of exposures. Changes in the average degree and in the completeness index are related to the incorporation of new banks to the Mexican banking system and also to the settlement of FX operations by the CLS and the bankruptcy of Lehman Brothers.

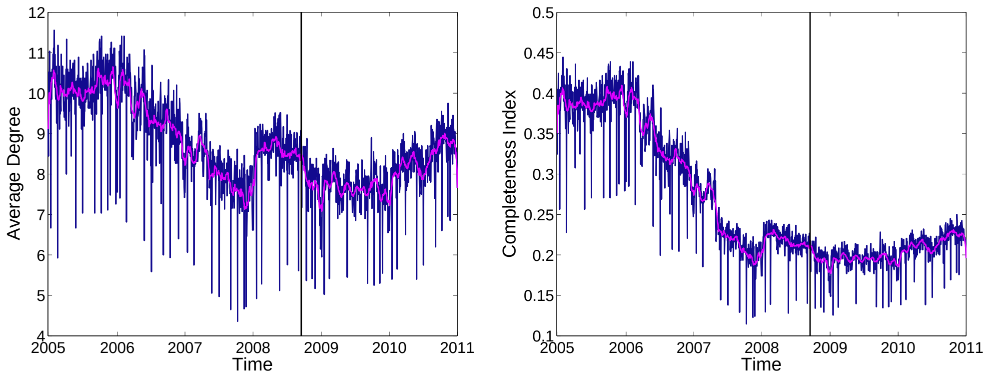
Table 2 shows the same structural properties reported for the SPEI networks for three different interbank networks: interbank exposures, interbank exposures eliminating the FX operations which are settled by the CLS and the interbank exposures without all the FX operations. From the table we can observe that the core of the interbank exposures network without FX operations is the smallest. It is also the smallest for the rest of the other measures. These results suggest that the inclusion or exclusion of FX settlement risk changes the network structural properties in a considerable way.

Figure 9 shows two relevant graphs: (a) the evolution of the size of the core and (b) the evolution of the relative clustering coefficient. From Figure 9(a) we can observe that the core is smaller than the core of the SPEI network. This means that the SPEI network is strongly connected in contrast to the interbank exposures network. Figure 9(b) shows the evolution of the relative clustering coefficient of the interbank exposures network. We can see that the interbank exposures network also deviates from a random graph.

Figure 10 shows the graph of the fitting to a power law of the degree and exposures distributions[11], for a particular day. On that day there were 40 active banks, and the

[10]Some of the sudden drops in the number of exposures are related to bank holidays in Mexico and the US. The rest are related to some drops in the Mexican stock exchange index, the IPC.

[11]The Matlab functions used to generate graphs to estimate the parameters and the computation of



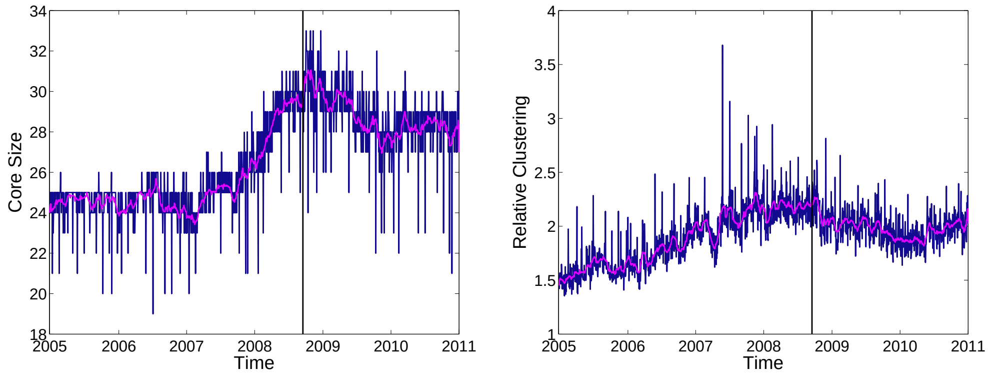
(a) Daily Average degree

(b) Daily completeness index

Figure 8: Evolution of the base case of the interbank exposures network.

Mean	Interbank	Interbank - CLS	Interbank - FX
Core Size	26.7	26.7	22.4
Completeness Index	0.3	0.3	0.2
Average Degree	9.0	8.7	5.7
Reciprocity	0.8	0.8	0.6
Average Distance	1.7	1.8	2.0
Total Arcs	279.7	262.2	145.0
Average Strength*	7.1	6.4	4.3
Total Volume*	125.5	110.8	77.2

Table 2: *Statistics of the interbank exposures network.*



(a) Core size

(b) Relative clustering

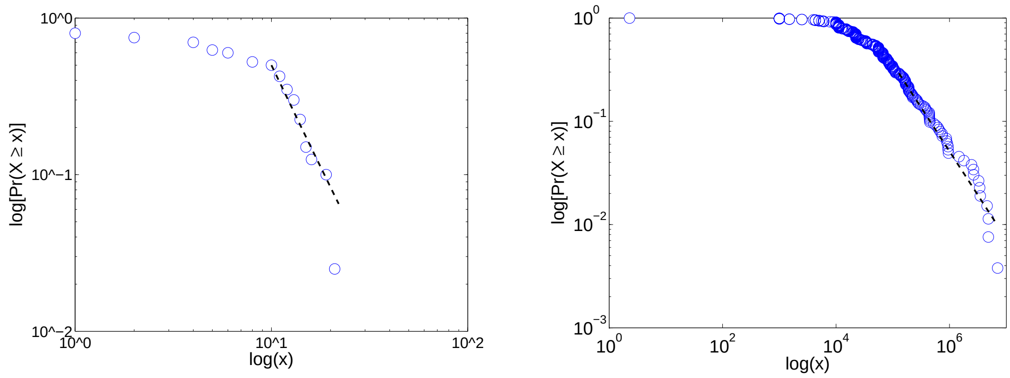
Figure 9: Evolution of the interbank exposures network.

estimated parameters were $\widehat{\eta} = 3.5$ and $\widehat{d}_{min} = 10$. The *p-value* obtained was 0.393; therefore, it can be concluded that the degree distribution follows a power law distribution. However, it is important to point out that fitting a power law distribution to a small sample (less than 50 data points) could result in an apparent good fitting even if the data is not distributed as a power law. Therefore, the results for the degree distribution should be considered with caution. Regarding the exposures, the distributional fitting allows us to reach a more confident conclusion, given that the daily samples consist of at least 126 data points.

Table 3 summarizes the percentage of days in which the hypothesis that the sample can be fitted by a power law is not rejected. The table shows two different criteria for the rejection of the hypothesis: if the *p-value* is less than 0.1 and a more relaxed criteria, if the *p-value* is less than 0.05. From Table 3 one can infer that, with a strict criterion, it is not possible to determine the scale free property for the interbank exposures network for the degree, in degree and out degree. However, if a more relaxed criterion is used, such networks would present such property most of the time. The parameters of the distributions for the base case and the case without CLS FX operations are similar. The parameters for the interbank exposures without the CLS are: for the degree $\widehat{\eta} = 2.96 \pm 1.24$ and $\widehat{d}_{min} = 8.64 \pm 3.71$, for the inner degree $\widehat{\eta} = 3.03 \pm 1.16$ and $\widehat{d}_{min} = 8.25 \pm 3.31$ and for the outer degree $\widehat{\eta} = 2.95 \pm 1.23$ and $\widehat{d}_{min} = 7.80 \pm 3.39$.

Another relevant finding is that, not considering the FX exposures changes the structure of the network considerably in a way that only the inner degree and the exposures exhibit the scale-free property. The inner degree parameters are: $\widehat{\eta} = 3.33 \pm 0.52$ and $\widehat{d}_{min} = 6.77 \pm 1.73$, and the parameters for the exposures: $\widehat{\eta} = 1.89 \pm 0.65$ and $\widehat{w}_{min} = 449,132.83 \pm 448,273.96$.

the *p-value* were obtained from www.santafe.edu/∼aaronc/powerlaws/. All of these functions use the methodology proposed in Clauset et al. (2009)



(a) Degree

(b) Exposures

Figure 10: Test for the power law distribution for the base case network (without CLS)

	Interbank		Interbank without CLS		Interbank without FX	
p-value	$< .05$	$< .1$	$< .05$	$< .1$	$< .05$	$< .1$
Degree	77%	68%	81%	65%	54%	41%
Inn degree	81%	66%	83%	67%	84%	76%
Out degree	80%	60%	80%	64%	56%	45%
Exposures	57%	50%	63%	54%	83%	76%

Table 3: Percentage of days in which the exposures network exhibited power law distributions.

Figure 11(a) shows the evolution of the *p-value* of the inner-degree distribution for the base case of the interbank exposures network. From this figure one can see that there is no trend regarding the network structure. If the *p-value* is above the horizontal lines, the power law distribution's hypothesis is supported. It is also observed that the *p-values* do not follow any pattern. Putting it another way, there are no particular periods of time for which the network exhibits the scale-free property. For the other degree distributions and exposures the graphs are similar.

Nevertheless, the evolution of the *p-value* of the degree distribution for the high-value payments network of SPEI unveils an increasing trend (figure 11(b)). The beginning of this trend takes place somewhere in 2008. The main reason for this structural change seems to be a change in the fees for sending payments through SPEI[12], rather than Lehman's failure, as some individual metrics suggest.

4.2. Individual measures

Before describing the individual measures we will examine two other relevant properties of a network. Figure 12(a) shows affinity vs degree. From this figure we can

[12] An assessment of this modification in regulation may be found in Negrín et al. (2009).

see that the SPEI network, for the base case, exhibits the "disassortative mixing" phenomenon, which means that the counterparts of highly connected nodes are low-degree nodes. Figure 12(b) shows the reciprocity in the SPEI network. From this figure we can infer that reciprocity is around 82%, which means that most of the payments flows are bilateral. The base case of the interbank exposures network exhibited a less pronounced "disassortative mixing" property than for the base case of SPEI network. Nevertheless, if a difference is made between large-value and low-value the payments in the SPEI, the results change. For example, reciprocity remains about the same for low-value payments but decreases for large-value payments.

There are many individual measures developed in this paper but we will show only some of them, which can describe interesting behavior by particular banks. Among the individual measures which will be reported in the next subsections are:

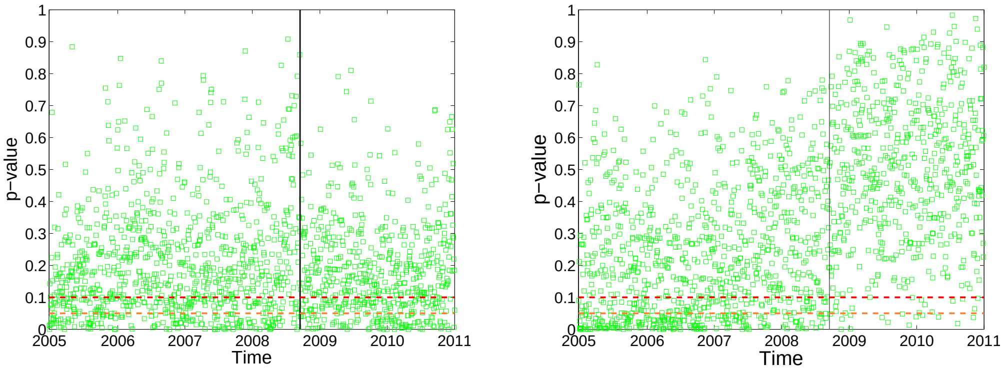
- The Herfindahl-Hirschman Index (HHI)
- Clustering coefficient
- Strength
- Volume
- Preference index
- In and Out degree

Only some of the above-described measures will be reported for either the SPEI or the exposures network. We are particularly interested in metrics which can tell us something about an institution in relation to systemic risk.

4.2.1. Payments system

In this subsection is shown the evolution of some relevant metrics which could help to describe individual bank behavior. Figure 13(a) shows the evolution of the inner HHI for one institution making a difference between large-value and low-value payments. The figure illustrates different behavior from the same institution for the two different types of payments. This institution is more concentrated in its incoming payment flows for large-value payments than for its incoming payment flows for low-value payments. On the other hand, Figure 13(b) depicts a bank whose concentration is sometimes higher for large-value payments and at other times higher for low-value payments. Therefore, it is impossible to tell in which type of payment it is more concentrated, and such payments are probably made based on its current strategy. It is important to point out that this specific behavior would be very difficult to reproduce with the current network formation models.

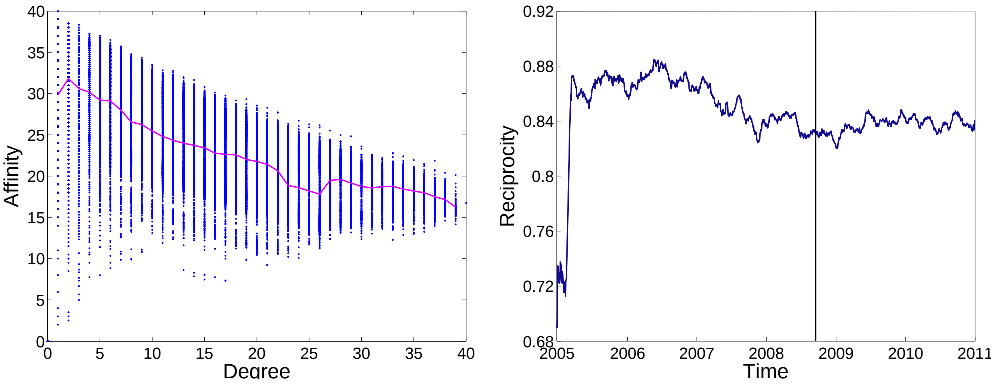
Figure 14 shows the evolution of the clustering coefficient for two different institutions. The clustering coefficient is calculated for the same usual separation of payments: low vs large value. This figure illustrates that the two banks exhibit different clustering coefficients for low-value and large-value payments. Moreover, while bank 19, Figure 14(a), exhibits a larger clustering coefficient for low-value payments, bank 8, Figure 14(b), presents the opposite behavior. This can be summarized in the following way: the counterparts of bank 19 are more connected in the network of low-value payments than in the network of large-value payments, while bank 8 exhibits the opposite behavior.



(a) Interbank

(b) SPEI

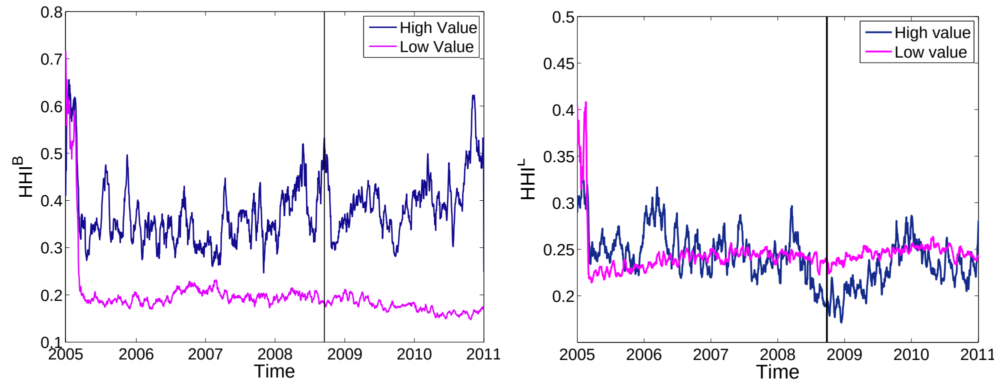
Figure 11: Evolution of the *p-value* of the inner degree .



(a) Affinity vs degree

(b) Reciprocity

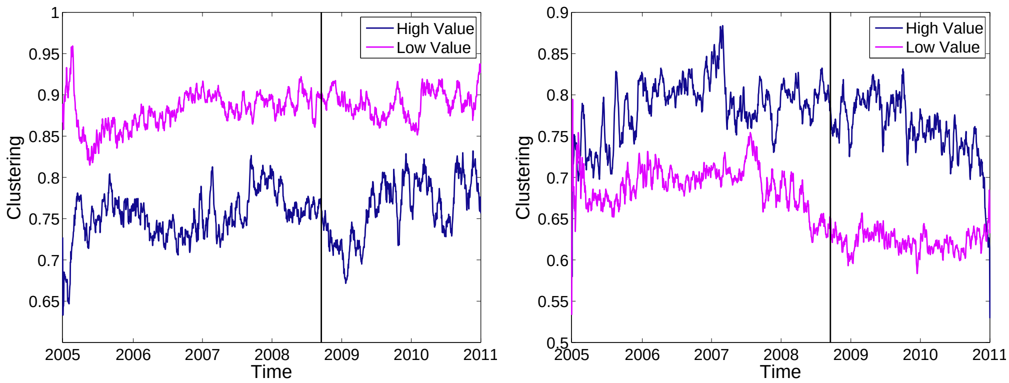
Figure 12: Evolution of the SPEI network.



(a) Borrowing Herfindahl-Hirschman Index for Bank 12.

(b) Lending Herfindahl-Hirschman Index for Bank 3.

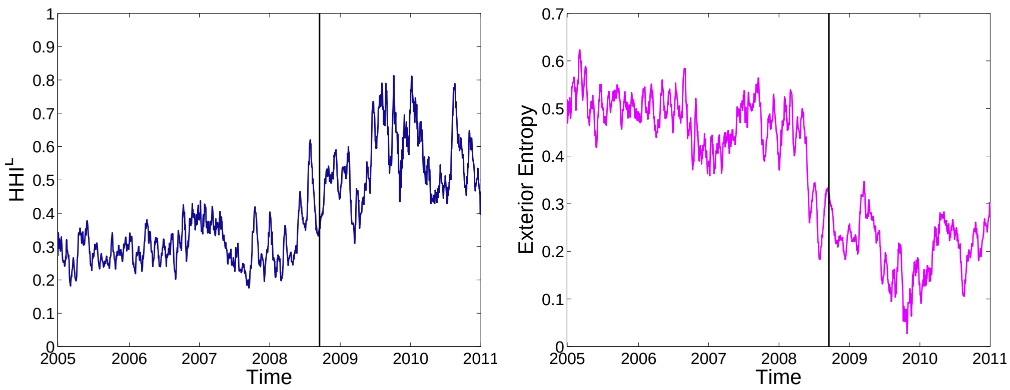
Figure 13: Lending and Borrowing Herfindahl-Hirschman Index.



(a) Clustering coefficient for Bank 25

(b) Clustering coefficient for Bank 35

Figure 14: Evolution clustering coefficient for two institutions.



(a) Lending HHI for bank B

(b) Exterior entropy for bank B

Figure 15: Concentration measures for bank B.

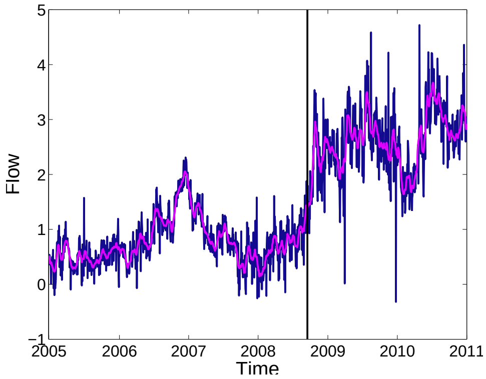
4.2.2. Interbank exposures

Among many relevant measures which can be useful to determine the vulnerability of an institution, one is concentration. Although the measurement of concentration is well known and quite simple, by means of the HHI, little attention has been given to it in the modeling of networks and systemic risk. Here we would like to emphasize again that assumptions which would imply diversification imply less concentration and, as a consequence, less risk.

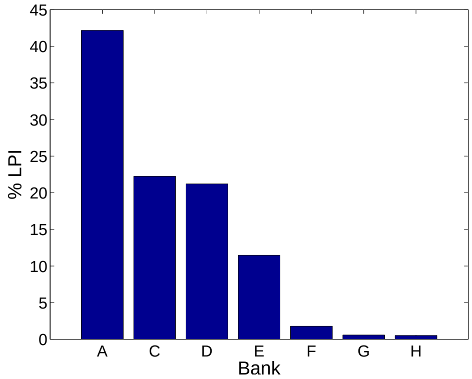
This contradicts the opinion expressed in Nacaskul (2010), in which a measure of centrality is defined in a way that banks with a higher diversification of assets and liabilities among its counterparts are considered more risky or systemically important.

When stress is building up in the financial system, concentration is harmful in both ways: to both lenders and borrowers. Figure 15 shows two concentration-related measures: the HHI and the entropy. In this figure one can observe that bank B changed its concentration as a lender after the failure of Lehman Brothers. After this event, this bank was lending in a more concentrated fashion. The entropy measure changes in the opposite way. The correlation between the two measures is -0.88, something which is very common in this network. Another relevant aspect shown here is change in the behavior of bank B; after an extreme event, bank B became more selective in lending, and we believe that this feature is impossible to replicate with random networks.

In addition to the above example, there are other important metrics which can be used to monitor the behavior of banks in the network (for example, the total, inner and outer strength, flow, etc.). These measures can indicate the role that a certain bank plays in the network and the way it changes its behavior during stressful periods. Figure 16(a) shows the flow of bank F, which increased after the fall of Lehman Brothers; meaning that this bank became a liquidity provider in the interbank exposures network. Figure 16(b) shows the preference index for several counterparts of bank I. This index points out that, in the period of maximum concentration, bank I had 40% of its funds loaned



(a) Bank F's flow



(b) Lending preference index for bank I

Figure 16: Flow and preference index for two different banks for the base case network.

to counterpart A alone and approximately another 40% jointly to banks C and D.

5. Centrality and systemic risk

This section presents the results on the centrality measures applied to all the different cases of the two networks considered in our study: the SPEI and the interbank exposures networks. Additionally, this section introduces a robustness study of such measures. As has been previously said, (see Henggeler-Muller (2006)) an institution is relevant in a network for several reasons: its degree, its closeness, its betweenness, its total strength, etc. We are convinced that instead of choosing a particular centrality measure to derive the importance of an institution in a network, it is possible to combine them in order to take full advantage of the information which is implicit on each measure. With that purpose in mind, we propose to use principal components (see Appendix B for a brief introduction to principal components) to combine all the previously mentioned centrality measures. Figures 17 and 18 show the different rankings of a bank by each different type of centrality measure for the interbank exposures network. From such figures one can see that, although there are some centrality measures which assign similar rankings (degree and closeness, for example), there are some others which behave in a very different way, like EEC and betweenness.

Figure 19 shows, in one single frame, how such measures of centrality are combined to make a single measure of centrality for bank F. A very interesting feature is that the principal components measure of centrality is not contaminated by the variability of the betweenness and closeness centralities. It is important to point out that the betweenness and closeness centralities changed dramatically after the failure of Lehman Brothers. Additionally, the figure shows that the ranking for this bank fluctuates around fifth place but is not static. This result has important implications for systemic risk purposes and for the determination of systemic relevance. For example, in this figure it is possible

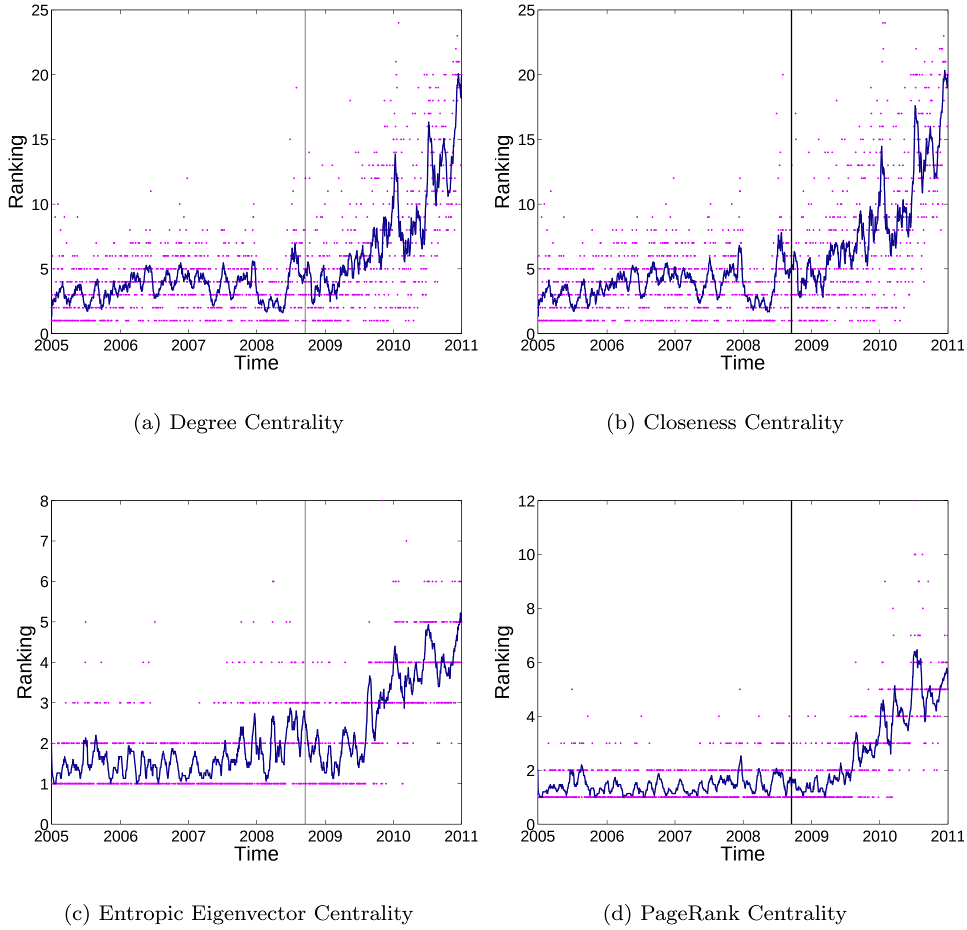
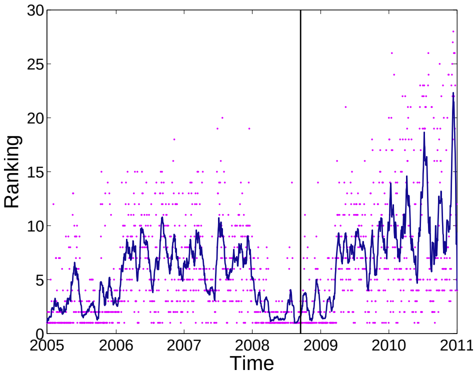
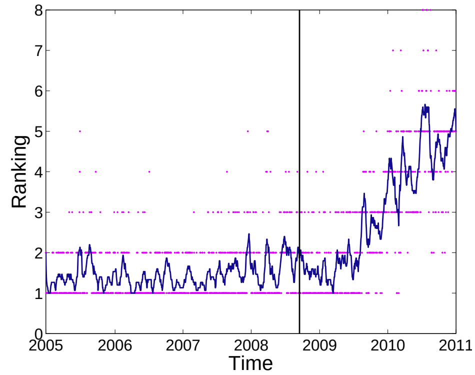


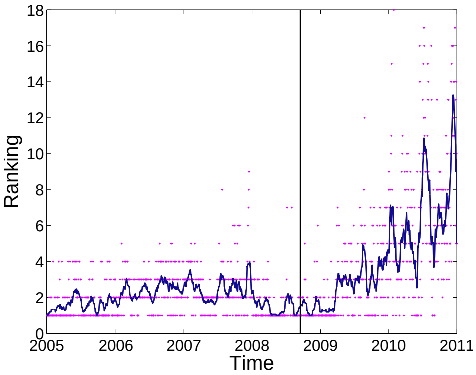
Figure 17: Individual centrality measures.



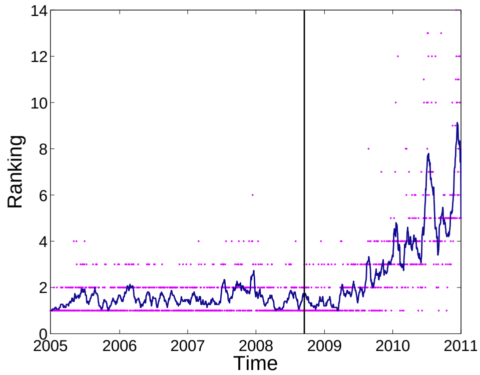
(a) Betweenness Centrality



(b) Strength Centrality



(c) Principal Components Centrality, three variables



(d) Principal Components Centrality, six variables

Figure 18: Individual centrality measures.

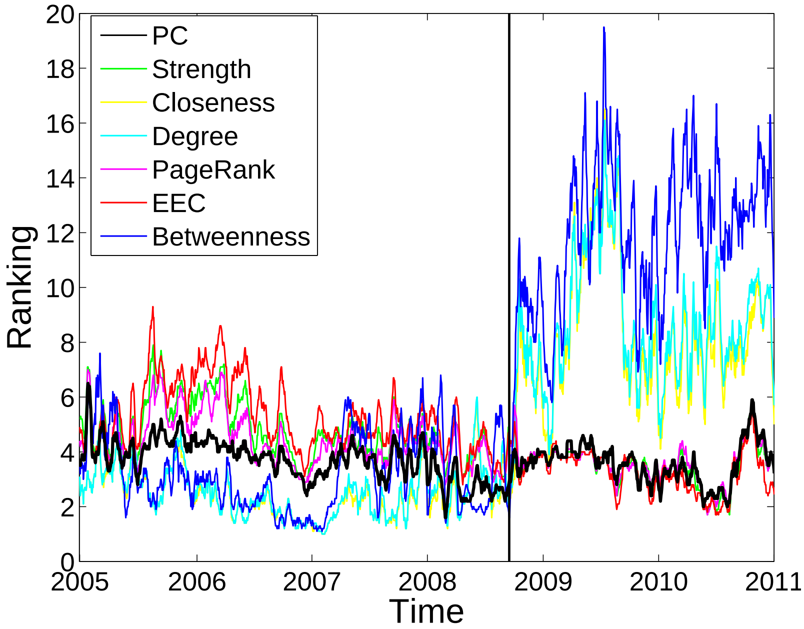


Figure 19: Principal Components Centrality.

to see that, according to the principal components measure of centrality, around July 2010 this bank could be considered as the third most important bank in the network, but around October 2010 this bank could be considered as the sixth most important bank in the network. Although, we use this example for illustrative purposes, there are other revealing examples of changes in importance on the studied networks.

The criteria that was used to decide which principal components measure of centrality to use was based on a correlation study. Figure 20 shows the pairwise correlations between some centrality measures. From this figure one can see that there are some centrality measures which have an almost perfect positive correlation like strength centrality and PageRank centrality. Nevertheless, there are some centrality measures which appear to contribute different information to the unified measure like betweenness centrality, closeness centrality and degree centrality. The highly correlated centrality measures, EEC, PageRank and strength can be used interchangeably, but we would recommend the measure with less computational complexity. However, since these measures are highly correlated in this particular network, things might turn out to be different in other types of networks. Finally, we will provide some evidence on the robustness of such centrality measures.

In the following subsections only the principal components centrality measure will be used for the interbank exposures and the SPEI networks. Some important centrality results for the SPEI payment flows will be reported and, afterward, the interbank exposures results will be also be reported.

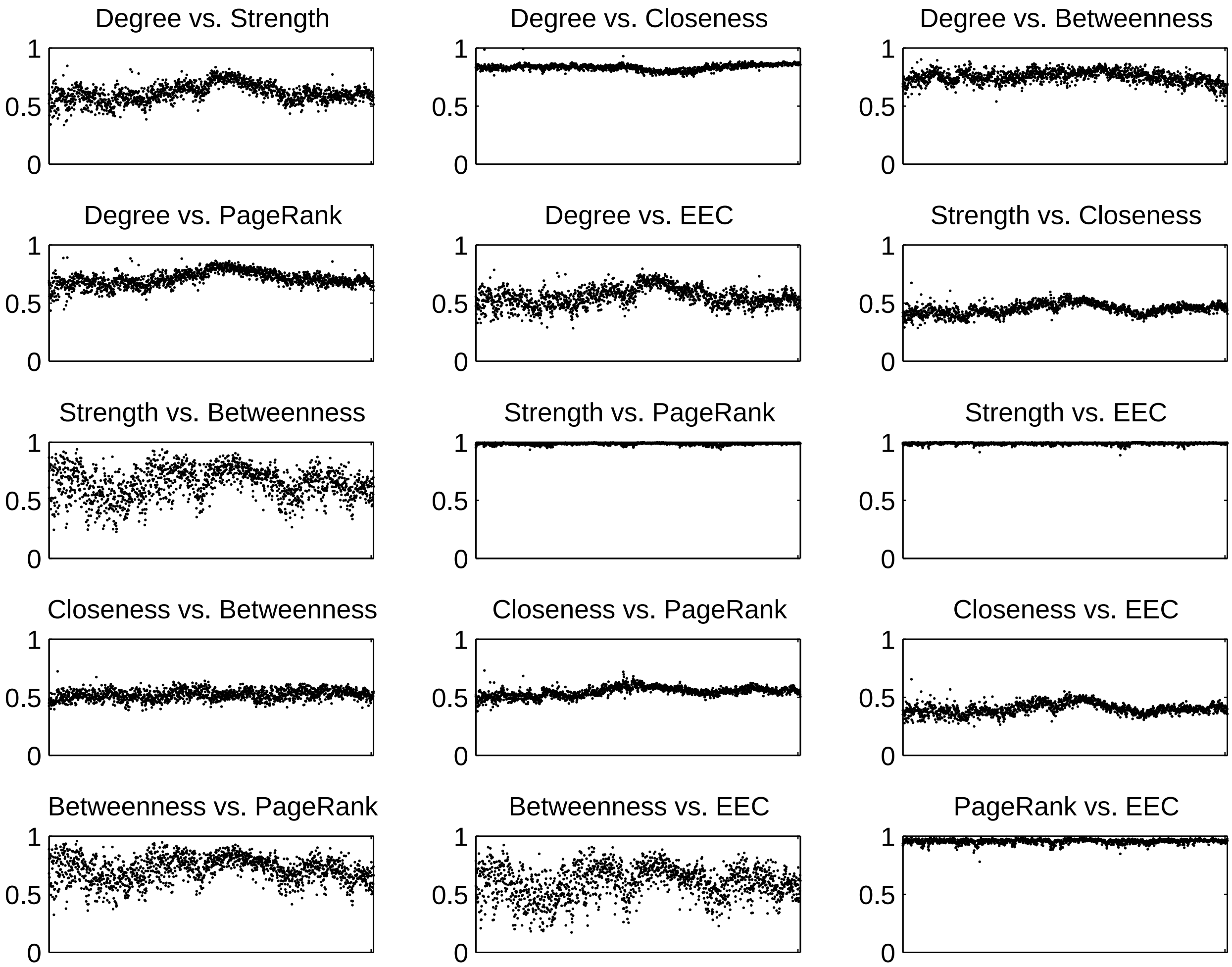
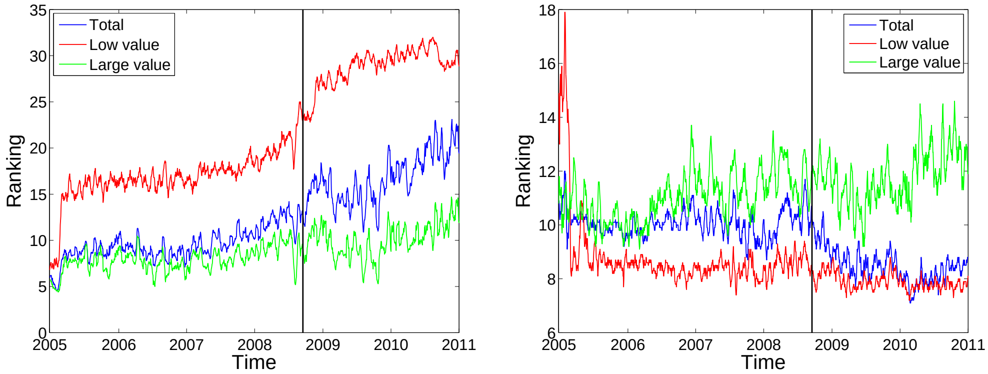


Figure 20: Pairwise correlations between centrality measures.



(a) Low-value vs large-value centrality for bank C

(b) Low-value vs large-value centrality for bank D

Figure 21: Centrality for low-value and large-value payments in SPEI.

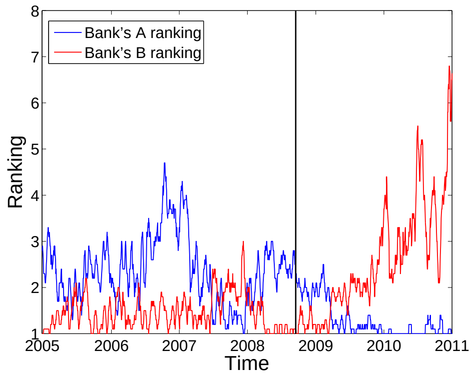
5.1. Payments system

This subsection will show some centrality measures findings for the SPEI networks. As was previously said on Section 3, with the SPEI data two networks were generated: a network only with low-value payments and a network with high-value payments. For both networks, some centrality findings are reported in order to discover if an institution plays a more important role for low-value or large-value payments. Figure 21(a) shows that this bank plays a more relevant role in the network of large-value payments, whereas Figure 21(b) shows that this bank plays a more important role in the network of low-value payments. By differentiating the size of payments and building three different networks one can understand why a bank is important in the payments network.

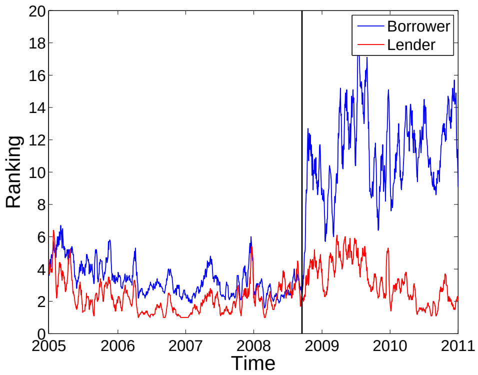
Summarizing: using the principal components approach to create a single centrality measure for the SPEI networks was highly effective in terms of capturing the variance of the data (in this case, the variance of the rest of the centrality measures.). In the total SPEI network, the average explained variance was around 73% with a PC index constructed with three variables and 77% with a PC index constructed with six variables. The PC index constructed with three variables, on its worst day, explained 50% of the variance and, on its best day, explained 86%. On the other hand, the PC index constructed with six variables explained 62% and 88% on its worst and best days, respectively.

5.2. Interbank exposures

The interbank exposures network also provides us with interesting findings. For example, Figure 22 shows some relevant discoveries for the interbank exposures network: Figure 22(a) shows changes in centrality for two different banks. In the figure, it is possible to see changes in the relevance of each institution in terms of the PC centrality. Figure 22(b) shows an interesting finding. After the failure of Lehman Brothers, bank



(a) Changes in centrality for banks A and B



(b) Changes in centrality behavior for bank C

Figure 22: Changes in centrality.

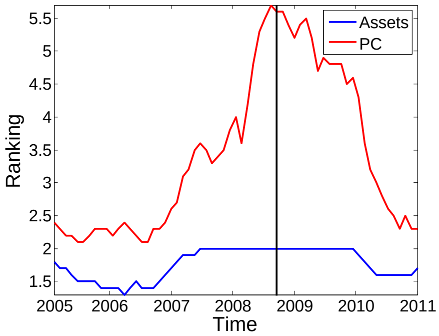
C started to play a less important role in the network as a borrower but preserved its importance as a lender. We consider both results to be very relevant as systemic importance on interconnectedness terms has not been studied with this frequency and under this perspective. This evidence would add more facts to our position that the behavior of banks changes and, in particular, such changes are more dramatic after a very important event.

Given some of the above-mentioned centrality measures, regulators can better estimate the importance of a bank in the exposures network. Such measures go beyond size and, in some cases, are not correlated or even negatively correlated with the size of an institution (see table 4). A negative correlation of interconnectedness measures and size, for a large bank, would indicate that such a bank is not that important in terms of interconnectedness or, at least, not to the same degree as for its size. A negative interconnectedness correlation with its size for a small or medium-sized bank would indicate that such a bank is more important based on its connectivity than based on its size. Figure 23(a) compares the PC centrality with the Assets centrality of a medium-sized bank. It can be seen that, even though based in its assets the bank does not seem important, based on the PC centrality, the medium-sized bank achieves a top-ten ranking. On the other hand, Figure 23(b) depicts a large bank whose relevance seems low in terms of PC centrality while in terms of assets it is the second most important bank in the system.

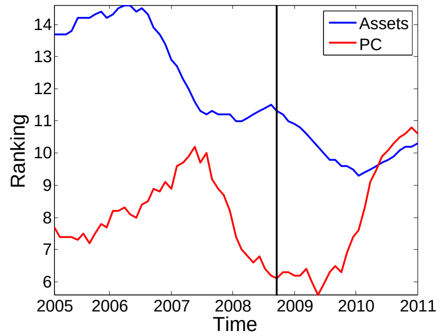
Minimum	Maximum
-0.109	0.764

Table 4: Correlation between PC centrality and Assets centrality.

Table 5 shows some basic statistics of the centrality for Bank E for the base case. From this table one can observe that Principal Components measures of centrality have



(a) PC centrality and Assets centrality of a large bank



(b) PC centrality and Assets centrality of a medium-sized bank

Figure 23: PC centrality and Assets centrality

a relatively low standard deviation, lower than for the betweenness, degree and closeness centrality measures.

	Mode	*Mean*	*Std. Dev.*
C_B	6	9.89	5.10
EEC	2	2.72	1.51
PR	2	3.16	1.80
C_D	6	8.12	4.37
C_C	6	8.78	4.94
C_S	2	2.96	1.68
$PC^{(3)}$	4	4.89	2.77
$PC^{(6)}$	3	3.44	2.07

Table 5: Statistics of centrality measures for Bank E.

5.3. Robustness of centrality measures

There are some desirable properties that a centrality measure should have. For example, in Sabidussi (1966), the author proposes that the centrality of a node should increase if an arc is added to such a node. Additionally, to add a node anywhere in the network should not decrease the centrality of any node. Betweenness centrality does not satisfy this criteria, but this centrality measure is important in contagion transmission terms. In Freeman (1979), the author proposes that a centrality measure must qualify the node at the center of a star network as the node with the highest centrality. This is because this node has the highest degree, is included in all the paths between the other vertices, and is the closest to the rest of the nodes.

Besides the above-mentioned desirable characteristics of a centrality measure, a centrality measure can be considered as robust if a small perturbation on the input data causes small changes in the output. This is important when there could exist errors in the data used to define the network[13] because centrality would be assigned in similar ways as if there were no errors on the input data.

In order to test the robustness of centrality measures Frantz and Carley (2005) perform simulations where an initial network is generated and a copy with perturbations is created. Perturbations consist of creation and elimination of arcs and nodes. Nevertheless, the networks simulated in this study do not include weights, and the centrality measures considered are only topological measures. Additionally, the authors also associate the robustness of the centrality measures considered with different topologies.

We believe that the robustness of centrality measures is important in our context, although we do not consider perturbations as errors but as changes on a dynamic network. Therefore, robustness is important in this context as such measures will be used by financial authorities which monitor important financial networks through time. The robustness of centrality measures for the interbank exposures and the SPEI networks will be associated with a similar centrality assignment from one day to another.

The study reported here is similar to the one proposed in Frantz and Carley (2005). Networks with 100 nodes were generated using the Bollobas model with parameters $\alpha = 0.2, \beta = 0.6, \gamma = 0.2, \delta^- = 0.01, \delta^+ = 4.45$. The arcs' weights are numbers from a power law distribution with parameters $\eta = 1.92, x_{min} = 297,330$. The perturbations to the network consisted of creation and deletion of a percentage of arcs. Such percentages determine an error level: 1%, 5%, 10% and 20%. The arcs for elimination were selected randomly from a uniform distribution, and created arcs were also randomly created from a uniform distribution. After the perturbation of the original network, the centrality measures were computed and, in order to compare such measures, we used the criteria proposed in Frantz and Carley (2005). Such congruence measures are:

- Top k vertices. This measure takes the value of 1 if the vertex with the highest centrality in the original network is among the k nodes with the highest centrality in the perturbed network. In this study k took the values 1, 3 and 10.

- Bottom k vertices. This measure takes the value 1 if the vertex with the lowest centrality in the original network is among the k nodes with the lowest centrality in the perturbed network. In this study k took the values 1, 3 and 10.

- Correlation. The correlation coefficient computed among the values of the centrality measures in the original network and the values of the centrality measure in the perturbed network.

- Overlap k. This measure takes values between 0 and 1. It reflects the extent to which the 10 vertices of category k in the original network matches with the 10 vertices of category k in the perturbed network. Category 1 corresponds to the top 10 vertices, Category 2 corresponds to the next 10 vertices below the Category

[13]This is more important for other social networks which are more prone to errors in the input data used to define the bilateral relationships.

1 vertices and so forth. Given that we generated 100 vertices, there will be 10 categories. The measure is computed in the following way:

$$O_k = \frac{|V_k^o \bigcap V_k^p|}{|V_k^o \bigcup V_k^p|}$$

where V_k^o is the set of vertices which belong to the category k in the original network and V_k^p is the set of vertices which belong to category k in the perturbed network. This measure is used to identify how much the vertices change categories once the network is perturbed.

For every original network, perturbed network and type of error, a thousand independent replications were made. The results for every congruence measure are reported below.

5.3.1. Top k vertices

All the centrality measures were robust under the Top k vertices congruence measure, even for the maximum error level. Table 6 shows the percentage of time the measure took the value 1 for the maximum error level. These results show that centrality measures classify approximately in the same way as the node with the highest centrality in both networks, the original and the perturbed one.

	Creation			Deletion		
Top k	$k=1$	$k=3$	$k=10$	$k=1$	$k=3$	$k=10$
C_D	99%	100%	100%	96%	100%	100%
C_S	94%	100%	100%	92%	99%	100%
C_C	98%	100%	100%	94%	100%	100%
C_B	97%	100%	100%	92%	100%	100%
PR	97%	100%	100%	96%	100%	100%
EEC	90%	95%	98%	90%	92%	95%
$PC^{(3)}$	97%	100%	100%	93%	100%	100%
$PC^{(6)}$	96%	100%	100%	93%	100%	100%

Table 6: Percentage of right classification under the Top k vertices congruence measure for an error level of 20 %

The slightly less robust measures were C_S and EEC. The latter even placed the node with the highest centrality in the original network outside the top 10 in the perturbed network. These measures depend on the weigths of the arcs. On the other hand, the measures created under the principal components methodology were relatively robust. Finally, all centrality measures were more sensitive to the removal of arcs than to the addition of arcs.

5.3.2. Bottom k vertices

The centrality measure, C_B, is very effective in identifying the least central vertex and the less central vertices. This measure is robust to perturbations of small error levels but affected at the highest level of error. Additionally, this centrality measure is more sensitive to the creation of arcs than to the deletion.

The rest of the centrality measures change the result for the least central vertex, even with the lowest error level. Therefore, measure C_B is the most robust centrality measure under this type of congruence measure. Finally, perturbation by elimination has a greater effect than perturbation by creation of arcs.

	Creation			Deletion		
Bottom k	$k=1$	$k=3$	$k=10$	$k=1$	$k=3$	$k=10$
C_D	75%	99%	99%	4%	100%	100%
C_S	75%	99%	99%	4%	100%	100%
C_C	75%	98%	99%	4%	94%	98%
C_B	99%	99%	99%	100%	100%	100%
PR	74%	98%	99%	5%	99%	100%
EEC	74%	99%	99%	4%	99%	100%
$PC^{(3)}$	75%	99%	100%	4%	99%	100%
$PC^{(6)}$	75%	99%	100%	3%	98%	100%

Table 7: Percentage of right classification under the bottom k vertices congruence measure for an error level of 1%

5.3.3. Correlation

When the correlation congruence measure is considered, the error by arc creation causes measures C_D, C_C and C_B to suffer slight alterations in their results with the lowest error level (superior blue line in Figure 24 (a)). When the error level increases correlations decrease, in proportion with the error level. This is in contrast to the rest of the measures, which with the lowest error level show bigger alterations in their results (Figure 24 (b)). Among them, the most robust is EEC, whose changes are less dramatic than the ones of *PageRank*, C_S, $PC^{(3)}$ and $PC^{(6)}$.

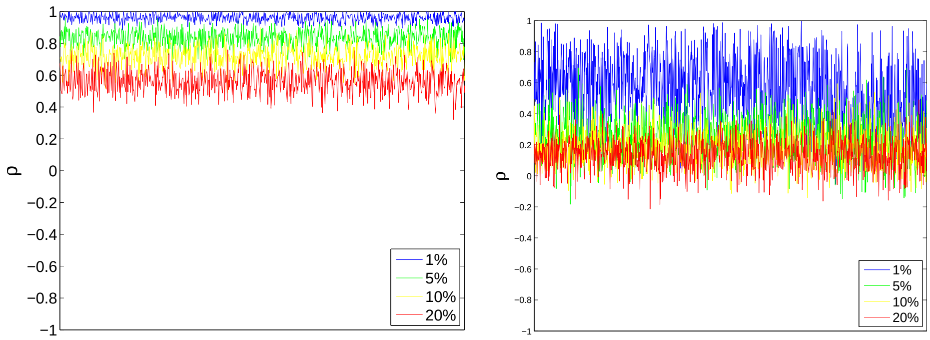
For the arc elimination perturbation, the most robust centrality measure was C_B, with correlations above 0.9, even for the maximum error level. The centrality measure C_D had a similar performance in the arc creation perturbation (similar to Figure 24 (a)). Measure C_C had more severe alterations than C_D. Finally, the rest of the measures had a similar performance: changing the error level causes important changes in results having correlations around 0.4, similar to Figure 24 (b)).

Measures C_B and C_D were affected more by the arc creation than by the arc elimination, whereas the other measures had the opposite behavior.

5.3.4. Overlap k

Arc creation affected all centrality measures non-monotonically (Figure 25 (a)). Measures were more affected in the central categories than in the extreme categories. Each measure presents different effects regarding the first category, and the degree of change converges for the last category exept for C_B. In particular, $PC^{(6)}$ was more robust than $PC^{(3)}$ for the first two categories. Considering all categories, the most robust measures under this criteria were EEC and C_B.

Arc elimination had an almost monotonic decreasing effect for most of the measures (Figure 25 (b)). In general, all measures present the same degree of impact except for the C_C. Measure C_B had very different behavior from the rest of the measures, being slightly affected in the first categories.



(a) C_D (Arcs creation)

(b) $PC^{(6)}$ (Arcs deletion)

Figure 24: *Correlations for the centrality measures* C_D *y* $PC^{(6)}$ *plotted against iteration.*

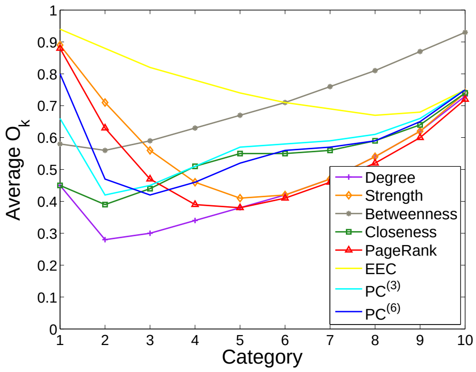
6. Conclusions and further work

The most important conclusions in this paper are the following:

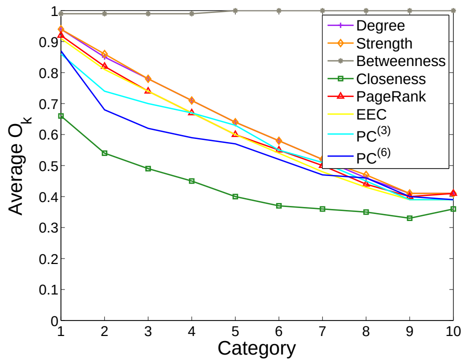
- Several centrality measures can be taken into account simultaneously by generating a single centrality measure. Such a measure can be used for regulatory purposes. This is important because taking a decision on which measure to use or which measure is more important can be very difficult. Moreover, if a measure like betweenness is chosen, its variability could make regulation impossible. However, such a measure should not be disregarded because of its importance on the study of contagion. Finally, if one studies the networks generated by type of exposure or by size of payment, one can identify the different roles that banks (or financial institutions) play in the financial system.

- More behavior-related measures can be useful in studying the financial system, such as the HHI or the preference index. Additionally, such measures provide some evidence of the difficulty generating meaningful financial networks by using random graphs theory. These behavioral aspects are crucial to the monitoring and measurement of systemic risk. In our study we observed that the preference index can change dramatically (from 0 to 1) only by changing the time window used to compute the index. We believe that by adding the frequency to the intensity of the relationship a more meaningful measure can be obtained.

- Network formation models might not be fully appropriate because the weights on the network are very important for economic reasons. Additionally, our experience with the data is that when a new bank enters the financial system, this bank does not establish a lending/borrowing relationship with the most connected nodes; this could be in contradiction to the preferential attachment models. More work on the generalization of such models is necessary in order to be useful in financial



(a) Arcs creation



(b) Arcs deletion

Figure 25: *Average Overlap (O_k) by category for an error level of 5%*

networks because such models do not provide a means to model bank behavior or to an understanding of the incentives of the participants.

- The robustness study is the first of its type on financial networks. The work which inspired this paper was done on unweighted networks and mostly topological measures were used. This paper provides some good evidence of the robustness of more sophisticated centrality measures, including the principal components composite index.

To summarize: in this paper a centrality measure is proposed which can be used for regulatory purposes. This measure estimates the interconnectedness of a bank in the best possible way. Moreover, this unified measure of interconnectedness can be employed on the methodology proposed by the BCBS to determine G-SIBs but can also be extended to the context of Domestic-SIBs. A wide range of empirical measures is also provided as is a complete description of two very important networks for the Mexican banking system: the network of interbank exposures and the network of the payments system flows. Some measures, beyond the topological ones, which can be very useful for financial stability monitoring, are also proposed. In addition, this paper includes a simulation study on the robustness of some centrality measures which have not been tested in the past. Finally, some of the main findings of this study are: the payments system network is strongly connected; in contrast to the interbank exposures network, connectivity of the payments system network increases with time; and last but not least, the proposed unified measure of interconnectedness for banks is uncorrrelated with assets size for some specific cases.

Possible lines of extensions to this work include the development of more general network formation models, and the study of other financial networks like the securities settlement network. Additionally, more can be done to test the robustness of centrality measures by exploring more exhaustively the parameters of the Bollobás model, including different network architectures or considering perturbations similar to the ones described by the evolution of the SPEI or the interbank exposures network, in which arcs are created and eliminated at the same time.

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Appendix A. Entropy

The entropy concept was originated in physics and has also been used in information theory. In this appendix we will provide some basic definitions of entropy and how this concept is being used in the context of systemic risk. In particular, we will review briefly the concept of entropy from information theory's point of view.

Let's assume that we have a set of messages $X = \{x_1, x_2, ..., x_n\}$ or information sources, with probability distribution $P = \{p_1, p_2, ..., p_n\}$ which associates with each x_i a probability p_i of appearance.

The quantity of information obtained from a message x_i with probability p_i is defined as:

$$I(x_i) = \log\left(\frac{1}{p_i}\right). \tag{A.1}$$

Shannon proposed in 1948, (see Shannon (2001)) to characterize the information gained from the reception of a message with a function which depends on the likelihood of the messages to appear. The most likely messages provide less information, whereas the least likely messages provide more information.

Entropy is the expected information of X. Assuming that X is a discrete random variable, then the entropy is the expected value of the random variable $I(X)$:

$$H(X) = \mathbb{E}(I(X)) = \sum_{i=1}^{n} p_i\, I(x_i) = -\sum_{i=1}^{n} p_i \log(p_i) \tag{A.2}$$

with the convention that $0 \cdot \log(0) := 0$. This is known as the discrete Shannon entropy.

The minimum value for the entropy is reached when $p_j = 1$ for some $j \in \{1, 2, ..., n\}$ and $p_i = 0$ for all $i \neq j$. In this case

$$H(X) = p_j \log\left(\frac{1}{p_j}\right) = 0. \tag{A.3}$$

Entropy is maximized when the appearance of any symbol is equiprobable; that is, when $p_i = \frac{1}{n}$ for all $i \in \{1, 2, ..., n\}$, then

$$H(X) = -\frac{1}{n} \sum_{i=1}^{n} \log \left(\frac{1}{n} \right) = \log(n). \tag{A.4}$$

The use of entropy in the context of systemic risk is widespread, to the point of being computed over sets which are not probability distributions. Entropy has been used several times in this paper: on the centrality measures, as a principle to estimate the network of exposures, and as a measure of heterogeneity and concentration.

Appendix A.1. Entropy for centrality measurement

In Section 2.3 it was mentioned how in Nacaskul (2010) the author used the concept of entropy to define entropic eigenvector centrality. In this paper, the author argues that a vertex, k, with a higher degree and higher diversification of its weights to its counterparts should be considered as more important than the other vertex, l, with few counterparts and higher concentration of weights. The computation of the entropy along the row of the exposures matrix, W^{norm}, corresponding to vertex l is larger than the entropy calculated over the row corresponding to vertex k.

As was previously mentioned, we believe that a bank with more concentration in its funding and loans has more risk and is more likely to be affected by contagion than a bank which is less concentrated. Concentration in lending or borrowing would be reflected in a higher HHI and a lower entropy. Therefore, the EEC could be weighting more on vertices which are less important than nodes lending and borrowing in a more concentrated fashion.

Nevertheless, considering again the same vertices, k and l, the vertex with the highest entropy, k, also has more counterparts and its failure could impact more participants in the system. This could also imply that, to a certain extent, the measure EEC relies on or at least incorporates the degree centrality measure.

Summarizing: Wether EEC is assigning importance in an adequate manner or not should be studied with more care in the future.

Appendix A.2. Entropy as a measure of concentration

A very important characteristic of scale-free networks is the heterogeneity of the arcs' distribution. In Wang et al. (2006) the authors use entropy to measure heterogeneity. They claim that, making a scale-free network more robust to the random deletion of vertices is equivalent to maximizing the entropy of the degree distribution. After expressing the entropy of the degree distribution for a scale-free network in terms of the number of vertices of the network and the parameters of the power law distribution, the authors obtain expressions for the values of the parameters which maximize entropy.

In Saltoglu and Yenilmez (2010) the authors interpret entropy as a measure of heterogeneity, but they compute the entropy over the vector used for the modified PageRank algorithm, which they propose as a centrality measure. In the most homogeneous form of this vector, institutions would have similar connectivity and, as a consequence, entropy should be high. The existence of an institution which is becoming more important would cause the entropy to decrease. The authors compute this measure with data from the Turkish financial system and they identify, by means of the entropy, a deviation on the homogeneity at the beginning of a financial crisis in Turkey. This crisis led to the failure of Demirbank, which became more central at this time. Finally, the authors conclude that these measures can be used to forecast a financial crisis.

Let's consider the computation of entropy over a vector which contains the information about the loans given by bank i to the system. Such a vector is the w_i^L row of the matrix W^L standardized (in such a way that the sum of the entries is 1); that is, the entries of the vector are $w_{ij}^L = \frac{w_{ij}^L}{\sum_{j=1}^n w_{ij}^L} \quad \forall j \in \{1, ..., n\}$.

If the exposures of such a bank are distributed homogeneously among its counterparts, then one would have large values for the entropy, and if some of the exposures are more concentrated in a few counterparts, the entropy would take smaller values. In this specific case, the entropy can be interpreted as an indicator of concentration, in the opposite sense to the HHI. In fact, the correlation obtained in one of the examples presented here is close to -1.

Appendix A.3. Maximum entropy for network estimation

Some central banks do not have complete information regarding the exposures between banks. Typically, only total exposures or total loans are known and, in the best cases, some central banks have information about the counterparts whose exposures are the largest. In matrix terms, for a matrix W^L, such central banks do not know the individual w_{ij}^L but only the total exposure of bank i to the system, $\sum_j w_{ij}^L$, and total borrowing of bank i from the system, $\sum_i w_{ij}^L$.

It is common in several papers on the study of contagion through the interbank lending market to assume that banks distribute their loans in an equitable way (see Upper (2007) for a summary on several papers and the assumptions made). This corresponds to maximizing the entropy among the interbank loans.

This assumption relies on the relative entropy, also known as the Kullback-Leibler divergence. Given the distributions $P = \{p_1, p_2, ..., p_n\}$ and $Q = \{q_1, q_2, ..., q_n\}$ for a discrete random variable, X, which takes values $\{x_1, x_2, ..., x_n\}$, the relative entropy of X over P with respect to Q is defined by[14]:

$$H_{P|Q}(X) = \sum_i p_i \log \frac{p_i}{q_i}. \tag{A.5}$$

Relative entropy can be interpreted as a measure of deviation between two probability distributions. If this entropy is large then both distributions are "distant". In this context, the probability distributions used are the standardized entries of the matrix W^L and the matrix associated with the maximum entropy assumption.

The unknown entries one wishes to compute are the solutions to an optimization problem whose objective is to minimize an expression similar to (A.5), subject to some constraints, including the condition that the diagonal has zero values. The reader may find a detailed description of this methodology in Sachs (2010).

Nevertheless, there are some empirical examples which have shown that exposures in banking systems are not homogeneously distributed, and such an assumption might

[14]This is only defined if $q_i > 0$ for any i such that $p_i > 0$ and it is also agreed that $0 \cdot \log(0)$ y $0 \cdot \log(\frac{0}{0})$ are interpreted as zero.

distort the estimation of contagion risk. Moreover, in Mistrulli (2007) the author shows that the maximum entropy assumption underestimates, in some cases, and overestimates, in others, the risk of contagion.

Appendix B. Principal Components Analysis

The goal of this procedure is to use an orthogonal transformation to transform a set of observations, possibly correlated, into an uncorrelated set of observations. The transformation is defined such that the first principal component has as high variance as possible and each succeeding component in turn has the highest variance possible. The methodology describing the solution can be found in Jolliffe (2002). Formally speaking, given p random variables $X^1, X^2, \ldots, X^p$ such that,

$$X = (X^1|X^2|\ldots|X^p), \quad \text{and } X^j = \begin{pmatrix} X_1^j \\ X_2^j \\ \vdots \\ X_n^j \end{pmatrix} \forall j,$$

we wish to find a linear combination, c, such that it has maximum variance. We define the set of all linear combinations, $\mathcal{W}_x$, as follows:

$$\mathcal{W}_x = \{c \mid c = Xa,\ a \in \mathcal{R}^p\}. \tag{B.1}$$

We assume that all variables are standardized. That is

$$\hat{X_i^j} = \frac{X_i^j - \bar{X}^j}{S_j}$$

where $\bar{X}^j = \sum_{i=1}^n p_i X_i^j$, $S_j = \sum_{i=1}^n (X_i^j - \bar{X}^j)^2$ and $\sum_{i=1}^n p_i = 1$.

For convenience, we define the following metric to measure the distance between variables.

$$d_D^2(X^j, X^k) =< X^j - X^k, X^j - X^k >_D, \tag{B.2}$$
$$\text{with } D = diag\{p_1, p_2, \ldots, p_n\}$$

Such that

$$< X^j, X^k >_D = X^{j^T} D X^k = S_{j,k} \tag{B.3}$$
$$||X^j||_D^2 =< X^j, X^j >_D = S_j^2 \tag{B.4}$$
$$\cos(X^j, X^k) = \frac{S_{j,k}}{S_j S_k} = \rho_{j,k}. \tag{B.5}$$

Therefore, the covariance matrix is defined by $V = X^T D X \in \mathcal{R}^{p \times p}$ and the first principal component is obtained by solving the following optimization problem,

$$\max_a a^T V a \quad \text{s.t. } ||a||_2^2 = 1 \tag{B.6}$$

whose solution is the eigenvector associated with the maximum eigenvalue.