Banco de México

Documentos de Investigación

Banco de México

Working Papers

N° 2016-12

Functional Systemic Risk, Complementarities and Early Warnings

Carlos Cañón
Banco de México

Santiago Gallón
Universidad de Antioquia

Santiago Olivar
Banco de México

July 2016

Functional Systemic Risk, Complementarities and Early Warnings*

Carlos Cañón[†]
Banco de México

Santiago Gallón[‡]
Universidad de Antioquia

Santiago Olivar[§]
Banco de México

Abstract: This paper proposes a systemic risk index based on Functional Data Analysis (FDA), overcoming salient shortcomings of standard methodologies related to data usage, data sparseness, and high dimensionality issues. Using Mexican data, a set of systemic risk indexes are constructed and we show that the proposed functional index captures new information, and through simulations, that it outperforms previous methods when the indicators become increasingly nonlinear. Finally, we show which indexes serve as complements, and which are the best early warning indicators.

Keywords: Systemic Risk; Functional Data Analysis; Dimensionality Reduction; Signal Informativeness

JEL Classification: G01, G10, G17, G18

Resumen: Este documento propone un índice de riesgo sistémico basado en Análisis Funcional de Datos (AFD) que proporciona una alternativa a limitantes, frecuentes en otros métodos estándar, asociados al uso de los datos, presencia de datos dispersos y alta dimensionalidad. Se construyó un conjunto de índices de riesgo sistémico utilizando datos mexicanos. El índice funcional propuesto captura nueva información y, simulaciones muestran, que, en presencia de no linealidades, es mejor que los demás métodos. Finalmente, mostramos cuáles índices se complementan entre sí y evaluamos su desempeño como indicadores de alerta temprana.

Palabras Clave: Riesgo Sistémico; Análisis Funcional de Datos; Reducción de Dimensionalidad; Informatividad de una Señal

1 Introduction

Since the 2008-09 financial crisis, which took a significant toll on governments, financial institutions and the worldwide economy, there exists a growing interest by policy makers and the academic community in measuring and monitoring systemic risk in order to anticipate the occurrence of costly events, thus safekeeping financial stability. Several measures have been proposed during the last few years, some of them improving the way weights assignment on indicators used as inputs, others tackling the question of which inputs should be used, and others proposing a succinct definition of systemic risk. Additionally, other attempts have been directed towards reducing the dimensionality of the initial set of indexes or indicators, or finding the best set of Early Warning Indicators (EWI).

A crucial question arises: *Do standard methodologies to measure financial systemic risk use all available information?* The answer to this question relies principally on how the information is extracted from the data, the set of indicators used to construct the indexes, and which indexes are combined into a superior index. However, despite the large number of proposed indexes there is no consensus about how the financial systemic risk should be measured (see Bisias et al. (2012) for an extensive discussion).[1] On the one hand, the lack of consensus is not unexpected as some indexes are tailored to a narrow context. For example, some indexes seek to measure systemic risk, while others measure the contribution of a particular financial institution to an aggregate measure of systemic risk. On the other hand, we cannot discard that standard methodologies can be improved. For example, current indexes have difficulties in incorporating large number of indicators, data gaps or to deal with nonlinearities.

In relation to the data needed to construct Financial Systemic Stress Indexes (FSSIs), commonly used methodologies do not appropriately use the information contained at the structural features of the sample data. In other words, the fact that an index has, for instance, an increasing or decreasing trend, or a periodic shape, or a local volatile behavior is itself informative. Up to our knowledge this information is not used in the systemic risk literature[2], and we propose the use

---

[1] Bisias et al. (2012) argue on the desireability of not having a single systemic risk as it has different sources.

[2] Current methodologies do not extract information from the order of an indicator. For them,

of statistical methodologies that allow to extract a meaningful common pattern that summarizes the information conveyed by all variables in the sample.

Additionally, there are two common data restrictions implicit on the methods to construct FSSIs. The first one is related to the no sparseness assumption in the data, in the sense that all inputs must be calculated over all measured points. However, in practice it is often the case that some inputs have only been observed at a relatively small number of points (i.e. sparseness in the inputs). The second restriction involves a relatively low or medium-scale input data. Nevertheless, due to the explosion of available information, the statistical accuracy and computational efficiency of methods are seriously affected. Therefore, a possibly nonlinear dimensionality reduction technique becomes necessary in order to handle high-dimensional data preserving the data structure as much as possible.

In this paper we propose a novel methodology to obtain a financial systemic risk index, which is simple to implement, and captures more information from the data with respect to standard methods. The new index can also deal with data sparseness and high dimensionality. With this new index at hand, we compare popular indexes (and combinations of them) used by policy makers, and asses their performance as EWIs.

In particular, we appeal to a functional framework by assuming that the sample of indicators required to construct the index is a sample of random functions rather than a set of random vectors as is usual in other methodologies for constructing FSSIs. For functional data statistical analysis see Ramsay and Silverman (2005, 2002), and Horváth and Kokoszka (2012).

Our functional methodology is a three-stage method. In the first stage, a functional clustering method is applied to identify a number of homogeneous groups of functions in the sample which share similar shape patterns. In the second stage, for each cluster a template function is estimated in order to summarize the information conveyed by all functions (i.e. preserving the structural features of curves in the corresponding cluster). Finally, based on the template functions obtained for each cluster, a functional index is built by the application of an appropriate functional dimensionality reduction technique.

By comparing the proposed index with a wide range of indexes common in

---

the fact that x_t precedes x_{t+1}, where x_t is a realization of a random variable X at time t, is no more informative than if x_{t+1} precedes x_t.

the literature and to policy makers, we obtain several results. First, we show that the functional index captures new information. In particular, we compare different methodologies with exactly the same data, and the Functional Index is not discarded by variable selection methods. This result confirms that relevant information is embedded into the shape of the variables, and that standard methods are unable to capture it. Second, we find an optimal partition of indexes and identify which of them serve as complements. Finally, we assess the signal quality of the indexes as EWIs for Mexican data.

A second set of results is obtained through a simple simulation study to show the conditions under which the proposed methodology outperforms other methods. By constructing a two-sector economy we compare the Functional Index with the same group of standard indexes, and evaluate their fit given a set of common and idiosyncratic shocks. The main finding is that the new index outperforms all other indexes when the variables become increasingly nonlinear. This result is important if we accept the hypothesis that the higher the fragility of the financial system, the more nonlinear the market variables may become. Finally, and using variables with mild nonlinearities, the functional index has a very good fit in the presence of both common and idiosyncratic shocks.

The rest of this paper is structured as follows. Section 1 finishes with the literature review. Section 2 describes the data used to construct the systemic risk indexes. The methodology of the proposed functional systemic risk index is gathered in Section 3. Section 4 presents, the results in terms of the complementarities between a set of indexes, and which index, or combination of indexes, is considered as the best EWI. Additionally, results from a simulation study of a two-sector economy are presented. Finally, some conclusions are given in Section5.

1.1   Related Literature

A vast literature has emerged trying to explain the sources of systemic risk and the propagation of financial crises, showing us the difficulties in this subject. First, we have a group of papers such as, Allen and Gale (2000), Freixas et al. (2000) and Allen et al. (2010), that argue that the failure of one financial institution leads to the default of others through a domino effect. Another way of interpreting crises is presented by Boyd et al. (2009), which explains

3

them as a mix of economy-driven shocks and governmental responses. Hence, stress indicators are seen as a measure of lagged government responses to systemic bank shocks, rather than the occurrence of crises per se. But closer to this work, Acharya and Yorulmazer (2007a), Acharya and Yorulmazer (2007b), Acharya (2009) and Farhi and Tirole (2012), argue that systemic risk is preceded by common shocks that appear indiscriminately and potentially affecting the functionality and confidence of the entire system.

Since the Great Recession, there have been numerous efforts with different and more specific approaches studying systemic risk. Such attempts include direct interbank connections (Allen and Bali (2012)), competition (Anginer et al. (2014)), size and influence of institutions (Arnold et al. (2012)), firm's characteristics (Banulescu and Dumitrescu (2015)), spillovers into the real economy (Bernal et al. (2014)), connectedness (Billio et al. (2012) and Jobst (2013)), systemic relevance of an institution (Brechmann et al. (2013)), Systemic Expected Shortfall (Acharya et al. (2009) and Acharya et al. (2010)), topological (Kuzubaş et al. (2014) and Martínez Jaramillo et al. (2014)), unstable funding (Lopez-Espinosa et al. (2012)), contribution of a component to its sector via the exposure CoVaR's (Madan and Schoutens (2013)), institutional imbalances (Oet et al. (2013)), large banks' stock return correlations (Patro et al. (2013)), tail risk of the portfolio of the banking sector's liabilities (Puzanova and Düllmann (2013)), aggregated Distance-to-Default (Saldías (2013)), correlated default (Suh (2012)), clustering coefficient (Tabak et al. (2014)), banks' exposures to common risk factors (Trapp and Wewel (2013)), global systemic risk (Weib et al. (2014)) and marginal expected shortfall (Yun and Moon (2014)).

In the following lines we explain three ramifications in which the main contributions can be categorized.

First, the most recent financial crises have highlighted that the causes of stress events evolve over time and that inaccurate signals may lead to inconsistent policy recommendations. This dilemma has awoken the interest on developing indexes that can measure and predict systemic risk events. Such examples are Lo Duca and Pletonen (2013), Hakkio and Keeton (2009), Carlson et al. (2012), Drehmann and Juselius (2012), Kritzman et al. (2010) and Hollo et al. (2012). However, as every index should be conceived as a trade off between user objectives, model complexity and data availability,

among this literature there is no evidence of a unique correct methodology for combining all the independent variables into a single overall index. Our paper proposes a systemic risk index based on a Functional Data Analysis framework, which provides an alternative to overcome common shortcomings of current indexes, as it allows for the use and analysis of additional information not captured by previous methodologies, while dealing with data sparseness and high dimensionality.

Second, from the wide range of Financial Stress Indexes that have been constructed recently, the question whether they complement each other arises. In order to respond to that, we take advantage of the methodologies of the aforementioned indexes and construct them for the case of Mexico and use the two-step methodology posited by Aramonte et al. (2013). This two-step methodology consists firstly of a test of the adjusted R^2 from regressions of changes in the principal component of all indexes - except for index i - on changes in index i. Instead of comparing the R^2 we apply the variable selection technique called Least Absolute Shrinkage and Selection Operator (LASSO) by Tibshirani (1996), which reduces the number of predictors in a penalized regression model. This method has been applied recently on various papers, such attempts include bankruptcy forecast (Tian et al. (2015)) and cross-volatility (Aboura and Chevallier (2015)).

Third, there is a literature that investigates leading indicators and their signal quality at forecasting systemic risk events. Candelon et al. (2014) evaluates a dynamic logit early warning system, which exhibits significantly better predictive abilities than their static equivalents. Laina et al. (2015) uses univariate signal extraction and multivariate logit analysis to assess the factors that drive systemic crises. These methodologies suppose a non-linear relationship between the leading indicator and the event, unlike Drehmann and Juselius (2013) which evaluates the leading indicators on the basis of their performance relative to the macroprudential policy maker's decision problem with a methodology that supposes a linear relationship between de dependent and independent variables. Using the latter methodology we evaluate and discuss the signal quality of a set of indexes constructed as leading indicators for the Mexican case.

Our paper is related to Arsov et al. (2013) and Giglio et al. (2016) in that we recognize that systemic risk indexes should not be regarded as good EWI, but we are still interested in assessing their performance as such. With respect to

Arsov et al. (2013), we also focus on "near-coincident" indicator, but the main difference is that our new index is capable to include sparse inputs, and we argue in favor of combining different indexes to improve the performance as EWI. With respect to Giglio et al. (2016), while we coincide in the necessity of combining systemic risk indexes, we are not interested in proposing a new method to combine them. Our paper is focused on proposing a new index that extracts more information from the data.

2 Data and Additional FSSIs

2.1 Data

The data has three characteristics. First, it is on a weekly basis. The indexes mainly use market based indicators which are typically available at this frequency. Second, we only include public information in the construction of Mexican FSSIs. Finally, each of the variables are available at least since 2005 in order to identify the recent financial crisis.

2.2 Additional FSSI

To measure the systemic risk we constructed several FSSIs based on the methodologies proposed by Lo Duca and Pletonen (2013), Hakkio and Keeton (2009), Carlson et al. (2012), Drehmann and Juselius (2012), Kritzman et al. (2010) and Hollo et al. (2012). For simplicity, from heron we will call them as *Lo Duca*, *KCFSI*, *Carlson*, *Debt Service Ratio*, *Absorption Ratio* and *CISS*, respectively. Despite we do not modify the original methodologies, due to data limitations we adapted some indicators to the Mexican case. Table 9 at Appendix D describes in greater detail the data.

We selected a group of indexes based on three criteria. First, we are interested on measuring systemic risk alone. Thus, we are not interested on measuring the contribution of a particular financial institution to systemic risk, nor to what extent a financial institution can be affected by a systemic episode. Second, we want to compare our Functional Index with other indexes extensively used by policy makers, or cited by academic papers. Finally, we concentrate on

those indexes that basically require having access to Bloomberg. Each index is calculated with Mexican data.[3]

The Debt Service Ratio, proposed by Drehmann and Juselius (2012), is an index specially tailored to track the indebtedness of the economy's private sector, which is a variable that the literature finds highly correlated with financial stress episodes. Another issue is the definition itself of a stress episode, and Hakkio and Keeton (2009) propose a definition, and subsequently an index.

The next index, proposed by Lo Duca and Pletonen (2013), contributes to a more accurate identification of the starting point of an episode of financial stress, strange as it sounds this is a relevant research question. Later we present an index proposed by Carlson et al. (2012), it investigates alternatives to standard weighting methods, e.g. using principal component analysis, for a given set of indicators.

Kritzman et al. (2010) propose the Absorption Ratio, which measures the systemic risk by the extent to which the market is more coupled, causing shocks to propagate faster and broadly along the system. The last index we use, proposed by Hollo et al. (2012), is the first to provide a statistically founded measure of systemic risk. The authors use portfolio theory to develop the index, and assume that systemic risk arises from different sectors of the economy. Appendix B provides a detailed description about these FSSIs, and Table 8 at Appendix D presents a short description of all of them.

We present all the indexes on one spot, see Figures 1-6. The peaks in all the indexes capture episodes of high financial stress. For example, *Carlson*, *CISS*, *Debt Service Ratio* and *KCFSI* capture the uncertainty from the Greek Rescue Plan. Also, all indexes reflect high levels of stress during the Lehman Brothers bankruptcy.

---

[3] All the Matlab and Stata codes are available upon request.
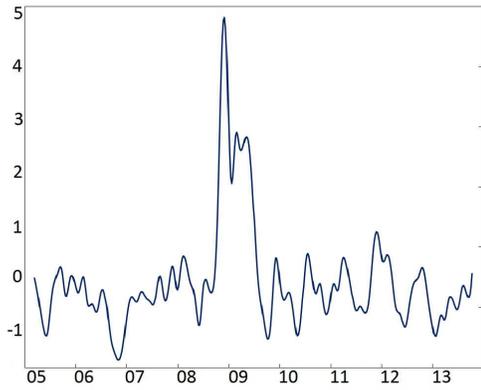
Figure 1: Carlson
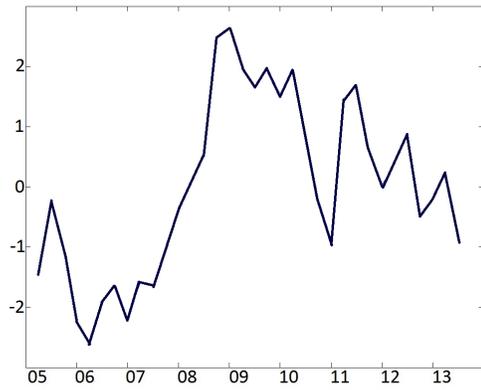

Figure 2: Debt Service Ratio
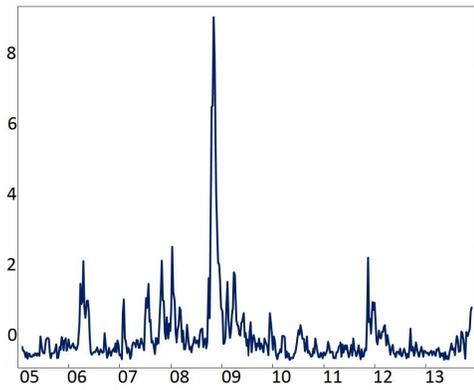

Figure 3: CISS
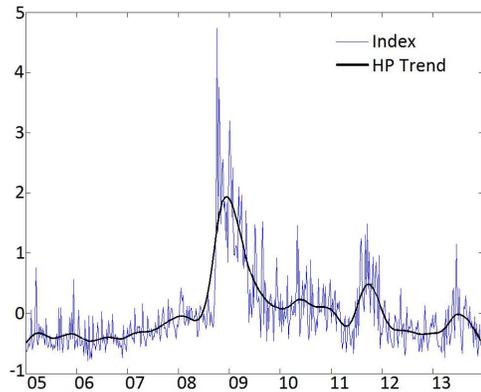
Figure 4: KCFSI
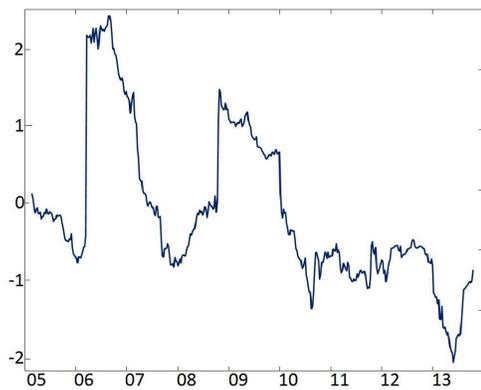
Figure 5: Absorption Ratio
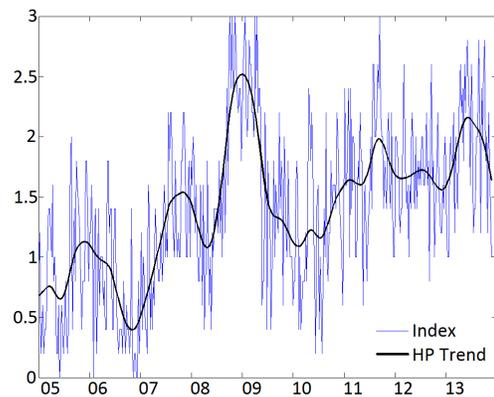
Figure 6: Lo Duca

3 The Functional Approach

3.1 Functional Systemic Risk Index

In the present section we argue in favor of a general systemic risk index appealing to a functional framework by assuming that the (possibly high-dimensional) sample of p random indicator variables used to construct the index is a set of random functions belonging to some infinite-dimensional (functional) space $\mathcal{F}$ instead of a finite-dimensional one as is usual. Thus, the variables are viewed as functions or realizations of a continuous multivariate stochastic process $\boldsymbol{X} = \{\boldsymbol{X}(t) \in \mathbb{R}^p \colon t \in \mathcal{T} \subset \mathbb{R}^+\} \in \mathcal{F}$ defined on some index set $\mathcal{T}$, where the data set is obtained from observations of a smooth random process[4] observed at discrete time points $t_1, \ldots, t_{n_i}$. Usually, the functional data setting is appropriate in different cases including, for instance, irregularly spaced measurements, high-frequency data, sparsely observed curves, analysis where the derivatives of underlying functions are important, among others. Indeed, in the last twenty years, the statistical literature has witnessed numerous advances about statistical analysis of functions with its subsequent applications in a wide variety of scientific areas (e.g. in bioinformatics, medicine, economics, finance, marketing, meteorology, geology, physiology, etc.), constituting itself as an important and dynamic area of modern statistics.[5]

In the classic FDA situation with densely sampled grid curves a common approach involves forming a fine grid of time points and sampling the curves at each time point. Therefore, one reason to adopt this approach relies on the goal of building a functional index that, for instance, allows the policy maker to obtain risk measurements at any time, and not necessarily at equidistant time points as usual.

In order to build the functional systemic risk index, and given that the number of indicator variables could be huge, our functional methodology is a three-stage method. In the first stage, a functional clustering method is applied to identify K homogeneous groups of functions in the sample that share similar shape patterns. In this stage, there exists the possibility that

---

[4] By *smooth* we denote the absence of kinks.

[5] Textbooks by Ramsay and Silverman (2005, 2002), and Horváth and Kokoszka (2012) offer detailed introductions to the branch of functional data analysis.

the curves into a particular cluster belong to an a-priori particular economic or financial sector. In the second stage, for each cluster a template (representative) function $Y_k(t)$, $k = 1, \ldots, K$ is estimated in order to summarize the information conveyed by all functions (i.e. preserving the structural features of curves in the corresponding cluster). Finally, based on template functions obtained for each cluster, a functional index $I(t)$ is built by the application of an appropriate functional dimensionality reduction technique such as functional principal component analysis or adopting a geometric-based approach by using manifold embedding methods for instance.

What type of "systemic risk" our index measures? This methodology, bottom line, performs a dimensionality reduction. As the inputs we use are market variables, the systemic risk we measure is tightly linked to the volatility of all indicators. Thus, with the exception of Hollo et al. (2012), our index and the other indexes are measuring roughly the same thing.[6] The difference of our index is that, even if we do not use sparse inputs or a very large number of them, as with the other standard methods, this new method does captures new information. Moreover, if we use exactly the same information we show that still we are capturing more information.

3.2 Functional Model-Based Clustering

Although there exists numerous functional data clustering approaches, we adopt two popular methods: the classical and well-known two-stage approach which consists in reducing the dimension of data in a first step by approximating the curves into a finite basis of functions, for instance Fourier, splines or wavelets basis, or by using the dimension reduction method of functional principal component analysis. Next, in the second step, any usual heuristic or geometric clustering algorithm such as the K-means one is used on the basis coefficients or on the principal component scores. The other approach, known as the model-based clustering procedure, is defined in a probabilistic setting by assuming a probability distribution on the data $\boldsymbol{x} \in \mathbb{R}^p$, where the data are sampled from a finite mixture of K parametric probability densities $f_k(\boldsymbol{x}|\boldsymbol{\theta}_k)$, $k = 1, \ldots, K$, i.e. $r(\boldsymbol{x}) = \sum_{k=1}^{K} \pi_k f_k(\boldsymbol{x}|\boldsymbol{\theta}_k)$, where $\pi_k \in [0, 1]$ (with $\sum_{k=1}^{K} \pi_k = 1$) is the mixture

---

[6] Bisias et al. (2012) group these indexes as Forward-Looking Risk Measures. These indexes measure the fragility of the financial system.

proportion and $\boldsymbol{\theta}_k$ is a parameter vector for the kth mixture component. One of the advantages of the model-based approach is that, in contrast with the two-stage methods, the two tasks of dimensionality reduction and clustering can be performed simultaneously. See James and Sugar (2003) for a flexible model-based procedure for clustering functional data, and Jacques and Preda (2013) for a survey on functional data clustering.

It is important to remark that in practice it is often the case that each function, or some number of them in the sample of functions, have only been observed at a relatively small number of points and possibly observed at different time points. Hence, any fixed grid will involve a relatively sparse set of points, i.e. many missing observations for each curve.[7] James and Sugar (2003) develop a model-based method for clustering all type of functional data including sparsely sampled functional data, where a mixed effects framework is used instead of the basis approximation approach, in which the latter fails due to the sparseness of the sampled functional data.

Following James and Sugar (2003), the problem setup may be formulated as follows. Let a collection of $N \geq 2$ units for which n_i observations on any variable X at $t_{ij} \in \mathcal{T} \subset \mathbb{R}$ are available, denoted by X_{ij}, $j = 1, \ldots, n_i$, $i = 1, \ldots, N$. The observations are considered as realizations generated by evaluating the set of unknown smooth functions $X_i(t)$ at points $t_{i1}, \ldots, t_{in_i}$, i.e. $X_{ij} = X_i(t_{ij})$. In presence of errors, a natural way to model the unknown smooth functions $X_i(t)$ by means of a finite q-dimensional basis approximation is given by $g_i(t) = \boldsymbol{s}(t)^\top \boldsymbol{\eta}_i$, where $\boldsymbol{s}(t)$ and $\boldsymbol{\eta}_i$ are q-dimensional spline basis and unknown spline coefficients vectors, respectively.[8] Letting $\boldsymbol{X}_i$, $\boldsymbol{g}_i$ and $\boldsymbol{\epsilon}_i$ be the corresponding vectors of observed, true, and measurement error values at times $t_{i1}, \ldots, t_{in_i}$, then

$$\boldsymbol{X}_i = \boldsymbol{g}_i + \boldsymbol{\epsilon}_i, \qquad \boldsymbol{\epsilon}_i \sim \mathcal{N}\left(\boldsymbol{0}, \sigma^2 \boldsymbol{I}\right), \ i = 1, \ldots, N,$$

and where the measurement errors are assumed to be uncorrelated with $\boldsymbol{g}_i$.

The spline coefficients vectors $\boldsymbol{\eta}_i$'s are treated as random effects and modelled using a Gaussian distribution,

$$\boldsymbol{\eta}_i = \boldsymbol{\mu}_{\boldsymbol{w}_i} + \boldsymbol{\gamma}_i, \qquad \boldsymbol{\gamma}_i \sim \mathcal{N}\left(\boldsymbol{0}, \boldsymbol{\Gamma}\right),$$

---

[7] (See James and Sugar (2003), James (2011)) for more information.
[8] Other appropriate basis function systems include, for instance, Fourier and wavelets bases.

where $\boldsymbol{w}_i \in \{1, \ldots, K\}$ denotes the unknown cluster membership. James and Sugar (2003) propose a further parameterization for the cluster means by rewritting $\boldsymbol{\mu}_k$ as

$$\boldsymbol{\mu}_k = \boldsymbol{\delta}_0 + \boldsymbol{\Delta}\boldsymbol{\alpha}_k$$

where $\boldsymbol{\delta}_0$ and $\boldsymbol{\alpha}_k$ are respectively q- and h-dimensional vectors, and $\boldsymbol{\Delta}$ is a $q \times h$ matrix with $h \leq \min(q, K-1)$. This parameterization proves useful for producing low-dimensional representations of the curves.

Under this formulation, the functional clustering model can be expressed as

$$\boldsymbol{Y}_i = \boldsymbol{S}_i\left(\boldsymbol{\delta}_0 + \boldsymbol{\Delta}\boldsymbol{\alpha}_k + \boldsymbol{\gamma}_i\right) + \boldsymbol{\epsilon}_i, \quad \boldsymbol{\epsilon}_i \sim \mathcal{N}\left(\boldsymbol{0}, \sigma^2 \boldsymbol{I}\right), \ \boldsymbol{\gamma}_i \sim \mathcal{N}\left(\boldsymbol{0}, \boldsymbol{\Gamma}\right), \ i = 1, \ldots, N$$

where $\boldsymbol{S}_i = \left(\boldsymbol{s}(t_{i1}), \ldots, \boldsymbol{s}(t_{in_i})\right)^\top$ is the basis matrix for the ith curve, and with restrictions $\sum_{k=1}^K \boldsymbol{\alpha}_k = \boldsymbol{0}$ and $\boldsymbol{\Delta}^\top \boldsymbol{S}^\top \boldsymbol{\Sigma}^{-1} \boldsymbol{S} \boldsymbol{\Delta} = \boldsymbol{I}$, with $\boldsymbol{\Sigma} = \sigma^2 \boldsymbol{I} + \boldsymbol{S}\boldsymbol{\Gamma}\boldsymbol{S}^\top$, where $\boldsymbol{S}$ is the basis matrix evaluated over a fine lattice of time points that encompasses the full range of the data.

The estimation of the model involves estimating $\boldsymbol{\delta}_0$, $\boldsymbol{\alpha}_k$, $\boldsymbol{\Delta}$, $\boldsymbol{\Gamma}$ and σ^2 by the maximization of the mixture likelihood function $L(\boldsymbol{\theta}_k) = \prod_{i=1}^N \sum_{k=1}^K \pi_k f_k(\boldsymbol{x}_i|\boldsymbol{\theta}_k)$, $\boldsymbol{\theta}_k = (\boldsymbol{\delta}_0, \boldsymbol{\alpha}_k, \boldsymbol{\Delta}, \boldsymbol{\Gamma}, \sigma^2, \pi_k)$ through the Expectation-Maximization (EM) algorithm treating the cluster memberships as missing data, where $\boldsymbol{w}_i$ is multinomial with parameters π_k, $k = 1, \ldots, K$. Details of the algorithm are provided in James and Sugar (2003).

3.3 Functional Principal Component Analysis

The setup of the Multivariate Functional Principal Component Analysis relies on the spectral analysis of the covariance operator $\mathcal{K}$ of $\boldsymbol{X}(t)$, in the class $\mathcal{L}_2(\mathcal{T})^p$ of multivariate square-integrable functions, given by

$$\mathcal{K} \colon \mathcal{L}_2(\mathcal{T})^p \to \mathcal{L}_2(\mathcal{T})^p$$
$$\boldsymbol{\varphi} \mapsto \mathcal{K}\boldsymbol{\varphi} = \int_\mathcal{T} K(\cdot, t)\boldsymbol{\varphi}(t)\mathrm{d}t, \qquad \boldsymbol{\varphi} = (\varphi_1(t), \ldots, \varphi_p(t))^\top,$$

where $K(s, t)$ is the continuous covariance (kernel) function (i.e. symmetric, real-valued and positive definite) of $\boldsymbol{X}(t)$, with mean function $\boldsymbol{\mu}(t) = (\mu_1(t), \ldots, \mu_p(t))^\top = \mathbb{E}\left[\boldsymbol{X}(t)\right]$, defined as

$$K(s, t) = \mathbb{E}\left[(\boldsymbol{X}(s) - \boldsymbol{\mu}(s)) \otimes (\boldsymbol{X}(t) - \boldsymbol{\mu}(t))\right], \qquad s, t \in \mathcal{T},$$

where $\otimes$ is the tensor product on $\mathbb{R}^p$.[9]

Thus, let an orthonormal sequence of continuous eigenfunctions $\{\boldsymbol{\varphi}_n \in \mathcal{L}_2(\mathcal{T})^p\}$ and a sequence of corresponding nonnegative eigenvalues $\{\lambda_n\}$, the spectral decomposition of $\mathcal{K}$ is given by (i.e. the Mercer's Lemma)

$$\mathcal{K}\boldsymbol{\varphi}_n = \lambda_n\boldsymbol{\varphi}_n \longrightarrow \int_{\mathcal{T}} K(s,t)\boldsymbol{\varphi}_n(t)\mathrm{d}t = \lambda_n\boldsymbol{\varphi}_n(s), \qquad n = 1, 2\ldots,$$

which is a Fredholm integral equation of the second kind, with $\int_{\mathcal{T}} \sum_{j=1}^{p} \varphi_{j,n}(s)\varphi_{j,m}(s)\mathrm{d}s = \delta_{n,m}$, where $\delta_{n,m}$ is the Kronecker delta, $\delta_{n,m} = 1$ for $n = m$ and $\delta_{n,m} = 0$ for $n \neq m$.

The principal components $\{Z_n\}$ of $\boldsymbol{X}(t)$ are given by

$$Z_n = \int_{\mathcal{T}} \sum_{j=1}^{p} \left[\left(\boldsymbol{X}_j(t) - \boldsymbol{\mu}_j(t) \right) \right] \varphi_{j,n}(t)\mathrm{d}t,$$

which are zero-mean uncorrelated random variables where $\mathbb{E}\left(Z_n Z_m\right) = \lambda_n\delta_{n,m}$.

For the empirical implementation of the multivariate functional principal component analysis see Ramsay and Silverman (2002) or Jacques and Preda (2014).

3.4 Functional Index

The Functional Systemic Risk Index (FFSSI) for the Mexican data is on a weekly basis. We select 44 sparced indicators, and applying the methodology by James and Sugar (2003) we construct eight clusters, and for each one a functional index using functional principal components.

At Figure 7 we plot the indicators at each cluster and we observe several regularities. First, clusters do not have the same number of indicators. Second, sparced indicators are not gathered at one cluster. Finally, some indicators that have a similar shape do not necessarily belong to the same cluster.

The FFSSI we propose has exactly the same interpretation of a standard index that uses principal components. The advantage of our methods relies on the fact that we alleviate common data restrictions, namely, that all indicators need to start and finish at the same date, and we cannot allow for data gaps.

---

[9] For any $s, t \in \mathcal{T}$, $K(s,t)$ is a $p \times p$ matrix with elements $K(s,t)\,[j,l] = \text{Cov}\,[X_j(s), X_l(t)]$, $j, l = 1, \ldots, p$.
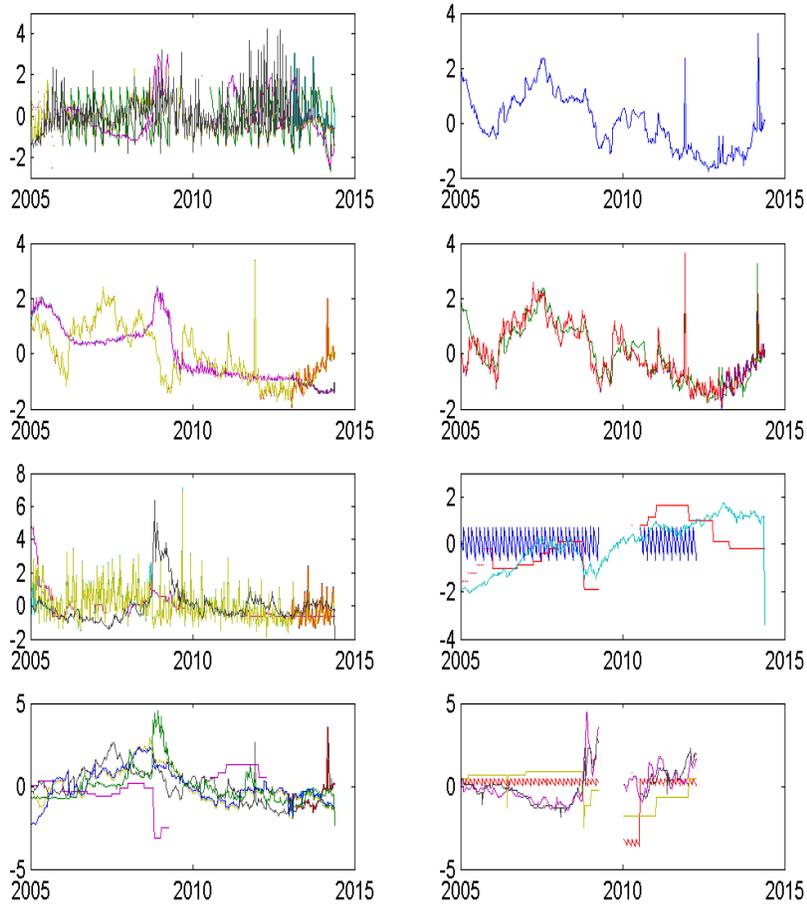
Note: All figures present years at the horizontal axis, and the normalized indexes at the vertical axis.

Figure 7: Functional Clusters

Figure 8 plots the FFSSI for Mexico. In the following Section we discuss the performance of this index versus all the other indexes described in Section 2. Moreover, and for the Mexican data, we identify which indexes better complement each other, and which are better EWIs.
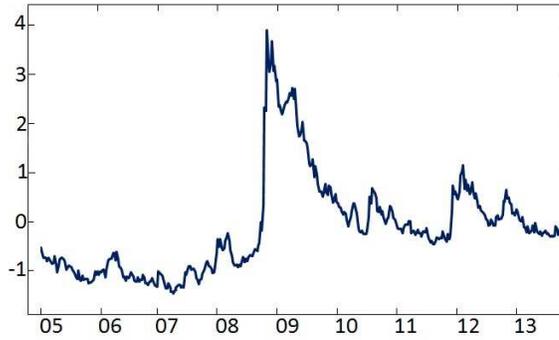
Note: We present years at the horizontal axis, and the index at the vertical axis.

Figure 8: Functional Systemic Risk Index

4   Results

Can the Functional Index capture relevant information about the systemic risk from the shape of the data? Is this extra information captured by standard systemic risk indexes? Which indexes better complement each other? Which are good Early Warning Indicators? With these questions in mind we divide this section in two. First, we use Aramonte et al. (2013) and Rothman et al. (2010) to answer the first three questions, and then we use Drehmann and Juselius (2013) to answer the last question.

Before continuing, we want to spend a few words explaining how the methodologies operate. At Appendix A we thoroughly explain how we apply these methodologies to the set of indicators used by the Mexican Central Bank to construct their systemic risk index. At Appendix C all methodologies are explained in detail.

The methodology in Aramonte et al. (2013), which involves three steps, identifies the smallest set of indexes that gather as much information as the full set of indexes. In the first step, we evaluate if the indicator helps to predict rough measures of stock market volatility. Then, we evaluate the fit of every index on the first principal component of all other indexes, and we rank them accordingly. Finally, using the same principle of the second step, we evaluate the fit of the first principal component of each element of the power set of the indexes on the first principal component of the remaining indexes.

From a statistical point of view, Aramonte et al. (2013) has some drawbacks

which we solve. Namely, the first two steps involve a variable per variable analysis, and use a simple adjusted R^2 criteria to rank the indexes. We propose to use a multivariate LASSO approach, following Rothman et al. (2010), to replace the first two steps. The refinement we propose jointly analyzes all indexes, and makes the ranking more reliable. This methodology is explained in detail at Appendix C.

To asses all indexes' performance as EWIs we use Drehmann and Juselius (2013). In this methodology we sort the indexes according to an *ad-hoc* rule. Namely, the quality of the index's signal must be above some threshold, the quality should increase as one approaches to a systemic episode, and the indicator should warn "with enough time." For example, we will discard indexes whose signal arrives, for example, two months before the crisis occurrence or whose signal's intensity does not increase the closer we get to the beginning of the crisis.

In the remainder of the section we proceed as follows. First, we evaluate if the Functional Index captures relevant information not available with previous methodologies. Also, we identify the best partition of indexes, and which serve as good EWI. Second, via simulations we compare the Functional Index with other systemic risk indexes, and study the role of data nonlinearity, and common and idiosyncratic shocks.

4.1   Complementarities and EWIs

We apply Aramonte et al. (2013), Rothman et al. (2010) and Drehmann and Juselius (2013) to the set of FSSIs using the Mexican data. The indexes to be considered are: *Carlson*, *KCFSI*, *Lo Duca*, *Absorption Ratio*, *CISS*, the Mexican Central Bank's index (*DEF*), the Functional FSSI with our own collected data (*FFSSI*), and variations of the CISS and FFSSI with the data described in Table 7 at Appendix D (*CISS_DEF* and *FFSSI_DEF*, respectively)

4.1.1   Complementarities.

We study the complementarities of all considered Financial Systemic Stress Indexes (FSSI). For the sake of clarity we begin analyzing a situation where

the data is the same, but the methodologies differ. Later we compare all the indexes.[10]

Same Data, Different Methodologies. In this exercise we consider the indicators used by the Mexican Central Bank to construct the FSSI. We only compare those indexes that could use exactly the same information, namely DEF, CISS_DEF and FFSSI_DEF.

Two questions can be answered, namely: *Which is the optimal partition of the set of indexes? Which indexes complement each other?* For this case the power set is small.

The table below shows the adjusted R^2 (aR^2) calculated for each element of the power set. The first column has the indexes in the set, the second the aR^2, the third the aR^2 where principal components are replaced by one-lag autoregressive residuals, and the fourth the average of columns two and three.

Table 1: Same Data, Different Methods

	aR^2	aR^2 Rob Check	Average	Ranking
DEF	0.799	0.799	0.799	1
CISS_DEF, FFSSI_DEF	0.799	0.799	0.799	2
FFSSI_DEF	0.606	0.604	0.605	3
CISS_DEF, DEF	0.606	0.604	0.605	4
DEF, FFSSI_DEF	0.435	0.432	0.433	5
CISS_DEF	0.435	0.432	0.433	6

Table 1 shows that the index constructed with the standard principal component method explains as much variability as the other two indexes. We cannot conclude from this that DEF index will be either a better Early Warning, nor a better forecaster. Further analysis remains to be done.

We observe a complementarity between the CISS_DEF and the FFSSI_DEF methodologies. As the only difference between FFSSI_DEF and DEF is that the former uses more information (that is contained on the shape of the data), we

---

[10]We do not consider the situation with the same methodology but different datasets because we are not advocating in favor of a particular set of indicators. Our goal is to propose a new methodology, and compare it with other FSSIs.

can conclude that the extra information is responsible for the complementarity. This fact will hold further robustness checks.

Can we eliminate an index with Rothman et al. (2010)'s multivariate LASSO? Using as dependent variables the volatility of the exchange rate, the EMBI, the consumer price index, and the VIMEX we could not eliminate any. This result implies that indeed the Functional Index is capturing additional information versus the principal component methodology. Still, this extra information captured by the Functional Index does not make it necessarily a better EWI.

All Indexes   Now we look at the big picture and evaluate the relevance of the Functional Index vis-a-vis other indexes, and seek the optimal partition of the power set of indexes. Once we find a partition we can infer a rule about which indexes should be combined. Finally, we can identify the *smallest* set of indexes that predict the variability of all others.

Before starting the analysis a clarification is due regarding the use of FFSSI or FFSSI_DEF. We will present our results with FFSSI_DEF, rather than with FFSSI for two reasons. The data used in FFSSI_DEF is very well selected,[11] has no gaps and has the same periodicity. Our goal is not to advocate in favor of a particular set of indicators but in favor of a new methodology. A second reason is that if we include new sounded indicators with data gaps or different periodicities, that will only play in our favor. We believe using FFSSI_DEF is a conservative approach. All the results we present are robust using instead the FFSSI.

The first question we ask is if we believe any of the indexes should be eliminated. In other words, do we have *a priori* any doubt about one, or some, of the indexes? We proceed assuming the answer is No.

If we assume all indexes should not be discarded then the power set has a cardinality equal to $2^7 = 126$. The table below presents the aR^2 for the six highest ranked power sets.

Table 2 shows that the smallest partition is constituted by DEF and Carlson indexes. While the former is constructed using standard principal components,

---

[11]Actually it is the same used by the Mexican Central Bank (DEG index). See Lopez Chuken (2014).

Table 2: All Indexes

	aR^2	aR^2 Rob Check	Average	Ranking
Carlson, DEF	0.409	0.407	0.408	1
Complement of Carlson, DEF	0.409	0.407	0.408	2
Carlson, DEF, KCFSI	0.404	0.401	0.402	3
Complement of Carlson, DEF, KCFSI	0.404	0.401	0.402	4
Carlson, DEF, Abs Ratio	0.399	0.396	0.397	5
Complement of Carlson, DEF, Abs Ratio	0.399	0.396	0.397	6

the latter adjust the weights assigned to the indicators (or inputs) according to previous interventions of policy makers. Again, we should not conclude from here that one partition is better than the other. For this we need to evaluate their predictability, or the performance as EWIs.

Analyzing the complementarities along all indexes we observe a clear partition. On the one hand, we have the DEF and Carlson indexes. On the other, we have the complement, namely KCFSI, Lo Duca, AR, CISS, and FFSSI_DEF indexes. This partition confirms the complementarity between the CISS and the FFSSI indexes.

To further test the robustness of the results we start eliminating, in a first stage, those indexes that explain the least of the variability of the remaining indexes. From hereon we refer to them as the *worst indexes*.

A clear pattern eventually arises. As an example, we focus on a case where the worst four indexes are eliminated. The cardinality of the power set is $2^4 = 16$. The table below shows the aR^2 of the ten highest ranked sets.

Table 3 confirms the pattern of complementarities. The set with the highest aR^2 is constituted by the DEF and the Carlson indexes. The set constituted by CISS and FFSSI_DEF follows. At this point calling one of the sets as superior is inaccurate because they share the same cardinality.

The above results suffer minor changes once we apply Rothman et al. (2010)'s multivariate LASSO. The set of indicators that survive the filter is constituted by the KCFSI index, the Lo Duca index, the CISS index, the FFSSI_DEF index and the DEF index. Then, we apply Aramonte et al. (2013)'s last step and obtain an optimal partition where DEF and KCFSI indexes constitute the first subgroup, and Lo Duca, CISS and FFSSI_DEG indexes

	aR^2	aR^2 Rob Check	Average	Ranking
Carlson, DEF	0.333	0.330	0.332	1
CISS, FFSSI_DEF	0.333	0.330	0.332	2
DEF	0.328	0.325	0.327	3
Carlson, CISS,FFSSI_DEF	0.328	0.325	0.327	4
Carlson, FFSSI_DEF	0.242	0.238	0.240	5
CISS,DEF	0.242	0.238	0.240	6
Carlson, CISS,DEF	0.204	0.200	0.202	7
FFSSI_DEF	0.204	0.200	0.202	8
Carlson	0.104	0.100	0.102	9
CISS,FFSSI_DEF, DEF	0.104	0.100	0.102	10
Carlson, FFSSI_DEF,DEF	0.092	0.088	0.090	11
CISS	0.092	0.088	0.090	12
FFSSI_DEF, DEF	0.082	0.078	0.080	13
Carlosn, CISS	0.082	0.078	0.080	14

Table 3: All Indexes: Best 4

the second subgroup.

The previous paragraph confirms that the Functional Index captures relevant information. The fact that FFSSI_DEF and DEF use exactly the same information, and that the multivariate LASSO selects them both, confirms that relevant information about the systemic risk is embedded into the shape of the data. Our argument will be strengthened if indicators present data gaps, or start at different points in time.

4.1.2 EWIs

FSSIs commonly are not regarded as good EWIs. Those constructed with financial institutions' equity returns and book data have low predictive power of macroeconomic downturns. Giglio et al. (2016) construct a set of indexes for the US, EU and UK, verify the claim for individual indexes, and propose a way to combine them to obtain a new systemic risk index with better predictive power.

FSSIs constructed with market-based data are usually better "near-coincident"[12] indicators than EWIs. Likewise, Arsov et al. (2013) construct a set

---

[12]These indicators are very useful. Policy makers can determine extraordinary regulatory

of indexes for the US and Euro area, verify the claim for all indexes, and propose a new "near-coincident" index. The authors do not envision the possibility of combining individual indexes into a superior index.

The Functional Index, together with the other indexes constructed with the Mexican data, heavily rely on market-based data. In the last part of this section we apply Drehmann and Juselius (2013) to assess the performance of individual FSSIs as EWIs, and argue in favor of combining a subset of them according to their performance.

We use the EMBI as a financial activity index. The higher the volatility with respect to the historical volatility, the more likely we are to face a systemic episode. The results are robust if we replace the EMBI by the VIMEX, the exchange rate or the consumer price index (see Appendix D for the corresponding figures).

Figure 9 shows that the performance as EWIs of the indexes using principal components is heterogenous. On the one hand, the performance of the KCFSI index satisfies all desired requirements. Remarkably, the quality of the signal almost ten weeks before the systemic event is very close to one. On the other hand, the performance of DEF and FFSSI indexes exceeds the 0.5 threshold twenty weeks before the systemic event. However, the quality of the signal is lower compared to KCFSI's. While KCFSI has a stronger signal (in the sense of Drehmann and Juselius (2013)), the DEF and FFSSI better explain the variability of the remaining indexes.

The quality of the signal, for the KCFSI, DEF and FFSSI, has an advantage over other indexes. Although their quality in average is lower, the rate of change (their slope) is the highest. This implies that if these indexes start increasing with a high pace, then there is a high probability that a systemic event may materialize.

The CISS index has the highest average signal quality. First, the CISS and CISS_DEF AUC is on average higher than 0.7 over all the fifty weeks prior to the systemic episode. Second, the AUC is smooth and increasing at least thirty periods prior to the episode. Finally, when we focus on the ten to fifteen weeks prior to the systemic episode, these indexes outperform any other index in the sample.

measures to be implemented if they cross a predetermined threshold.
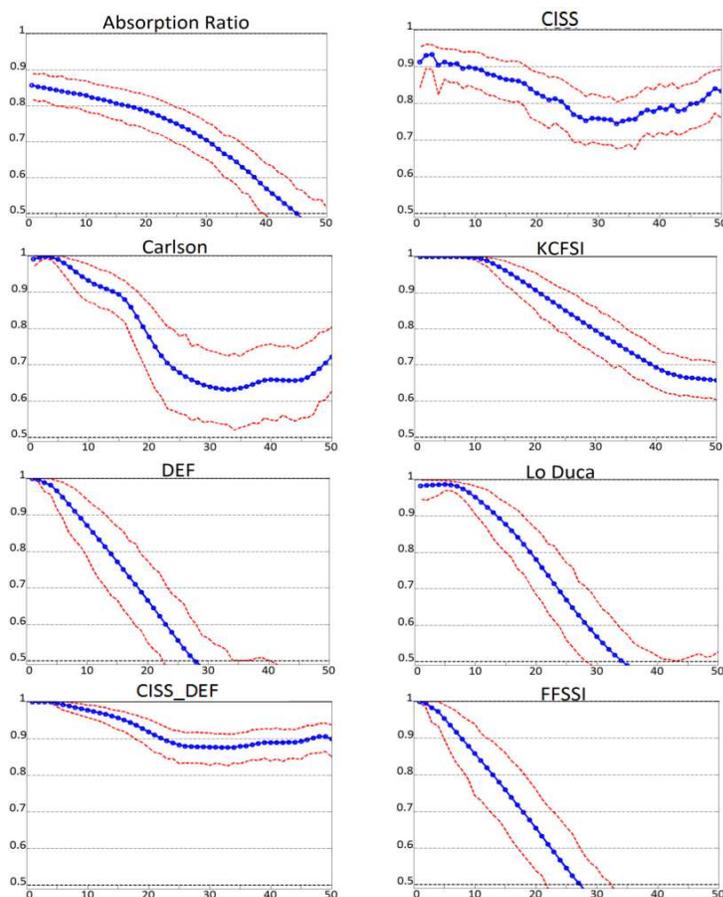
21

Note: We present the AUC at the vertical axis, and the number of weeks before a crisis occurrence at the horizontal axis. Red dashed lines represent 95% confidence intervals.

Figure 9: Indexes' AUC using EMBI as the dummy indicator

The performance of Carlson and Lo Duca indexes is also interesting. The quality of both signals is almost equal to one several weeks before a systemic episode. Also, the quality of the signal is over the 0.5 threshold almost forty weeks before.

Finally, the performance of the Absorption Ratio is discarded because the AUC does not reach to one, not even when the systemic episode occurs.

How should we use all indexes? From the previous subsection we learned that the DEF and the FFSSI explain better the variability of the remaining indexes. Also, we learned that the FFSSI and the CISS complement each other.

On this subsection we learned that, for the Mexican data, the CISS is the best EWI, and that the KCFSI, DEF and FFSSI have the highest AUC's slope.

In order to keep things simple we should monitor the FFSSI and the CISS, either separately or by combining them into another index.[13] First, these indexes, in particular the FFSSI, brings into the table all the information embedded into the other indexes. Additionally, even if the data is not sparced, with the FFSSI we capture information that no other index is capable of capturing. Second, with the CISS we guarantee using the best EWI, and with the FFSSI we use an index with a high AUC's slope.

4.2   Simulations

Now we consider a two-sector economy and construct a systemic risk index using variables from both sectors. The objective is to compare the Functional Index with other FSSIs under different environments, namely, under shocks affecting both sectors, and others affecting just one of them. To follow a conservative approach we only use the true variables that constitute the systemic risk index, all of them will be continuous, and will start and finish at the same dates.

Benchmark.   The simulated data is obtained from a mixture of two Gaussian distributions. At the benchmark, both Gaussian distributions have an equal weight, and equal covariance matrices. The difference lies on the mean's sign. Figures 10 - 11 show a scatterplot of the sample of random variables, and the systemic risk variable.

At Appendix D, Figures 21 - 26, we present the systemic risk index calculated with all FSSIs. Aside from the Absorption Ratio index, the FSSIs achieve to resemble the true systemic risk variable. From the figures we observe that the indexes that use Principal Components, i.e. *KCFSI* and *FFSSI*, have the closest fit. However, other indexes also perform well.

---

[13]We already showed these two indexes complement each other and capture most of the variability of the other indexes. In an scenario without data gaps, and without important nonlinearities, we can replace the FFSSI with the DEF index.
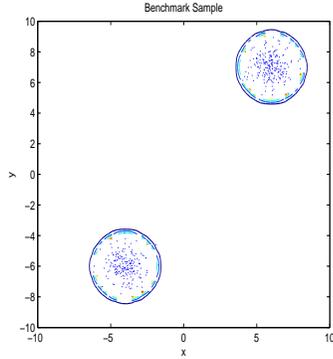
Note: We present the support of both random variables at the axis.

Figure 10: Benchmark Sample
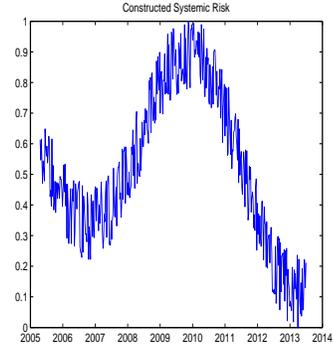
Note: Years at the hori. axis, and the systemic risk index at the vert. axis.

Figure 11: Constructed Systemic Risk

Common and Idiosyncratic Shocks. The objective now is to observe how the fit changes with different types of shocks to our simulated two-sector economy. We consider four common shocks and two idiosyncratic shocks. The former are shocks to all the elements of the diagonal of both covariance matrices; shocks to the mean of both Gaussian distributions; to the weights assigned from the mixture distribution; and to the number of variables that conform the true systemic risk variable. The idiosyncratic shocks, are on the off-diagonal elements of the covariance matrix and on the mean of one of the Gaussian distributions.

Table 4 presents the results for the common and idiosyncratic shocks. The main result is that the Functional Index outperforms other FSSIs when the input variables become increasingly nonlinear. In other words, we find evidence in favor of the importance of capturing the information embedded into the shape of the variables. This result is relevant if we accept the hypothesis that as the fragility of the financial system increases, the variables used to construct the index become more nonlinear.

The results for the other shock assume that the variables used to construct the FSSI are "fairly linear".[14] The first insight is that the Functional Index's fit is very good compared to the other FSSIs. In many cases, the difference with the best fitted index is minimal. Second, the fit of the Functional Index is particularly good when we modify the number of variables that compose the

---

[14]By "fairly linear" we mean that the variables are not lines, but rather polynomials with mild nonlinearities. For example, none of our variables are polynomials of degree two.

systemic risk variables. Finally, the fit is very stable for the shocks on the variance and on the mean. Our results do not qualitatively change for common or idiosyncratic shocks.

Wrapping up, under a conservative setting, the simulations show that the Functional Index is well suited to capture the information embedded into the shape of the data. In addition, the proposed index, given a set of shocks, has a good fit, specially when the systemic risk is determined by a greater number of variables.

5   Conclusions

In this paper we propose a new systemic risk index tailored to deal with frequent shortcomings of similar indexes prevalent in the literature and used by policy makers. We argue our methodology allows to capture new and relevant information contained in the data vis-a-vis standard indexes. In particular, and using a Functional Data Analysis framework, our index incorporates the information contained in the *shape* of the inputs to construct the index. Standard methods do not include this information.[15]

We also compare the index with other systemic risk measures widely used by policy makers using Mexican data. First, using Aramonte et al. (2013), we find a robust partition of the set of indexes that determines the optimal way to combine them. Second, using Rothman et al. (2010)'s multivariate LASSO, we show that the new index captures important information embedded into the shape of the variables. Third, following Drehmann and Juselius (2013), we compute the quality signal of each index and rank them accordingly. We conclude arguing that the best systemic risk index, given the Mexican data, is a combination of our index and the one proposed by Hollo et al. (2012).

However, if all relevant indicators are well behaved (not sparsed data) and without important nonlinearities, the performance the Functional Index is similar to that of any standard principal component indexes.

Finally, through simulating a two-sector economy, we show that the Functional Index has the best fit when the data becomes increasingly nonlinear.

---

[15]The index can be easily calculated using R, code is available upon request.

Table 4: Common and Idiosyncratic Shocks

	Common Shocks on the NonLinearity						
	NonLinearity	**KCFSI**	**CISS**	**Lo Duca**	**Abs. Ratio**	**Carlson**	**FFSSI**
MAE	0.1	73.93	73.56	82.87	133.63	126.70	29.17
MAE	1.1	74.20	125.36	98.54	81.36	94.00	117.29
MAE	2.1	125.76	160.13	122.85	120.94	127.83	118.22
MAE	3.1	94.19	110.24	112.56	173.69	106.35	74.71
MAE	4.1	92.32	129.69	101.68	132.31	98.58	85.01

The shock is on the nonlinearity of the variables

	Common Shocks on the Variance						
	Variance	**KCFSI**	**CISS**	**Lo Duca**	**Abs. Ratio**	**Carlson**	**FFSSI**
MAE	0.1	36.52	171.61	87.75	173.47	91.97	48.18
MAE	0.3	37.80	52.93	87.11	166.41	90.29	48.42
MAE	0.5	38.62	71.06	89.98	160.79	88.98	48.63
MAE	0.7	39.26	67.97	87.89	145.07	87.82	48.83
MAE	0.9	159.74	56.98	87.43	144.94	86.75	49.02

The shock is at the diagonal elements of the covariance matrices

	Idiosyncratic Shocks on the Variance						
	Variance	**KCFSI**	**CISS**	**Lo Duca**	**Abs. Ratio**	**Carlson**	**FFSSI**
MAE	0	38.62	71.06	89.98	160.79	88.98	48.63
MAE	0.1	37.26	57.13	90.38	146.95	90.35	51.40
MAE	0.2	35.93	55.17	91.55	146.93	91.77	61.95
MAE	0.3	34.65	62.60	87.48	150.74	93.23	79.91
MAE	0.4	33.42	69.17	85.90	148.13	94.73	96.61

The shock is at the off diagonal elements of the covariance matrix of on Gaussian distribution

	Common Shocks on the Mean						
	Mean Increase	**KCFSI**	**CISS**	**Lo Duca**	**Abs. Ratio**	**Carlson**	**FFSSI**
MAE	0	38.62	71.06	89.98	160.79	88.98	48.63
MAE	0.05	38.53	54.18	89.41	138.34	89.12	48.52
MAE	0.1	38.43	56.07	89.21	151.97	89.26	48.42
MAE	0.15	38.34	48.60	89.61	137.22	89.39	48.32
MAE	0.2	38.25	68.75	89.10	141.17	89.53	48.22

The shock is on both Gaussian distributions

	Idiosyncratic Shocks on the Mean						
	Mean Increase	**KCFSI**	**CISS**	**Lo Duca**	**Abs. Ratio**	**Carlson**	**FFSSI**
MAE	0	38.62	71.06	89.98	160.79	88.98	48.63
MAE	0.05	39.34	54.73	89.10	151.53	88.65	48.58
MAE	0.1	40.10	54.23	88.55	146.60	88.30	48.52
MAE	0.15	40.91	55.95	88.31	138.74	87.91	48.47
MAE	0.2	41.77	69.85	87.54	145.39	87.49	48.42

The shock is on one of the Gaussian distributions

	Common Shocks on the Weight						
	Weight	**KCFSI**	**CISS**	**Lo Duca**	**Abs. Ratio**	**Carlson**	**FFSSI**
MAE	0.1	153.95	166.58	57.40	159.92	13.61	16.82
MAE	0.3	153.88	167.87	51.69	151.03	14.36	15.14
MAE	0.5	38.62	71.06	89.98	160.79	88.98	48.63
MAE	0.7	55.48	84.32	46.42	195.73	86.11	47.51
MAE	0.9	151.67	153.04	55.26	205.08	8.85	136.81

The shock is on the weight of the first Gaussian distribution

	Common Shocks on the Number of Variables Conforming the Systemic Risk						
	No. Variables	**KCFSI**	**CISS**	**Lo Duca**	**Abs. Ratio**	**Carlson**	**FFSSI**
MAE	29	158.33	53.69	42.09	192.43	22.82	69.74
MAE	22	29.30	71.61	88.08	151.38	96.13	21.39
MAE	18	38.62	71.06	89.98	160.79	88.98	48.63
MAE	15	151.78	152.58	60.97	202.63	8.70	69.65
MAE	13	158.68	54.44	61.66	157.28	38.72	17.13
MAE	11	81.36	85.11	111.78	171.79	114.24	19.68

The shock is on the number of variables used to compute the systemic risk index

This result is important to policy makers because the fragility of the financial system is often associated with higher nonlinearities in market variables.

References

S. Aboura and J. Chevallier. A cross-volatility index for hedging the country risk. *Journal of International Financial Markets, Institutions and Money*, 38: 25–41, 2015.

V. Acharya. A theory of systemic risk and design of pudential bank regulation. *Journal of Financial Stability*, 5:224–255, 2009.

V. Acharya and T. Yorulmazer. Too many to fail - an analysis of time inconsistency in bank closure policies. *Journal of Financial Intermediation*, 16(1):1–31, 2007a.

V. Acharya and T. Yorulmazer. Cash-in-the market pricing and optimal resolution of bank failures. *Review of Financial Studies*, 21:2705–2742, 2007b.

V. Acharya, L. Pedersen, T. Philippon, and M. Richardson. Regulating systemic risk. In V. Acharya and M. Richardson, editors, *Restoring Financial Stability: How to Repair a Failed System*, pages 283–304, New Jersey, 2009. John Wiley & Sons, Inc.

V. Acharya, L. Pedersen, T. Philippon, and M. Richardson. Measuring systemic risk. Working paper, New York University, 2010.

L. Allen and T. Bali. Does systemic risk in the financial sector predict future economic downturns? *The Review of Financial Studies*, 25(10):3000–3036, 2012.

L. Allen and D. Gale. Financial contagion. *Journal of Political Economy*, 108: 1–33, 2000.

L. Allen, A. Babus, and E. Carletti. Financial connections and systemic risk. *Economic Working Papers European University Institute*, ECO2010/26, 2010.

D. Anginer, A. Demirguc-Kunt, and M. Zhu. How does competition affect bank systemic risk? *Journal of Financial Intermediation*, 23(1):1–26, 2014.

S. Aramonte, S. Rosen, and S. W. Assessing and combining financial conditions indexes. Technical report, Divisions of Research & Statistics and Monetary Affairs, Federal Reserve Board, Washington, D.C., 2013. Finance and Economics Discussion Series.

B. Arnold, C. Borio, L. Ellis, and F. Moshirian. Systemic risk, macroprudential policy frameworks, monitoring financial systems and the evolution of capital adequacy. *Journal of Banking and Finance*, 36(12):3125–3132, 2012.

I. Arsov, E. Canetti, L. Kodres, and S. Mitra. "near-coincident" indicators of systemci stress. Technical Report 13/115, IMF, 2013. Working Paper.

G. Banulescu and E. Dumitrescu. Which are the sifis? a component expected shortfall approach to systemic risk. *Journal of Banking and Finance*, 50: 575–588, 2015.

O. Bernal, J. Gnabo, and G. Guilmin. Assessing the contribution of banks, insurance and other financial services to systemic risk. *Journal of Banking and Finance*, 47:270–287, 2014.

M. Billio, M. Getmansky, A. Lo, and L. Pelizzon. Econometric measures of connectedness and systemic risk in the finance and insurance sectors. *Journal of Financial Economics*, 104(3):535–559, 2012.

D. Bisias, M. Flood, A. Lo, and S. Valavanis. A survey of systemic risk analytics. *The Annual Review of Financial Economics*, 4:255–296, 2012.

M. Bordo, M. Dueker, and D. Wheelock. Aggregate price shocks and financial instability: A historical analysis. *Economic Inquiry*, 40:521–538, 2002.

J. Boyd, G. De Nicolo, and E. Loukoianova. Banking crises and crisis dating: Theory and evidence. Working paper, University of Minnesota, June 2009.

S. Brave and A. Butters. Gathering insights on the forest from the trees: A new metric for financial conditions. Working Paper 2010-07, Federal Reserve of Chicago, 2010.

E. Brechmann, K. Hendrich, and C. Czado. Conditional copula simulation for systemic risk stress testing. *Insurance: Mathematics and Economics*, 53(3): 722–732, 2013.

B. Candelon, E. Dumitrescu, and C. Hurlin. Currency crisis early warning systems: Why they should be dynamic. *International Journal of Forecasting*, 30:1016–1029, 2014.

R. Cardarelli, S. Elekdag, and S. Lall. Financial stress, downturns, and recoveries. Working Paper 09/100, International Monetary Fund, 2009.

M. Carlson, K. Lewis, and W. Nelson. Using policy intervention to identify financial stress. Working paper, Division of Monetary Affairs, Board of Governors of the Federal Reserve System, 2012.

M. Drehmann and M. Juselius. Do debt service costs affect macroeconomic and

financial stability? *BIS Quarterly Review*, September 2012.

M. Drehmann and M. Juselius. Evaluating early warning indicators of banking crises: Satisfying policy requirements. Working Paper 421, Monetary and Economic Department, 2013.

E. Farhi and J. Tirole. Collective moral hazard, maturity mismatch, and systemic bailouts. *American Economic Review*, 102(1):60–93, 2012.

X. Freixas, B. Parigi, and J. Rochet. Systemic risk, interbank relations, and liquidity provision by the central bank. *Journal of Money*, 32:611–638, 2000.

S. Giglio, B. Kelly, and S. Pruitt. Systemic risk and the macroeconomy: An empirical evaluation. *Journal of Financial Economics*, 119:457–471, 2016.

M. Grimaldi. Detecting and interpreting financial stress in the euro area. Working Paper 1214, European Central Bank, 2010.

C. Hakkio and W. Keeton. Financial stress: What is it, how can it be measured, and why does it matter? Technical Report Second Quarter, Federal Reserve of Kansas City, 2009.

E. Hanschel and P. Monnin. Measuring and forecasting stress in the banking sector: evidence from switzerland. *BIS Papers*, 22:431–449, 2005.

J. Hatzius, P. Hooper, F. Mishkin, K. Schoenholtz, and M. Watson. Financial conditions indexes: A fresh look after the financial crisis. Working Paper 16150, National Bureau of Economic Research, 2010.

D. Hollo, M. Kremer, and M. Lo Duca. Ciss: A composite indicator of systemic stress. Working paper, Financial System. ECB, 2012.

L. Horváth and P. Kokoszka. *Inference for Functional Data with Applications*. Springer Series in Statistics. Springer, 2012.

M. Illing and Y. Liu. Measuring financial stress in a developed country: An application to canada. *Journal of Financial Stability*, 2(4):243–265, 2006.

J. Jacques and C. Preda. Functional data clustering: a survey. *Advances in Data Analysis and Classification*, pages 1–25, 2013.

J. Jacques and C. Preda. Model-based clustering for multivariate functional data. *Computational Statistics and Data Analysis*, 71:92–106, 2014.

G. James. Sparseness and functional data analysis. In F. Ferraty and Y. Romain, editors, *Oxford Handbook on Statistics and Functional Data Analysis*, Oxford handbooks, pages 298–326. Oxford University Press, New York, 2011.

G. James and C. Sugar. Clustering for sparsely sampled functional data. *Journal of the American Statistical Association*, 98(462):397–408, 2003.

A. Jobst. Multivariate dependence of implied volatilities from equity options as measure of systemic risk. *International Review of Financial Analysis*, 28: 112–129, 2013.

M. Kritzman, L. Yuanzhen, S. Page, and R. R. Principal components as a measure of systemic risk. Working paper, Financial Economics, 2010.

T. Kuzubaş, I. Ŏmercikoḡlu, and B. Saltoḡlu. Network centrality measures and systemic risk: An application to the turkish financial crisis. *Physica A: Statistical Mechanics and its Applications*, 405:203–215, 2014.

P. Laina, J. Nyholm, and P. Sarlin. Leading indicators of systemic banking crises: Finland in a panel of EU countries. *Review of Financial Economics*, 24:18–35, 2015.

M. Lo Duca and T. Pletonen. Assessing systemic risks and predicting systemic events. *Journal of Banking and Finance*, 37:2183–2195, 2013.

Y. Lopez Chuken. Una medida de estrés financiero para méxico, su relación con la actividad económica y los mecanismos de transmisión del estrés. Working paper, Instituto Tecnológico y de Estudios Superiores de Monterrey, Campus Ciudad de México, 2014.

G. Lopez-Espinosa, A. Rubia, L. Valderrama, and A. Moreno. Short-term wholesale funding and systemic risk: A global covar approach. *Journal of Banking and Finance*, 36:3150–3162, 2012.

D. Louzis and A. Vouldis. A methodology for constructing a financial systemic stress index: An application to Greece. *Economic Modelling*, 29:1228–1241, 2012.

D. Madan and W. Schoutens. Systemic risk tradeoffs and option prices. *Insurance: Mathematics and Economics*, 52(2):222–230, 2013.

S. Martínez Jaramillo, B. Alexandrova Kabadjova, B. Bravo Benítez, and P. Solórzano Margain. An empirical study of the mexican banking system's network and its implications for systemic risk. *Journal of Economic Dynamics and Control*, 40:242–265, 2014.

A. Morales and Ŏ. Estrada. A financial stability index for Colombia. *Annals of Finance*, 6:555–581, 2010.

W. Nelson and R. Perli. Selected indicators of financial stability. *Irving Fisher Committee's Bulletin on Central Bank Statistics*, 23:92–105, 2005.

M. Oet, T. Bianco, D. Gramlich, and S. Ong. Safe: An early warning system for systemic banking risk. *Journal of Banking and Finance*, 37(11):4510–4533,

2013.

D. Patro, M. Qi, and X. Sun. A simple indicator of systemic risk. *Journal of Financial Stability*, 9(1):105–116, 2013.

M. Pepe, G. Longton, and H. Janes. Accommodating covariates in receiver operating characteristic. *The Stata Journal*, 9:17–39, 2009a.

M. Pepe, G. Longton, and H. Janes. Estimation and comparison of receiver operating characteristic curves. *The Stata Journal*, 9:1–16, 2009b.

N. Puzanova and K. Düllmann. Systemic risk contributions: A credit portfolio approach. *Journal of Banking and Finance*, 37(4):1243–1257, 2013.

J. O. Ramsay and B. W. Silverman. *Applied Functional Data Analysis: Methods and Case Studies*. Springer-Verlag, 2002.

J. O. Ramsay and B. W. Silverman. *Functional Data Analysis*. Springer Series in Statistics. Springer-Verlag, 2nd edition, 2005.

A. J. Rothman, E. Levina, and J. Zhu. Sparse multivariate regression with covariance estimation. *Journal of Computational and Graphical Statistics*, 19 (4):974–962, 2010.

M. Saldías. Systemic risk analysis using forward-looking distance to default series. *Journal of Financial Stability*, 9:408–517, 2013.

A. Stuart and K. Ord. Distribution theory. *Kendall's Advanced Theory of Statistics*, 1, 1994.

S. Suh. Measuring systemic risk: A factor augmented correlated default approach. *Journal of Financial Intermediation*, 21:341–358, 2012.

B. Tabak, M. Takami, J. Rocha, D. Cajueiro, and S. Souza. Directed clustering coefficient as a measure of systemic risk in complex banking networks. *Physica A*, 394:211–216, 2014.

H. Theil. *Principles of Econometrics*. John Wiley & Sons, New York, 1971.

S. Tian, Y. Yu, and H. Guo. Variable selection and corporate bankruptcy forecasts. *Journal of Banking and Finance*, 52:89–100, 2015.

R. Tibshirani. Regression shrinkage and selection via the lasso. *Journal of the Royal Statistical Society, Series B*, 58(1):267–288, 1996.

M. Trapp and C. Wewel. Transatlantic systemic risk. *Journal of Banking and Finance*, 37:4241–4255, 2013.

G. Weib, D. Bostandzic, and S. Neumann. What factors drive systemic risk during international financial crises? *Journal of Banking and Finance*, 41: 78–96, 2014.

J. Yun and H. Moon. Measuring systemic risk in the korean banking sector via dynamic conditional correlation models. *Pacific-Basin Finance Journal*, 27: 94–114, 2014.

6 Appendix

Appendix A

The objective now is to apply Aramonte et al. (2013) and Rothman et al. (2010), and Drehmann and Juselius (2013) to the set of indicators used by the Mexican Central Bank to construct the systemic risk index. At Appendix C we describe in detail all methodologies.

Complementarities. The Mexican FSSI is calculated using 34 indicators clustered into six groups, see Table 7 at Appendix D. The reason for this division obeys the assumption that systemic risk factors may arise from different parts of the economy. A similar idea is adopted by Hollo et al. (2012).

The question now is which of these 34 indicators are really crucial. Or paraphrasing, is it possible to obtain "the same" index using a smaller set of indicators? This is a fair question at least for two reasons. First, policy makers can be interested in the general public's capability to asses the systemic risk of the financial sector. Second, even if policy makers do not pursue this, it is easier to follow a smaller set of indicators.

The following table summarizes the chosen set of indicators for different thresholds,

Table 5: Indicators chosen by Aramonte et al. (2013)

Threshold	Variables	Sectors
2	cds_gobmex	Country Risk
3	cds_gobmex	Country Risk
4	cds_gobmex	Country Risk
5	$volimpl\_tc, cds\_privmex, embi\_mex$	Exchange Rate, Country Risk
6	tdc, cds_gobmex	Exchange Rate, Country Risk
7	$vix, tdc, cds\_privmex, embi\_mex$	Equity, Exchange Rate, Country Risk

Table 5 illustrates which sectors contribute the most to the systemic risk for the Mexican case. The *Country Risk* and the *Exchange Market* are the two main sectors, and in a smaller scale we have the *Equity Market*.

Figures 12 - 16 at Appendix D present both the composite index for Mexico using the initial set of variables, and the index with the chosen set of variables[16] for different threshold values.

Rothman et al. (2010) requires a set of dependent variables, we used the volatility of the EMBI and the consumer price index. Conclusions are robust to using additional variables.[17]

The set of indicators that survive the robustness check excludes some of the indicators chosen with Aramonte et al. (2013). The indicators that survive the multivariate LASSO are $vmex\_10a$, $spread\_sup$, vol_tc, $swapito$, fwd_sprd_3m, $embi\_mex$, and the vix. Given this set of seven indicators we proceed with the last step of Aramonte et al. (2013) and obtain the best partition of the set. The first subgroup is given by $swapito$ and vix, and the other subgroup is constituted by the rest of indicators.

EWIs. To implement Drehmann and Juselius (2013) we need an indicator of stress events. We construct four indexes for stress events, two representative of the economic activity, and two representative of the financial sector. For the former group we use the volatility of the exchange rate and of the consumer price index, and for the latter the volatility of the *EMBI* and of the *VIMEX*.[18] With each of these variables we construct a stress dummy variable where a one represents a situation where the current volatility exceeded in 1.5 standard deviations the historical volatility. Figure 17 at Appendix D shows the historical behaviour of the stress dummy variables since 2005.

A good EWI should satisfy three criteria. First, the AUC[19] must be above 0.5 for a predetermined window, which in our case is between 20 and 6 weeks before any stress episode. Second, the AUC should be monotonically increasing in that same window. And finally, one should pick the EWI with the highest AUC.

Figure 27 shows the AUC for the best candidates to be an EWI of a systemic episode for Mexico. Each figure on the vertical axis has the AUC for a given period before a systemic episode starting at 0.5. The horizontal axis plots the weeks priors to

---

[16]We constructed the new composite index using standard principal component analysis.

[17]We also experimented with the volatility of the VIMEX and the exchange rate, but opted not to include them because some indicators used them.

[18]The EMBI is an country-based index, and the VIMEX is the implicit volatility of the stock market.

[19]AUC stands for Area Under the Receiver Operating Characteristic Curve. In short, the closer AUC is to 1 the more informative is the signal of the indicator. See Appendix C for details.

such an episode. As zero represents the moment where the systemic episode occurs, we must interpret them from right to left. For example, a good early warning has an AUC equal, or very near, to one a few periods before to the systemic episode, and it should monotonically decrease as we move to the right.

Not every indicator has the same signal quality. On the one hand, the AUC for the *spread sup* (upper right) and the *volimpl tc* (lower left) do not exceed 0.5 until twenty periods prior to the systemic episode. On the other hand, we have several indicators with an AUC over 0.6, even for fifty periods before.

Appendix B - FSSIs' Extended Explanation

Financial Systemic Stress Indexes (FSSI) aim to measure the prevailing state of instability in the financial system. They evaluate the current levels of frictions, stress and strains, and use distinct methodologies to summarize this information into a single statistic. The use and interpretation of FSSIs must be taken with caution because given the composition of the indicators, the complexity of systemic risk and data limitations, FSSIs provide imperfect views of the state of instability in the system. Nevertheless, *a good* FSSI allows real time monitoring and assessment of the stress level in the system, and help us study past crisis episodes.

The Debt Service Ratio (DSR) is proposed in Drehmann and Juselius (2012) as an indicator of the economic constraints created by the indebtedness of the private sector. An increase in the DSR suggests that a higher fraction of the population is over-indebted. We can interpret a high level of the DSR as a situation where if an economic downturn occurs, or in particular, if borrowers' repayment capacity is diminished, the stability of the financial system can be seriously affected.

The DSR is defined as interest payments and debt repayments divided by income. The authors argue that this formula captures the burden imposed by debt more accurately than other established leverage measures, such as the debt-to-GDP, because it takes into account factors that affect directly the borrowers' repayment capacity, such as interest rates and the maturity in the stock. The DSR is computed as follows:

$$DSR_t = \frac{i_t D_t}{(1 - (1 + i_t)^{-s_t})Y_t},$$

where Y_t denotes the quarterly aggregate income, D_t denotes the aggregate credit stock, i_t denotes the long term interest rate, and s_t denotes the average remaining maturity in quarters in the stock. To measure Y_t we will take as a proxy the quarterly GDP, and use the total credit portfolio as D_t.

An issue when measuring the financial stress is that no episode is identical to other previous episodes. The following index proposes a way to deal with this. Hakkio and Keeton (2009) enumerate and describe certain characteristics that episodes with high financial stress share, and they propose a FSSI that encompasses all these characteristics. The value added of this FSSI is that we are certain about what are we measuring, and in principle we should expect a reduction in the Type I error.

This FSSI is constructed utilizing the principal component method, a tool that, through the structure of correlations of different variables, provides different weights to each of them, giving more weight to those that are more informative. The final weight for each variable is chosen so the overall index explains the maximum variation in all the variables, subject to the standard deviation of the index being equal to one.

To solve the problem of combining all the different variables into a general index Theil (1971) proposes the next methodology. Let n be the number of independent variables; $X_{i,t}$ the value of the i^{st} standardized variable in time t; $[a_1, \ldots, a_n]$ be the set of weights for the n variables. The values solving the aforementioned problem are the elements of the first eigenvector divided by the first eigenvalue (ψ) for the sample correlation matrix of all the variables. Therefore, the FSSI is computed as follows.

$$FSI_t = (a_1/\psi)X_{i,t} + \cdots + (a_n/\psi)X_{n,t}.$$

The identification of the starting date of a financial crisis has been the subject of various research papers. Lo Duca and Pletonen (2013) note that several studies use an approach that relies on qualitative information and judgment (increases in non-performing loans, exhaustion of the banking system's capital, raises on defaults, etc.). As an alternative methodology they use a composite index to detect the starting point of a crisis. This date coincides with the FSSI exceeding a predefined threshold, which in the past anticipated a negative GDP deviation from its trend.

The first step is to construct the index including variables from the main segments of the domestic financial market. The authors proceed like this because when the economy enters into a stress period, tensions in different and numerous markets appear.

The FSSI, is computed at time t as a simple average as follows:

$$FSSI_t = \frac{1}{5} \sum_{i=1}^{5} q_{i,t}(Ind_{i,t}),$$

where $q_{i,t}$ is the variable i transformed into an integer that ranges from 0 to 3 according to the quantile the observation belong to at time t. According to pre-existing

studies,[20] this way of standardizing variables based on quantiles is more robust than a standardization based on mean and variance.[21]

An outstanding contribution of Lo Duca and Pletonen (2013) is that it introduces a methodology which identifies truly systemic episodes that had real economic costs as consequence, and not only events affecting specific market segments. To categorize these events they make an analysis between different percentiles of the FSSI and the economic activity selecting as threshold a level of the FSSI which in the past anticipated an economic downturn.

The next FSSI improves on the standard weighting methods. While a variety of weighting mechanisms have been employed by different stress indexes, principal component based approaches being the most common, the stress index introduced by Carlson et al. (2012) uses observed policy interventions to establish adequate weights to the variables.

This FSSI uses variables that capture measures of risk pricing, uncertainty and liquidity on the financial market. More specifically, it is based on the index introduced by Nelson and Perli (2005), which is calculated with three sub-indexes, i.e. levels, volatility and co-movement, and then uses a logistic regression framework to relate the three sub-indexes into a single index. The main difference of this FSSI respect to Nelson and Perli (2005) is that the periods of crisis are not chosen based on opinions of when financial institutions where under stress, but they actually identify the different interventions of policy makers into the financial industry and use them as a proxy for periods of stress. To conclude, this FSSI indicates the degree to which actual financial conditions are similar to periods where policymakers intervened.

The data used to construct the index consists of several variables that cover punctual topics like liquidity, risk pricing and uncertainty in the financial market. As we mentioned before, the overall FSSI is composed by three sub-indexes which depict specific aggregate characteristics of the variables used.

As the methodology used to combine the three sub-indexes into a single overall index is a logistic regression, it is necessary to take into account how they behaved during previous stress episodes. For this, the event signaling a stress period will be the intervention of policymakers.

---

[20]Based on Stuart and Ord (1994), Hollo et al. (2012) propose a transformation of raw stress indicators based on their empirical cumulative distribution function.

[21]Robustness understood as the FSSI's signalling stability is an essential property in the day-by-day financial stress monitoring. An index that fulfills this property should always recognize past stress episodes regardless of whether new observations continue to incorporate themselves as time goes by.

We count as an intervention all the measures taken by the Mexican authorities to protect credit and stabilize the financial system during 2008 financial crisis. Table 10 presents the list of interventions considered. Finally, to account for the fact that stress episodes do last for several periods, the methodology assumes a stress episode starts four weeks before the intervention, and finishes four weeks after the intervention.

The logistic regression utilized to evaluate how the sub-indexes behaved during stress event takes the form:

$$p_t = P(\beta_0 + \beta_L L_t + \beta_V V_t + \beta_C C_t),$$

where L, V and C represent the level, volatility, and co-movement sub-indexes respectively.

To construct the FSSI we use the coefficients obtained in the last equation as weights to combine the three sub-indexes into the overall index. The latter can be interpreted as a measure of the probability that financial markets are currently experiencing conditions identical to those in previous periods of stress.

Kritzman et al. (2010) propose an alternative measure of systemic risk called the absorption ratio (AR). This index is defined as the fraction of the total variance of a set of assets returns which is explained or "absorbed" by a fixed number of eigenvectors. The AR measures the fragility of the system in the sense that negative shocks propagate faster and broader in a coupled market than in a loosely connected market.

The equation to calculate the AR is:

$$AR = \frac{\sum_{i=1}^n \sigma_{Ei}^2}{\sum_{j=1}^N \sigma_{Aj}^2},$$

where N is the number of assets; n is the number of eigenvectors used; σ_{Ei}^2 is the variance of eigenvector i; and $\sum_{j=1}^N \sigma_{Aj}^2$ is the variance of asset j.

In order to estimate the AR, we used a rolling window of 200 weeks to estimate the covariance matrix, and we fixed the number of eigenvectors to one. Also, given the nature of the variables we use, the eigenvectors may not be associated with an observable financial variable. This is not an issue because the goal of this index is not to interpret the sources of risk, but to measure the extent to which sources of risk are becoming more or less coupled.

The last index we analyze goes a step ahead and calculates the systemic risk of the financial industry using portfolio theory. Hollo et al. (2012) introduce a FSSI named Composite Indicator of Systemic Stress (CISS), its principal strength is that it focuses

on the systemic dimension of financial stress, and adopts a statistical measurement framework that captures some of the main characteristics of systemic crises.

This index comprises, from the point of view of the authors, the five most important segments of an economy's financial system: financial and non-financial intermediaries, money markets, equity and bond markets, and foreign exchange markets. For our application, as Mexico is sensitive to the financial stability at the United States and Europe, we decided to include an extra segment.

The principal innovation of the CISS is the way it combines the sub-indexes of distinct segments of the financial system into a single composite indicator using standard portfolio theory, reflecting their time-varying cross-correlation structure. The authors claim that the CISS is a more appropriate measure of systemic risk because it assigns more weight to situations where stress prevails simultaneously at different segments of the economy.

Considering the cross-correlation between all individual asset returns, and not only their variances, the CISS distinguishes between a "horizontal" and a "vertical" dimensions of systemic risk. According to Hollo et al. (2012) the first dimension confines attention to the financial system, meanwhile the latter is focused on the two-sided interaction between the financial system and the economy. Putting more weight on systemic events allows us to identify the prevalence of financial instability according to the "horizontal" view above-mentioned. Moreover, each sub-index weight can be determined on the basis of its relative importance for real economic activity; citing the authors, this "offers a way to capture the "vertical view" of systemic stress".

Formally, y_t and ω are two $K \times 1$ vectors, where K is the number of sectors, be the a time-varying vector of sectoral indexes, and a static vector of weights, respectively. Then, let $S_t = \omega \bigotimes y_t$ be a $K \times 1$ time-varying vector of weighted indexes. The FSSI will be calculated as

$$FSSI_t = (S_t' C_t S_t)^{-1/2},$$

where C_t is a time-varying $K \times K$ correlation matrix that could be easily evaluated using a mulit-valued GARCH model. Finally, the vector of weight could be the result of an optimization program that minimizes the difference between $\omega' y_t$ and an index of economic activity ($EcAct_t$),

$$\min_t \sum_{t=1}^{T} (EcAct_t - \omega' y_t)^2,$$

subject to $\sum_{i=1}^{K} \omega_i \geq 0$.

Appendix C - Methodologies

Aramonte et al. (2013)

The authors propose a simple methodology to consolidate an arbitrarily set of FSSI.[22] Its value added is two-fold. On the one hand, it allows us to deal and combine a wide variety of indexes or indicators, irrespective of how each of them was constructed. Additionally, as the proposed method reduces de number of indexes that a policy maker should focus on, the interpretation of the resulting index should be more straightforward. On the other hand, the methodology is simple and easy to code.

They use indexes from the financial market[23] that should be correlated with other macroeconomic activity indicators or with stock returns.

The methodology is comprised by three steps. Firstly, individually evaluate which of the FSSIs help predict the economic activity or the stock returns, and eliminate those that do not. Secondly, evaluate the fit of each FSSI, with a simple adjusted R^2, on the first principal component of all other FSSIs. With this step the authors wish to identify those FSSIs that better explain the common variability of all the other FSSIs that made through step one. The authors propose to rank all FSSIs according to their adjusted R^2, and only keep the best of them.[24] And thirdly, using the same principle of step 2, pick the set of FSSIs to construct a final index. In particular, the authors propose to construct the power set of the set of the FSSIs that made through step 2, and for each element of the power set evaluate the fit, again using a simple adjusted R^2, of its first principal component on the first principal component of the rest of FSSIs that do not belong to that element of the power set.

Predictive Ability of the FSSIs.

To undertake this first step the authors suggest that the FSSIs should be analyzed

---

[22]Instead of Financial Stability Index they use the term Financial Conditions Index. In abstract both terms are not necessarily identical, but from a practical point of view, their methodology can be applied to a mixed set of indexes and indicators.

[23]Aramonte, Rosen and Schindler used the following indexes: Bloomberg U.S. Financial Conditions Index, Bloomberg U.S. Financial Conditions Index Plus, Cleveland Financial Stress Index, Morgan Stanley Financial Conditions Index U.S., Financial Market Stress Index, National Financial Conditions Index, Adjusted National Financial Conditions Index, St. Louis Fed Financial Stress Index, Kansas City Financial Stress Index, Citi Financial Conditions Index, I.M.F. U.S. Financial Conditions Index, I.M.F. U.S. Financial Stress Index.

[24]Theoretically there is no rule about how many of them to keep. In practice, we took the upper half.

in levels[25] because only when they reach unusually high or low levels we will observe real effects on the economy. On a variable basis, one starts evaluating the predictability of lagged values of each FSSI on indicators of the macroeconomic activity, or of stocks returns, based on simple linear regression model.[26]

$$y_t = \alpha + \beta \times FSI_{t-1}^i + \epsilon_t, \qquad i = 1, \ldots, N, \ \ t = 1, \ldots, T$$

where y_t is an indicator for the economic activity, or for the stock returns, at time t, FSI_{t-1}^i is the one period lag of the ith financial stability index, and ϵ_t is the error term also at time t.

Combining FSSIs.

The objective of the last two steps is to filter the FSSIs given a predetermined rule, and then create a new index with them. In the second step the authors propose to individually sort all FSSIs based on the information contained in the remaining FSSIs. In the last step, they propose to identify the set of FSSIs that best summarizes the information contained at the remaining FSSIs.

Now getting into the details, in the second step the authors measure the capability of each FSSI to capture the information of all the other FSSIs, running the following regression model, and calculating the respective adjusted R^2,

$$\Delta PC_t^{-i} = \gamma + \delta \times \Delta FSI_t^i + \upsilon_t, \qquad i = 1, \ldots, N, \ \ t = 1, \ldots, T$$

where ΔFSI_t^i is the first difference of the ith financial stability index at time t, and ΔPC_t^{-i} is the first difference of the first principal component of all remaining FSSIs, $PC_t^{-i} = fpc(\{FCI_t^j\}_{j \neq i})$, and υ_t is the error term at time t.[27] Having sorted all FSSIs according to their adjusted R^2 the authors propose to dismiss those at the bottom of the list.

In the final step we select the set of FSSIs that jointly best capture the information in the rest of FSSIs. For this, the authors construct the power set of the group of FSSIs selected in step 2, and run the following linear regression

$$\Delta PC_{\notin C, t} = \lambda + \phi \times \Delta PC_{\in C, t} + \vartheta_t, \qquad i = 1, \ldots, N, \ \ t = 1, \ldots, T$$

---

[25]See Hatzius et al. (2010)

[26]On more technical grounds, one should correct for possible heteroskedastic standard errors, and local-to-unity asymptotics

[27]As a robustness check the authors propose to repeat the same procedure but replacing the dependent and independent variables by the one-lag autoregressive residuals of, respectively, PC_t^{-i} and FSI_t^i.

where C is an element of the power set, and $\Delta PC_{\notin C,t}$ and $\Delta PC_{\in C,t}$ are the first differences of the first principal components of the FSSIs that do and do not belong to C at time t, respectively, and ϑ_t is the error term at time t. Finally, elements of the power set are sorted according to their adjusted R^2, and we take the first on the list to construct the final composite index.[28]

Drehmann and Juselius (2013)

The second methodology intends to select EWIs for banking crises. Arguably there are a priori criteria to select EWIs, the authors posit that any EWI should satisfy four characteristics. Firstly, they should have sound statistical forecasting power for detecting unsustainable booms that may produce widespread disruptions to the provision of financial services that may have serious consequences to the real economy. Secondly, EWIs should have an appropriate timing, that is, their forecasting power neither should arrive too late nor too early. The authors do not elaborate much on what "too late or early" means. Thirdly, EWIs should be stable, that is, their forecasting power should be increasing the closer one gets to a episode of financial instability. Finally, EWIs should have a clear interpretation.

Ideally, we should know the costs and benefits of implementing different macroprudential tools in order to identify the best EWIs. The authors argue that currently this is unfeasible because we do not have a good understanding of the macroprudential tools used so far, and those that remain to be used. A solution for the impasse is to evaluate EWIs over a range of utility functions, that is, given a particular utility the optimal decision must balance out a trade-off between Type I and Type II errors generated by each EWI.

The methodology proposes to calculate the quality of the signal, which is inversely related to the magnitude of the Type II error, associated to each EWI. The authors use the Receiver Operating Characteristic (ROC) curve, which is a mapping between Type I and II errors, to calculate the signals' quality. In particular, the area under the ROC curve (AUC) provides a quality measure very easy to interpret. Additionally, the AUC is easy to calculate, and is already coded into common statistical softwares such as Stata.[29].

We will spend a few lines to formally define the AUC. The economy can be in one of three states, Normal ($B = 0$), Boom ($B = 1$) or Crisis ($B = 2$). The policy

---

[28]To reduce the risk of overfitting, the authors propose to repeat the same procedure on different subsamples and average the results.

[29]See Pepe et al. (2009a) and Pepe et al. (2009b)

maker is only capable to know when the economy is in a Crisis, and can implement a policy ($P = 1$) or not ($P = 0$). If he decides to implement it he must carry on with a nonnegative cost, but the losses from a Crisis will be lower. Assume a particular utility function of the policy maker $U(P, B)$, where P is the policy, and B is the state of the economy, that satisfies $U(1, 1) > U(0, 1)$ and $U(0, 0) > U(1, 0)$. The decision of the policy maker is to determine a threshold $\theta > 0$ such that, if the signal S is greater than the threshold, the policy must be implemented.

Under this framework, the True Positive Ratio (TPR) is defined as $TPR_S(\theta) = P(S > \theta \mid B = 1)$, and the False Positive Ratio (FPR) is defined as $FPR_S(\theta) = P(S > \theta \mid B = 0)$. For low enough values of θ, TPR and FPR are close to one, and viceversa for high enough values of θ. The ROC is the mapping from FPR to TPR for all possible values of θ. The optimal level of θ is pinned down when the expected marginal rate of substitution between the net marginal utilities of an accurate prediction in normal times ($B = 0$) and booms ($B = 1$) is equal the slope of the ROC curve.

The AUC of signal S is given by:

$$AUC(S) = \int ROC(FPR(S))dFPR(S).$$

To conclude the presentation of the methodology the set of conditions an EWI should satisfy are the following.[30]

Criterion 1: An EWI S_i has the right timing if: $AUC(S_{i,h}) > 0.5$ for some horizon $h \in [-20, -6]$.

Criterion 2: An EWI S_i is stable if: $AUC(S_{i,6-j}) > AUC(S_{i,-6}) > AUC(S_{i,-6+k})$ for $1, \dots, 14$ and $1, \dots, 5$.

Criterion 3: EWI S_i outperforms EWI S_j for horizon h if $AUC(S_{i,h}) > AUC(S_{j,h})$.

Rothman et al. (2010)

The authors propose a sparse estimator of a multivariate regression coefficient matrix that accounts for correlation of the response variables involving penalized likelihood with simultaneous estimation of the regression coefficients and the covariance structure.

The classical regression model with one single response variable with a set of prediction variables generalized to a set of multiple responses is given by

$$\boldsymbol{Y} = \boldsymbol{XB} + \boldsymbol{E}$$

---

[30]The reader should recall that before we described one more criteria. The one that is missing is "easy interpretation" and we do not include it explicitly.

where $\boldsymbol{Y}$ is $n \times q$ matrix of n measurements of q random responses, $\boldsymbol{X}$ the $n \times p$ predictor matrix, $\boldsymbol{B} \in \mathbb{R}^{p \times q}$ is an unknown regression coefficient matrix, and $\boldsymbol{E}$ the $n \times q$ random error matrix with $\boldsymbol{E_i} \overset{iid}{\sim} \mathcal{N}_q(\boldsymbol{0}, \boldsymbol{\Sigma})$.

Prediction requires to estimate pq parameters which becomes difficult when there are many predictors and responses. Thus, to directly exploit the correlation in the response variables to improve prediction performance, the authors propose a sparse estimator for $\boldsymbol{B}$ that accounts for correlated errors by penalizing the negative log-likelihood function by adding two penalties: i) one LASSO penalty on the entries of $\boldsymbol{B}$, and ii) a LASSO penalty on the off-diagonal entries of the precision matrix $\boldsymbol{\Omega} = \boldsymbol{\Sigma}^{-1}$,

$$\left(\widehat{\boldsymbol{B}}, \widehat{\boldsymbol{\Omega}}\right) = \underset{\boldsymbol{B}, \boldsymbol{\Omega}}{\arg\min} \left\{ g(\boldsymbol{B}, \boldsymbol{\Omega}) + \lambda_1 \sum_{j \neq j'} |\omega_{jj'}| + \lambda_2 \sum_{j=1}^{p} \sum_{k=1}^{q} |\beta_j k| \right\},$$

where $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ are the tuning parameters, and $g(\boldsymbol{B}, \boldsymbol{\Omega})$ the negative log-likelihood function given by

$$g(\boldsymbol{B}, \boldsymbol{\Omega}) = \text{tr}\left[n^{-1}(\boldsymbol{Y} - \boldsymbol{XB} + \boldsymbol{E})^{\top}(\boldsymbol{Y} - \boldsymbol{XB} + \boldsymbol{E})\boldsymbol{\Omega} \right] - \log(\boldsymbol{\Omega}).$$

The LASSO penalty on $\boldsymbol{B}$ introduces sparsity in $\widehat{\boldsymbol{B}}$, reducing the number of predictors in the model. When $\lambda_2 = 0$, $\widehat{\boldsymbol{B}} = (\boldsymbol{X}^{\top}\boldsymbol{X})^{-1}\boldsymbol{X}^{\top}\boldsymbol{Y}$. The LASSO penalty on $\boldsymbol{\Omega}$ has the effect of reducing the number of parameters in the precision matrix, which is useful when q is large.

Rothman et al. (2010) present an fast algorithm for solving the non-convex optimization problem, which is implemented in the R package MRCE.

Appendix D

Table 6: Literature review of financial stress indexes, obtained from Louzis and Vouldis (2012)

Authors	Country and Period	Methodology	Evaluation method
Bordo et al. (2002)	US 1790-1997	A yearly Financial Conditions Index (FCI) was constructed as the sum of standardized raw stress variables (using the median instead of the mean).	-
Hanschel and Monnin (2005)	Switzerland 1987-2002	The raw stress indicators were aggregated into a single index using the variance-equal weight method (taking the average of standardized variables).	The identification of crisis periods was based on known facts and the constructed index was compared with these periods of high stress.
Illing and Liu (2006)	Canada 1981-2005	Daily data from banking sector, foreign exchange, debt and equity markets were combined into a Financial Stress Index (FSI) using various methods (Principal Components Analysis, credit weights, variance-equal weights and transformations using sample CDFs).	The various indices were compared in terms of Type I and Type II errors in signaling a crisis episode.
Nelson and Perli (2005)	US 1994-2005	Twelve financial variables were utilized to construct three subindicators combined into a single probability index by estimating a logit model.	-
Cardarelli et al. (2009)	17 Advanced economies 1981-2009	A quarterly FSSI for each country was constructed as the average of seven equally weighted variables grouped into three subindices (banking sector, securities and foreign exchange).	The episodes of financial stress identified by the FSI were compared with major financial stress episodes identified in the literature.
European Central Bank (2009b)	World's main 29 economies 1994-2010	The raw stress variables for each country were standardized and converted through logistic transformation.	The identification of crisis periods was based on known facts and the GIFT was compared with these periods of high stress.
Hakkio and Keeton (2009)	US 1990-2009	Principal Component Analysis.	The index was compared to known periods of financial stress.
Brave and Butters (2010)	US 1970-2010	An unbalanced panel of 100 mixed frequency financial variables was used to construct the Financial Conditions Index. Kalman filter, EM algorithm and Harvey accumulator techniques were utilized to produce the index.	Markov-switching techniques were applied to the FCI to identify financial crisis periods.
Lo Duca and Pletonen (2013)	10 advanced and 18 developing economies 1990-2010	For each country the FSI was constructed as the average of five stress components transformed into an integer that ranged from 0 to 3 according to the country specific quartile of the distribution the observation belongs to.	The index was compared to known periods of financial stress.
Grimaldi (2010)	Euro area 1999-2009	A list of stressful events defining the crisis periods were linked with sixteen market variables through a logit model in order to construct the weekly FSI, which shows the probability of crisis.	The FSI was compared with the implied volatility VSTOXX index in order to assess its signal/noise content.
Hatzius et al. (2010)	US 1970-2010	A modified Principal Component Analysis was used in order to combine 44 financial stress indicators in a single FCI. The main differences compared with other methods are: (i) the use of an unbalanced panel of financial variables, (ii) elimination of the variability of financial variables that is explained by current and past real activity and (iii) the aggregation of variables was done using more than one principal component.	The FCI was evaluated in terms of the ability to forecast real economic activity.
Hollo et al. (2012)	Not reported	Five subindices consisting of money, bond, equity, foreign exchange market data and financial intermediaries data were used to construct the indicator. The aggregation of the subindices was based on the portfolio risk theory.	-
Morales and Estrada (2010)	Colombia 1995-2008	Three different weighting schemes (variance-equal weights, principal components and a qualitative response approach) were used to construct a single stress index.	Identification of known stress periods.

Table 7: Inputs for Mexican Systemic Risk Index (DEF)

Sector	Variable	Description	Per.	Units	Source
Credit Institutions	cuartil_prlv	Spread between the third and first quartile for the 7 biggest average weekly interest rate.	Weekly	BP	BM
	vol_prlv_bmu	Spread between the interest rate of long-term obligations in national currency.	Monthly	%	BM
	cds_matr_7g	Average premium of hedge contract on 5 years for BBV, Santander, HSBC and Citigroup.	Weekly	Bp	Bloomberg
	acc_matr_7g	Average premium of hedge contract on 5 years for Bank of America, Citigroup, Goldman Sachs, JP Morgan, Merril Lynch, Wachovia and Wells Fargo.	Weekly	BP	Bloomberg
	foban_fogub	Weekly average differential between bank funding interest rate and government funding interest rate.	Daily	%	BM and Bloomberg
	tiie_cetes	Weekly average differential between interest rates TIIE 28 days and CETES 28 days.	Daily	%	BM and Bloomberg
Country Risk	cds_gobmex	Weekly average of government credit default swaps.	Weekly	BP	Bloomberg
	cds_privmex	Weekly average of private credit default swaps.	Weekly	BP	Bloomberg
	embi_mex	Spread between the yields from the Mexican government bonds and the U.S. Treasury Bond.	Daily	BP	Bloomberg
	petroleo_mex	Oil Price, Mexican mix.	Weekly	BP	Bloomberg
	inv_ext_gub	Ratio of foreign investment in government instruments and the total investment in these instruments.	Weekly	%	BM
Debt	sws_5a	Real zero interbank swap (5 years) - Real zero with SMP tax (5 years)	Daily	BP	Valmer
	infsprd_10a	M bonds with taxes (10 years) - Real yield with taxes (10 years)	Daily	%	Valmer
	mex_10a6m	Pending YTM (10 years - 6 years)	Daily	BP	Valmer
	vmex_10a	Variation on the YTM's M bond (10 years)	Daily	BP	Valmer
	us_10a6m	Pending zero USGov (10 years - 6 years)	Daily	BP	Valmer
	vus_10a	Zero USGov volatility (10 years)	Daily	BP	Valmer
	ibs_5a	IPAB bonds (5 years) - Bondes (5 years)	Daily	BP	Valmer
	spread_sup	Short-term superior debt spreads	Weekly	BP	BM
	spread_inf	Short-term inferior debt spreads	Weekly	BP	BM
Equity	ipyc	Main indicator for the Mexican Stock Exchange	Daily	%	BM and Bloomberg
	vol_ipyc	Weekly average deviation of IPyC	Weekly	%	BM and Bloomberg
	ipycsf_ipyc	IPyC of financial services / IPyC	Weekly	%	BM and Bloomberg
	vix	Financial indicator of expected volatility	Weekly	%	BM and Bloomberg
	vimex	Expected volatility of Mexican short-term equity market	Daily	%	BM and Bloomberg
Exchange rate	tdc	Weekly average exchange rate between MXN and USD	Daily	Pesos	Bloomberg
	vol_tc	Weekly average deviation of MXN/USD exchange rate	Weekly	MXN/USD	Bloomberg
	volimpl_tc	Exchange rate's implicit volatility	Daily	MXN/USD	Bloomberg
Derivatives	pn_no_com	Net position for participant on exchange rate futures	Weekly	Contracts	Bloomberg
	pa_fut_tiie28	Weekly average of the aggregate long and short open positions	Daily	Contracts	BM
	swapito	Implicit interest rate in dolars at the 24 hours Peso/Dolar swap	Daily	%	Valmer, Bloomberg
	optasas_mkt	Interest rate derivative in national currency	Daily	Pesos	BM
	optasas_otc	Interest rate derivative in foreign currency	Daily	Pesos	BM

Table 8: Short description of all indexes used in the analysis

Source	Type of inputs	Main characteristics of the methodology
Drehmann and Juselius (2012)	General statistics measuring the indebtedness of the private sector	Captures the burden imposed by debt more accurately than other established leverage measures because it takes into account factors that affect directly the borrowers' repayment capacity.
Hakkio and Keeton (2009)	Financial variables capturing key aspects of financial stress.	The variables are combined into an overall index assuming that financial stress is the main cause for the variance in the variables.
Lo Duca and Pletonen (2013)	Financial variables covering the main segments of the domestic financial market	The authors faced a trade-off between the degree of precision of the index at the country level, number of variables included, and the degree of homogeneity of the index across countries and time. The latter dimension prevailed for them.
Carlson et al. (2012)	Financial series that cover market liquidity, risk pricing, and uncertainty	Incorporates the levels, comovement and volatility of financial series. Its main contribution is in the way crisis periods are selected; for them a crisis period is when policy-makers responsible for regulating financial institutions intervened in the financial markets.
Kritzman et al. (2010)	Equity returns	Using eigenvalues and eigenvectors, they measure systemic risk as the extent to which equity return variance is explained by a fixed number of eigenvectors.
Hollo et al. (2012)	Five market-specific subindices created from individual financial stress measures	Generates the overall index using standard portfolio theory, reflecting the time-varying cross-correlation structure of different segments of an economy's financial system.

Table 9: Used data along the constructed indexes

Variable	Primary source	Indexes where it is used
Spread of the 3-month interbank rate from the 3-month Government bill rate	Bloomberg and Banco de Mexico.	Lo Duca KCFSI Carlson CISS
Realised volatility of the Mexican 2-year interest rate swap: realised volatility calculated as the weekly average of absolute daily rate changes	Bloomberg	KCFSI Carlson CISS
Spread between short-term corporate bond yields and 10-year constant maturity Treasury yield	Bloomberg and Banco de Mexico	KCFSI Carlson CISS
Volatility in short term corporate bond yields	Banco de Mexico	KCFSI
Implied/Expected volatility in the short term for the Mexican Stock Exchange at the end of the trading day (VIMEX)	Bloomberg	KCFSI Carlson
Negative quarterly equity returns (multiplied by minus one, so that negative equity returns increase financial stress; positive returns are disregarded and set to 0)	Bloomberg and Banco de Mexico	Lo Duca KCFSI Carlson
Realised volatility of the Mexican peso exchange rate vis-a-vis US dollar	Bloomberg and Banco de Mexico	Lo Duca KCFSI Carlson CISS
Realised volatility of the yield on the 3-month Government bill	Bloomberg and Banco de Mexico	Lo Duca Carlson
Realised volatility of the yield on the 3-month interbank rate	Bloomberg and Banco de Mexico	CISS
Realised volatility of the governmental funding rate	Banco de Mexico	CISS
Realised volatility of the 10-year Treasury yield	Bloomberg	CISS
Spread between the short term corporate bond yields and the 3-month Government bill rate	Bloomberg and Banco de Mexico	CISS
Realised volatility of the non-financial sector stock market index	Banco de Mexico	CISS
CMAX for the Datastream non-financial sector stock market index: maximum cumulated index losses over a moving 2-year window calculated as $CMAX_t = 1 - x_t / \max[x \in (x_{t-j} \mid j = 0, 1, ..., T)]$ with T = 104 for weekly data	Banco de Mexico	CISS
Realised volatility of the principal stock market index	Bloomberg and Banco de Mexico	CISS
Realised volatility of the equity return of the bank sector stock market index over the total market index	Bloomberg and Banco de Mexico	CISS
CMAX of the Mexican MEXBOL Price to book ratio	Bloomberg	CISS
Realised volatility of the Mexican exchange rate vis-a-vis Japanese Yen and European Euro	Bloomberg and Banco de Mexico	CISS
Stablished European and North American FSI's: SLF FSI, KCFSI, CFSI, EASSEQUI, EASSBOND and EASSFINI	Bloomberg	CISS

Date	Intervention
2008-2009	Sociedad Hipotecaria Federal (a financial institution belonging to the development banking sector) decided to support and refinance non-bank home mortgage institutions to meet their closest maturing liabilities. Furthermore, it extended medium-term credit lines for bridge funding, and long-term financing for individual loans.
October, 2008	Banco de México accepted new assets as collateral on liquidity loans at a rate of 1.1 times the one day target rate determined by Banco de México's Board of Governors. Monetary Regulation Deposits and other securities affected by the crisis were among the new type of collateral accepted.
October, 2008	Banco de México agreed to operate interest rate swap auctions to mitigate the impact of fluctuations in the long-term fixed rate yield curve.
October, 2008	Actions were taken to mitigate liquidity problems in domestic markets due to deteriorating conditions for accessing domestic and external financing.
October, 2008	The Federal Government announced a program to buy back Udibonos and Bonos M through Banco de México.
October 30, 2008	The National Banking and Securities Commission (CNBV) issued a new rule allowing the purchase and sale of government securities between mutual funds and financial firms belonging to the same financial group during a six-month period.
April 17, 2009	The International Monetary Fund approved a Flexible Credit Line for Mexico equivalent to 31.528 billion of Special Drawing Rights (around 47 billion dollars) over a one-year period with the possibility of renewal.
April 21, 2009	Through a dollar credit auction mechanism Banco de México offered commercial and development banks dollars from the swaps obtained from the U.S. Federal Reserve.

Table 10: Date Intervention

Source: Banco de México, 2009 Financial System Report.
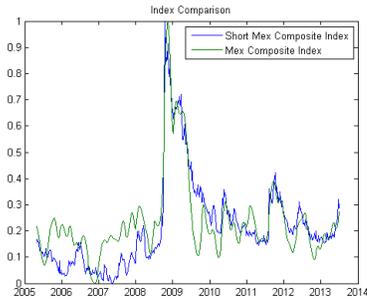
Figure 12: Threshold 2

Figure 13: Threshold 3
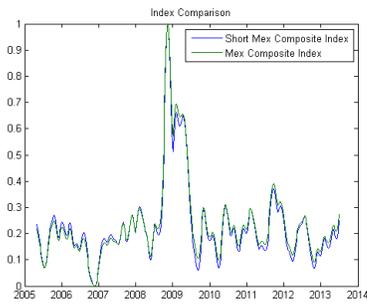
Figure 14: Threshold 4
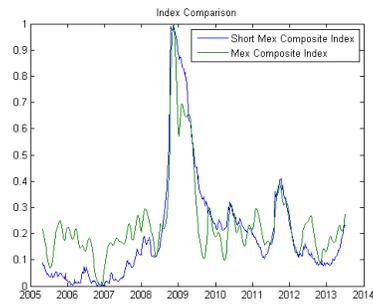
Figure 15: Threshold 5

Figure 16: Threshold 6
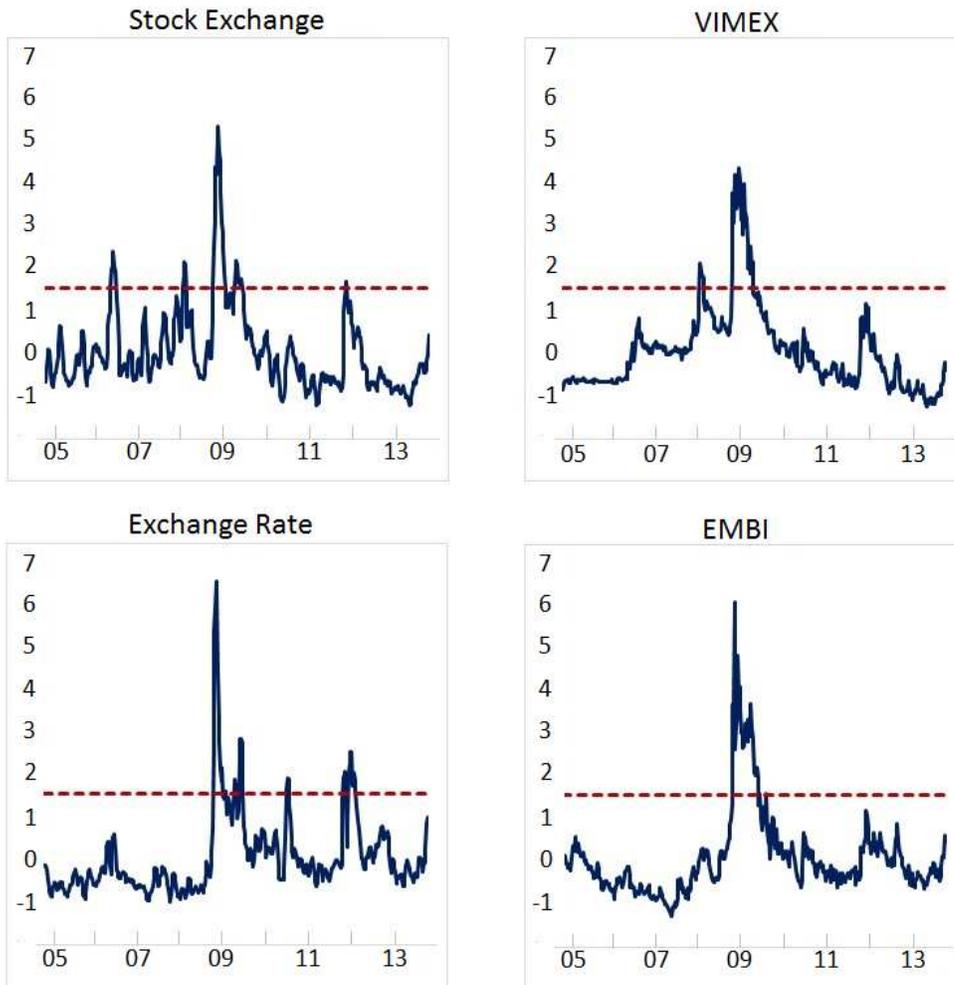
Figure 17: dummy indicators
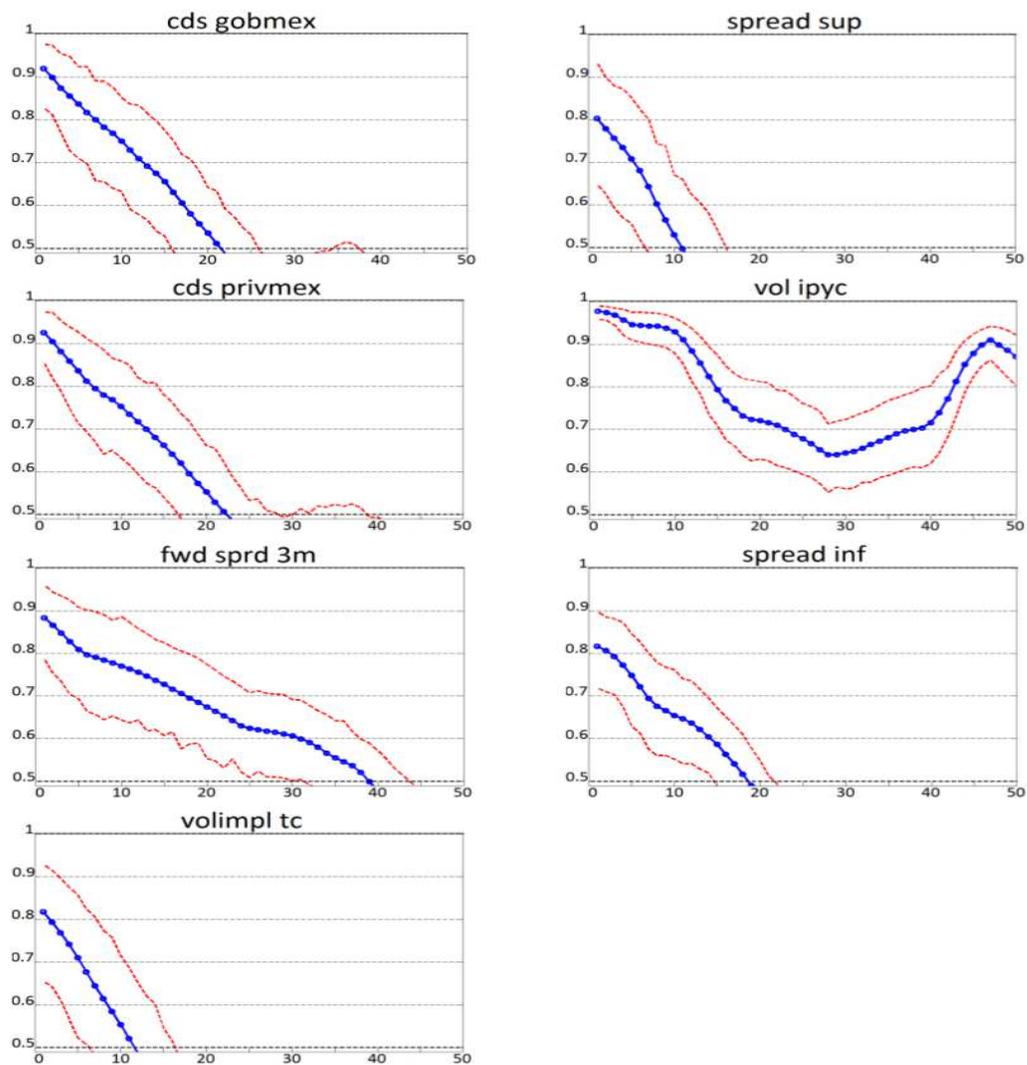
Note: We present the AUC at the vertical axis, and the number of weeks before a crisis occurrence at the horizontal axis. Red dashed lines represent 95% confidence intervals.

Figure 18: Indicators of DEF using VIMEX
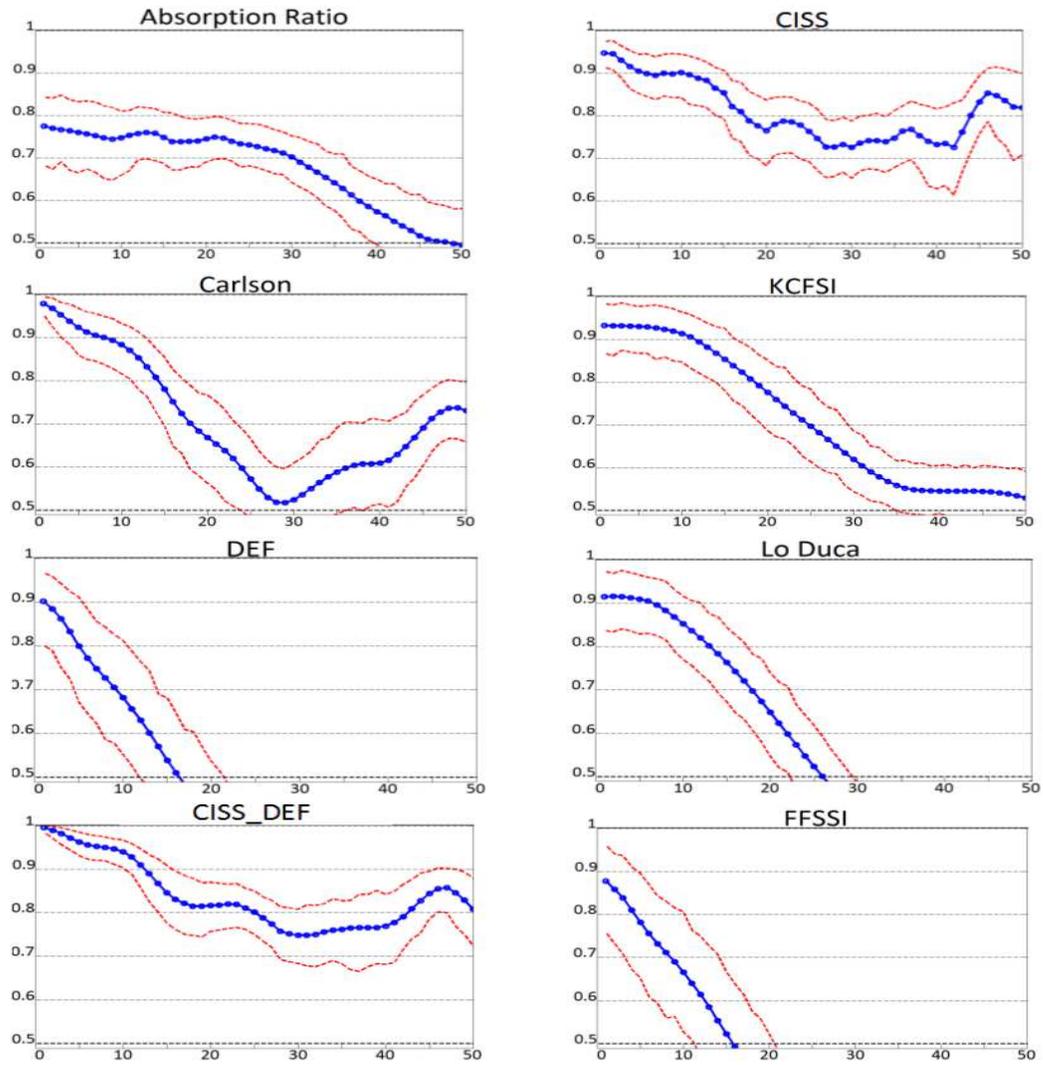
Note: We present the AUC at the vertical axis, and the number of weeks before a crisis occurrence at the horizontal axis. Red dashed lines represent 95% confidence intervals.

Figure 19: Indexes using VIMEX
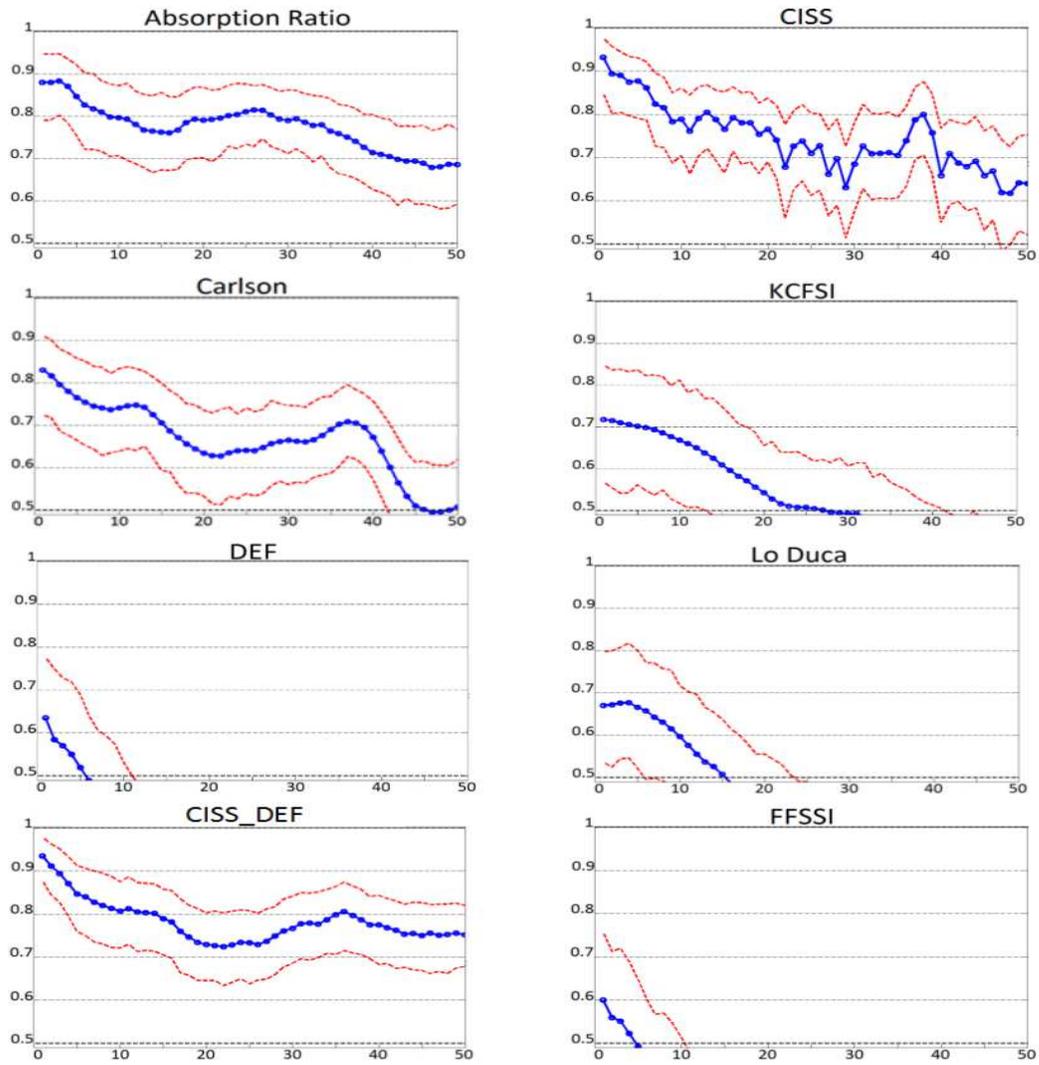
Note: We present the AUC at the vertical axis, and the number of weeks before a crisis occurrence at the horizontal axis. Red dashed lines represent 95% confidence intervals.

Figure 20: Indexes using IPC
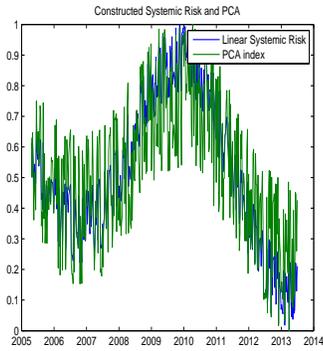
Figure 21: KCFSI
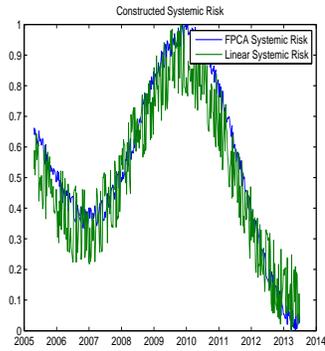
Figure 22: FFSSI
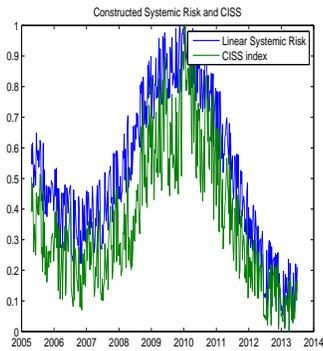
Figure 23: CISS
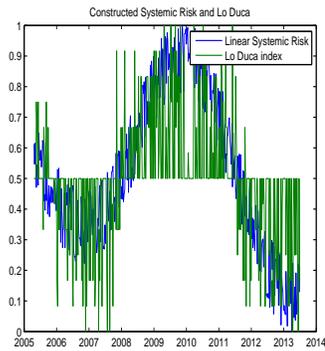

Figure 24: Lo Duca
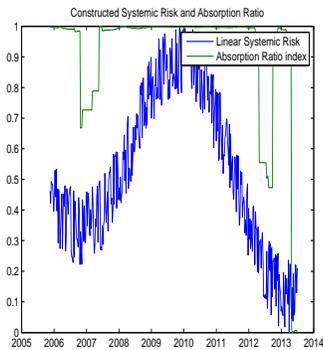
Figure 25: Absorption Ratio
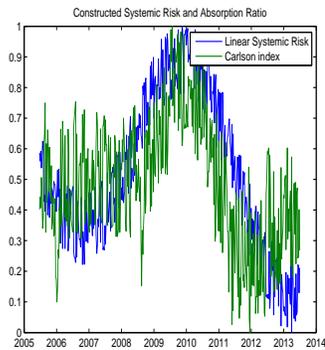
Figure 26: Carlson
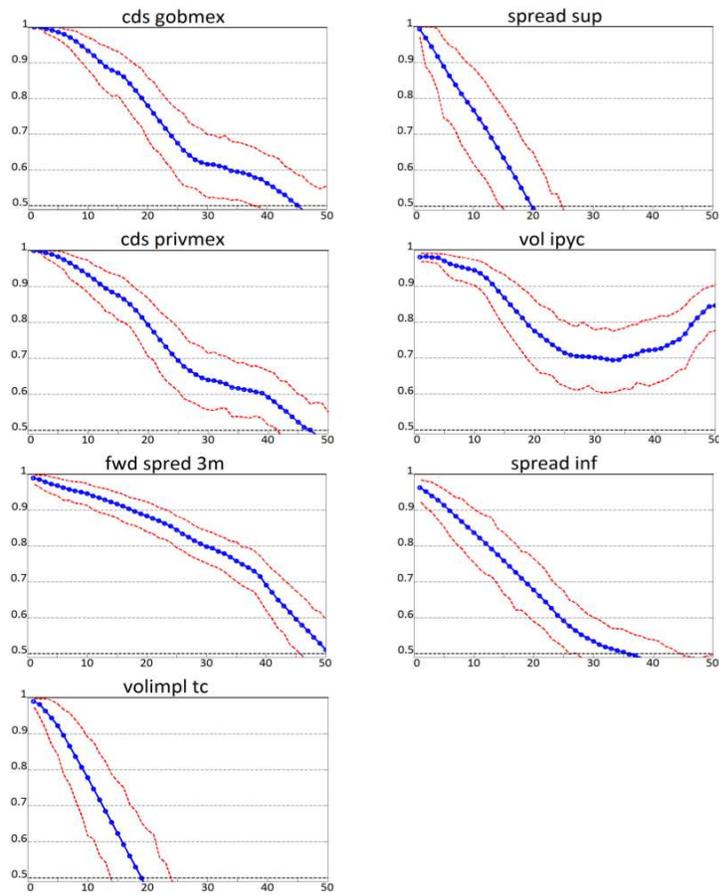
Note: We present the AUC at the vertical axis, and the number of weeks before a crisis occurrence at the horizontal axis. Red dashed lines represent 95% confidence intervals.

Figure 27: Indicators' AUC using EMBI as the dummy indicator