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How Robust Are SVARs at Measuring Monetary Policy in Small Open Economies?*

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Abstract: We study the ability of exclusion and sign restrictions to measure monetary policy shocks in small open economies. Our Monte Carlo experiments show that sign restrictions systematically overshoot inflation responses to the said shock, so we propose to add prior information to limit the number of economically implausible responses. This modified procedure robustly recovers the transmission of the shock, whereas exclusion restrictions show large sensitivity to the assumed monetary transmission mechanism of the model and the set of foreign variables included in the VAR. An application with Mexican data supports our findings.
Keywords: Exclusion Restrictions, Sign Restrictions, Small Open Economy, Monetary Policy Shock.
JEL Classification: C32, E52.

Resumen: Estudiamos la capacidad de las restricciones de exclusión y de signo para medir choques de política monetaria en economías pequeñas y abiertas. Nuestros experimentos de Monte Carlo muestran que las restricciones de signo sistemáticamente sobreestiman las respuestas de la inflación a dicho choque, por lo que proponemos incluir información a priori para limitar el número de respuestas económicamente inverosímiles. Este procedimiento modificado recobra robustamente la transmisión del choque, mientras que las restricciones de exclusión muestran gran sensibilidad al mecanismo de transmisión monetario supuesto del modelo y al conjunto de variables extranjeras incluidas en el VAR. Una aplicación con datos de México apoya nuestros resultados.
Palabras Clave: Restricciones de Exclusión, Restricciones de Signo, Economía Pequeña y Abierta, Choque de Política Monetaria.

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1 Introduction

The seminal contribution of Sims (1980) popularized the use of vector autoregressive models (VARs, hereafter) on macroeconomic data. Among other applications, we use VARs to measure the effects of different structural shocks on the economy, and we identify these shocks through restrictions that transform the reduced-form model into a *structural VAR* model, or SVAR. In particular, for a monetary policy shock,[1] we can classify the identifying restrictions into two great categories: those that exclude output and inflation to respond to the shock in the impact period, but permit lagged responses in subsequent periods; and those that allow all endogenous variables to respond in the impact period and thereafter. *Exclusion restrictions*, like those proposed by Bernanke and Mihov (1998), Christiano *et al.* (1999), or Sims and Zha (2006), belong to the first category, while *sign restrictions*, as introduced by Faust (1998), Canova and De Nicoló (2002), or Uhlig (2005), correspond to the second category. A vast amount of evidence has been gathered for large economies, while there are fewer studies focusing on small open economies (SOEs, for short). Among them, Cushman and Zha (1997), Gaytán and González (2006), and Berument (2007) use exclusion restrictions to measure monetary policy shocks in Canada, Mexico, and Turkey, respectively; while Ho and Yeh (2010) use sign restrictions to identify the said shocks in Taiwan; and Bjørnland and Halvorsen (2014) use a combination of sign and exclusion restrictions for Australia, Canada, New Zealand, Norway and the U.K.

While these applications are useful benchmarks, they do not study the advantages of using either exclusion or sign restrictions in SOEs. For instance, we do not know which class of identifying restrictions is more robust when home and foreign innovations are correlated, or how accurate the identified SVARs are when we include or omit certain foreign variables in estimation. In the same vein, we have not fully explored the consequences of using identifying assumptions that mismatch the timing of responses of the actual economy. The lack of thorough responses to these questions is surprising, given the number of SOEs that conduct monetary policy to keep home inflation on track.[2] In this context, inflation-targeting SOEs need reliable tools to measure the transmission mechanism of monetary policy on home variables, and the absence of papers on this topic is worrisome. An exception is the study of

[1] Other structural shocks have also deserved a fair amount of attention, like a technology shock, a government-spending shock, an oil price shock, etc.

[2] For instance, in 2012 there were 27 SOEs with an inflation-targeting regime. See http://www.bankofengland.co.uk/education/Documents/ccbs/handbooks/pdf/ccbshb29.pdf.

Jääskelä and Jennings (2011), who compare exclusion and sign restrictions on simulated data for Australia, and partially answer the questions stated above. In this paper, we seek to provide thorough answers to these questions.

In particular, we determine the conditions under which a SVAR, identified with exclusion or sign restrictions, reliably measures monetary policy shocks in SOEs. For exclusion restrictions, we focus on their *recursive* type, which builds on the Cholesky decomposition of the variance-covariance matrix of the VAR.[3] For sign restrictions, we evaluate two types: *standard sign restrictions*, which constrain the sign but not the size of the responses of endogenous variables; and *elasticity-bounded sign restrictions*, which impose the standard signs but also limit the size of some responses that otherwise would seem implausibly large. These type of *augmented sign restrictions*, for short, were introduced by Kilian and Murphy (2012) for oil price shocks. Further, Baumeister and Hamilton (2014) recommend to use prior beliefs regarding the elasticity of certain variables to discriminate among candidate sign-identified models. In our analysis, we take a similar approach for the case of a monetary policy shock, and we use prior information for the short-run interest-rate elasticity of inflation.

We split our analysis into two parts, one is a Monte Carlo exercise with simulated data for output, inflation, and the short-term nominal interest rate from a hypothetical small open economy, and the other is an application with Mexican data. In the case of the Monte Carlo exercise, we simulate a thousand of small samples in which we variate the timing of responses of the data generating process, or DGP. In one DGP, output and inflation react to the monetary policy shock in the impact period, while in the other DGP these variables react only with one period lag. Notice that sign restrictions match the timing of the first DGP and exclusion restrictions match the timing of the second one. For each simulated sample, we estimate a reduced-form VAR with foreign variables, as Cushman and Zha (1997), and another one without foreign variables, as Ho and Yeh (2010). Then, we identify SVARs using each set of identifying assumptions. Our intention is to measure the accuracy of a SVAR when its timing matches or mismatches the timing of the DGP, and when it includes or omits foreign variables. In the second part of our analysis, we apply the identifying assumptions to Mexican data, and document the similarities between the results of the first and the second part.[4]

[3] Although recursive restrictions are just one type among many other exclusion restrictions, our results might apply to all types of restrictions that preclude output and inflation to respond to a monetary policy shock in the impact period.

[4] Mexico is a fine example of a SOE, with a GDP nine times smaller than that of the U.S. (its major trading partner), a floating exchange rate, a dynamic exporting sector, and with an inflation-targeting regime.

Four main results stand out, of which the first three refer to the Monte Carlo experiment and the last one to the empirical application. First, when exclusion restrictions match the timing of the DGP, they accurately measure the actual responses of the economy to a monetary policy shock. However, the identification depends critically on whether foreign variables have been included in the estimation. This is the case because the variance-covariance matrix of the VAR without foreign variables is biased when foreign and home innovations are correlated. Adding relevant foreign variables, like those that highly correlate with home variables, corrects the bias and allows exclusion restrictions to identify the shock.

Second, we find that standard sign restrictions deliver responses of inflation that systematically overshoot those of the DGP, and responses of the nominal interest rate that undershoot those of the DGP. We fix these issues by adding a size restriction, in the spirit of Kilian and Murphy (2012), to the first response of inflation after the monetary policy shock. The size constraint builds on an estimated measure of the interest-rate elasticity of inflation, and helps to rule out sign-identified SVAR models that imply implausibly large responses of inflation with respect to the data. With this device active, augmented sign restrictions accurately measure the responses of inflation and the nominal interest rate of the DGP. Further, and in sharp contrast with exclusion restrictions, augmented sign restrictions seem to identify the monetary policy shock even when we omit foreign variables in estimation. We argue that this surprising result is explained by the fact that the imposed signs, which are supported by economic theory, act as auxiliary instruments that help to correct the bias of the VAR without foreign variables. Including foreign variables in the VAR is nevertheless useful because it shrinks the uncertainty bands of the responses of the sign-identified SVARs.

Third, augmented sign restrictions are more robust to measure the monetary policy shock when the timing of the DGP is unknown. This is the case because, when exclusion restrictions mismatch the timing of the DGP, they deliver responses of output and inflation that are in general smaller, and with delayed peak effects, as compared to those of the actual DGP. In addition, in this environment, exclusion restrictions do not rule out the presence of a *price puzzle*, although the actual DGP presents none. In contrast, when augmented sign restrictions mismatch the timing of the DGP, they predict responses of output and inflation that follow relatively close those of the actual DGP, and they correctly rule out the presence of a price puzzle.

Finally, the estimated SVARs with Mexican data yield similar results to those of the Monte Carlo experiment. For instance, the responses issued by sign restrictions are more stable than those of exclusion restrictions when we omit or include foreign variables, but in the latter case uncertainty bands are thinner. In the same vein, exclusion restrictions do not discard a price puzzle, while augmented sign restrictions reduce this possibility substantially. And last, the responses of inflation and the nominal interest rate from the standard sign and augmented sign restrictions differ in the same way as in the Monte Carlo experiment.

To the best of our knowledge, the closest analysis to ours is Jääskelä and Jennings (2011), who find caveats similar to ours for both exclusion and standard sign restrictions with simulated data in a monetary SVAR. For instance, exclusion restrictions point to a price puzzle when there is none in the model economy, while standard sign restrictions exaggerate the size of the responses of certain variables. Although our findings are complementary, a caveat of Jääskelä and Jennings's approach is that their DGP let all variables to react to the shock in the impact period, which invalidates by construction exclusion restrictions. In our framework, we give both sets of identifying assumptions equal chances of success *ex ante* by letting the timing of the DGP vary. This generalization allows us to test the robustness of each type of identifying restrictions. Also, Jääskelä and Jennings do not study the role of foreign variables on the performance of the SVARs, as we do here.

The remainder of the paper is as follows. Section 2 introduces the VAR model with block-exogeneity, the recursive type of exclusion restrictions, and some of the latest findings on the implementation of sign restrictions. Section 3 introduces the Monte Carlo exercise that we use to compare the accuracy and robustness of the identifying restrictions, and presents our main results. Section 4 discusses our estimated results with Mexican data and compares them with the Monte Carlo exercise. The final section concludes.

2 VARs and SVARs

In this section, we describe the reduced-form VAR models and the set of assumptions of the structural VARs that we use in this paper to identify a monetary policy shock, namely, exclusion restrictions and sign restrictions.[5] For the former, some variables are not allowed to respond on impact to the shock, while for the latter all variables may answer on impact to the shock. Since our focus is on small open economies (SOEs), we investigate the ef-

[5]The VARs models we use here do not distinguish differences between positive and negative shocks, which is a limitation of our analysis. However, the main objective of this document is not to identify potential asymmetries on the effect of a monetary policy shock, but to compare the ability of these methodologies to identify the said shock in equal circumstances.

fect of adding foreign variables on the performance of each set of identifying assumptions. Therefore, we consider two types of VAR models: a *simple VAR*, which includes only home variables; and a *VAR with exogenous variables*, which includes both home and a set of block-exogenous (foreign) variables. We start our description with the simple VAR.

2.1 Reduced-form VAR models

2.1.1 Simple VAR

Let Y_t be a vector of n endogenous variables at time t, and assume that the dynamics of Y_t follow a vector autoregression (VAR) of the form

$$Y_t = c + \sum_{\ell=1}^{p} A_\ell Y_{t-\ell} + \varepsilon_t, \tag{1}$$

where c is a vector of constants, A_ℓ denote $n \times n$ matrices of coefficients, and ε_t is a vector of reduced-form innovations, with mean zero (i.e., $\mathrm{E}\left\{\varepsilon_t\right\} = 0$), not autocorrelated ($\mathrm{E}\left\{\varepsilon_t \varepsilon_{t-\ell}'\right\} = 0_n$ for $\ell \neq 0$), and with a variance-covariance matrix given by $E\left\{\varepsilon_t \varepsilon_t'\right\} = \Omega_\varepsilon^s$, where the super index s stands for *simple VAR*. We estimate this system through maximum likelihood, which in this case implies to run an OLS regression on each equation. We refer to the joint distribution of parameters of the simple VAR as $\Gamma_s(\Theta_s)$, where Θ_s is a vector with all the parameters of the reduced-form model.

2.1.2 VAR with exogenous variables

Let Z_t be a vector of m foreign variables that affect the home variables in Y_{t+k}, for $k \geq 0$, but in contrast the elements of Y_t do not affect any of the values in Z_{t+k}. It is said that Z_t is *block-exogenous* to Y_t because the elements of the latter are uninformative about the values of Z_{t+k}. For a small open economy, this is likely to be the case. In addition, it is also likely that foreign shocks are correlated with home shocks, causing a business cycle co-movement between foreign and home economies.[6] Block-exogeneity is represented in a vector autoregression model as follows:

$$Y_t = c + \sum_{\ell=1}^{p} A_\ell Y_{t-\ell} + \sum_{\ell=1}^{p} B_\ell Z_{t-\ell} + \varepsilon_t, \tag{2}$$

$$Z_t = f + \sum_{\ell=1}^{p} D_\ell Z_{t-\ell} + \eta_t, \tag{3}$$

[6] For instance, strong trade ties might increase the correlation between foreign and home economic activities. See Blonigen *et al.* (2014).

where f is a vector of constants, and D_ℓ and B_ℓ are $m \times m$ and $n \times m$ matrices of parameters, respectively, and η_t is a $m \times 1$ vector with foreign innovations. For the error terms, let $\xi_t = [\eta_t \; \varepsilon_t]'$ be the vector of reduced-form innovations with mean zero ($\mathrm{E}(\xi_t) = 0_{(n+m)\times 1}$), not autocorrelated ($\mathrm{E}\left\{\xi_t \xi_{t-\ell}'\right\} = 0_{(n+m)}$ for $\ell \neq 0$), and with a variance-covariance matrix equal to

$$\mathrm{E}\left\{\xi_t \xi_t'\right\} \equiv \mathbf{\Omega} = \begin{bmatrix} \Omega_\eta & \Omega_{\eta\varepsilon} \\ \Omega_{\varepsilon\eta} & \Omega_\varepsilon^e \end{bmatrix},$$

where Ω_η is the $m \times m$ variance-covariance matrix of foreign innovations η_t, $\Omega_{\varepsilon\eta}$ is a $n \times m$ matrix with the covariances of home and foreign innovations (with $\Omega_{\varepsilon\eta}' = \Omega_{\eta\varepsilon}$), and Ω_ε^e is the $n \times n$ variance-covariance matrix of home innovations, where the super index e refers to the *VAR with exogenous variables*. Hamilton (1994), ch. 11.3, presents a 3-step algorithm to estimate this constrained VAR model through maximum likelihood. First, we obtain the estimates $\hat{f}$, $\hat{D}_\ell$, and $\hat{\Omega}_\eta$ by estimating system (3) as a simple VAR. Second, we set and estimate an auxiliary system, of the form

$$Y_t = g + G_0 Z_t + \sum_{\ell=1}^{p} G_{Y,\ell} Y_{t-\ell} + \sum_{\ell=1}^{p} G_{Z,\ell} Z_{t-\ell} + e_t, \tag{4}$$

through OLS equation-by-equation to obtain $\hat{g}$, $\hat{G}_0$, $\hat{G}_{Y,\ell}$, $\hat{G}_{Z,\ell}$, and $\hat{H} = \frac{1}{T}\sum_{t=1}^{T} \hat{e}_t \hat{e}_t'$, where T is the number of observations in the system. The latter corresponds to the residuals' variance-covariance matrix of the auxiliary system. Finally, the parameters of interest c, A_ℓ, B_ℓ, $\Omega_{\varepsilon\eta}$, and Ω_ε^e are obtained as follows:

$$\begin{aligned}
\hat{\Omega}_{\varepsilon\eta} &= \hat{G}_0 \hat{\Omega}_\eta, && (5)\\
\hat{c} &= \hat{g} + \hat{\Omega}_{\varepsilon\eta}\left(\hat{\Omega}_\eta\right)^{-1}\hat{f}, && (6)\\
\hat{B}_\ell &= \hat{G}_{Z,\ell} + \hat{\Omega}_{\varepsilon\eta}\left(\hat{\Omega}_\eta\right)^{-1}\hat{D}_\ell, && (7)\\
\hat{A}_\ell &= \hat{G}_{Y,\ell}, && (8)\\
\hat{\Omega}_\varepsilon^e &= \hat{H} + \hat{\Omega}_{\varepsilon\eta}\left(\hat{\Omega}_\eta\right)^{-1}\hat{\Omega}_{\eta\varepsilon}. && (9)
\end{aligned}$$

It is straightforward to verify that if $\hat{G}_0 = 0$, then the error terms η_t and ε_t are uncorrelated ($\hat{\Omega}_{\varepsilon\eta} = 0$) and the structural parameters can be obtained directly from the auxiliary system (4), as $\hat{c} = \hat{g}$, $\hat{B}_\ell = \hat{G}_{Z\ell}$, $\hat{A}_\ell = \hat{G}_{Y\ell}$, and $\hat{\Omega}_\varepsilon^e = \hat{H}$. However, when $\hat{\Omega}_{\varepsilon\eta} \neq 0$, it is important to follow the three steps suggested by Hamilton (1994) to avoid a bias in the estimated variance-covariance matrix of the home reduced-form errors, $\hat{\Omega}_\varepsilon^e$. This point is particularly important for the identifying assumptions embedded by exclusion restrictions, since in this approach the information required to identify shocks originates only from $\hat{\Omega}^e_\varepsilon$. We denote the joint distribution of parameters of the VAR with exogenous variables as $\Gamma_e(\Theta_e)$.

2.2 Structural VARs

It is well known that we cannot observe structural or fundamental shocks, such as a monetary policy shock, from the vector of reduced-form innovations ε_t of the VAR models (1) and (2). The reason is that each element of ε_t is a combination of structural shocks. The shock-identification strategy consists, thus, in finding a mapping $\mathcal{A}_k$ that links the reduced-form VAR home innovations, ε_t, to a set of fundamental innovations, υ_t, from a *structural VAR* (or SVAR) model, *i.e.*,[7]

$$\varepsilon_t = \mathcal{A}_k \upsilon_t.$$

In a monetary SVAR, it is typically assumed that vector ε_t can be stated as a contemporaneous, linear combination of υ_t, which is in turn a vector of n uncorrelated home structural shocks, with unit variance (i.e., $\mathrm{E}\left\{\upsilon_t \upsilon_t'\right\} = \mathbf{I}_n$). The reduced-form home innovations of both the simple VAR and the VAR with exogenous variables can be mapped to the structural shocks in such a manner. Notice that the variance of ε_t is equivalent to

$$\begin{aligned} \mathrm{E}\left\{\varepsilon_t \varepsilon_t'\right\} &= \mathcal{A}_k \mathrm{E}\left\{\upsilon_t \upsilon_t'\right\} \mathcal{A}_k', \text{ or simply} \\ \Omega_\varepsilon &= \mathcal{A}_k \mathcal{A}_k'. \end{aligned} \tag{10}$$

Since many matrices $\mathcal{A}_k$ solve the system of equations (10), the identification of an economically meaningful structural shocks requires to impose constraints on the elements of matrices $\mathcal{A}_k$, as we describe next.

2.2.1 Identification through exclusion restrictions

A simple and common approach is to assume exclusion restrictions, which impose enough zero entries in matrices $\mathcal{A}_k$ so that only one matrix $\mathcal{A}_{k'}$ solves equation (10). Interestingly, a matrix with a sufficient number of zeros is the lower-triangular Cholesky decomposition of matrix $\hat{\mathbf{\Omega}}_{\boldsymbol{\varepsilon}}$, which we denote by $\mathcal{A}_c = \underline{\text{chol}}(\hat{\mathbf{\Omega}}_{\boldsymbol{\varepsilon}})$. This structure is also a recursive one, since

[7] The relationship between the reduced-form VARs and the SVAR is evident if we pre-multiply system (2) with matrix $\mathcal{A}_k^{-1}$, i.e.,

$$\mathcal{A}_k^{-1} Y_t = \mathcal{A}_k^{-1} \left(c + \sum_{\ell=1}^{p} A_\ell Y_{t-\ell} + \sum_{\ell=1}^{p} B_\ell Z_{t-\ell} \right) + \upsilon_t$$

$\varepsilon_{1,t} = a_{11}\upsilon_{1,t}$, $\varepsilon_{2,t} = a_{21}\upsilon_{1,t} + a_{22}\upsilon_{2,t}$, and so on, where a_{ij} is the (i, j) entry of matrix $\mathcal{A}_c$. The implication of this assumption for the structural impulse responses is straightforward: variables that are ordered before a particular structural shock do not answer to this shock on impact.

Christiano *et al.* (1999, 2005) use a Cholesky decomposition to identify a monetary policy shock in the U.S. In this paper, we apply these recursive restrictions to a small open economy. However, other researchers have adopted different assumptions with exclusion restrictions. For instance, Cushman and Zha (1997) follow the identification strategy of Leeper *et al.* (1996), summarized in Sims and Zha (2006). These authors estimate a policy reaction function prior to the estimation of the reduced-form VAR, and then use the information of the former to identify the monetary policy shock. Cushman and Zha argue that the identification *à la* Sims and Zha helps to account for simultaneous reactions between the nominal interest rate and the nominal exchange rate. Cerdeiro (2010) applies the identification strategy of Bernanke and Mihov (1998) to the case of Argentina. The approach consists in using operating information of the central bank to produce a data-based index of policy. This index is used later in the reduced-form VAR to identify the monetary policy shock. In our framework, we stick to the Cholesky decomposition due to its simplicity, and because in our estimations with Mexican data in Section 4 we did not find an "exchange rate puzzle" after a shock in monetary policy, as it was the case in other studies (see Jääskelä and Jennings, 2011, and the refereces therein). Similarly, Berument (2007) uses the recursive Cholesky approach for Turkey, and exploits the spread between the short-term nominal interest rate and the nominal exchange rate depreciation as a measure of monetary policy tightness.

2.2.2 Identification through sign restrictions

Faust (1998), Canova and De Nicoló (2002), and Uhlig (2005) first used sign restrictions to identify monetary policy shocks in the U.S.[8] Sign restrictions do not impose zero entries in matrix $\mathcal{A}_k$. Instead, they constrain the sign of certain elements of $\mathcal{A}_k$ to obtain outcomes predicted by economic theory. It is common to constrain the sign of the response of endogenous variables to a structural shock in the impact period. For instance, we look for a matrix $\mathcal{A}_k$ so that $a_{ij}\upsilon_{j,t} > 0$ (or < 0), which means that variable i rises (or decreases) at time t after a positive innovation of structural shock $\upsilon_{j,t}$. Typically, researchers choose to restrict

[8]Other applications for sign-identified SVARs can be found in Farrant and Peersman (2006), Peersman and Straub (2009), or Kilian and Murphy (2012).

responses that are sign-robust across different models, and avoid to impose restrictions on ambiguous responses (see Canova and Paustian, 2011). As a sign restriction is more flexible than an exclusion or zero restriction, there exist potentially many matrices $\mathcal{A}_k$ that satisfy the required signs and solve equation (10). Thus, sign restrictions weakly identify SVARs, and achieve only set-identification. This property poses problems to decide which one of all sign-identified matrices $\mathcal{A}_k$ may be the most representative to capture the aggregate dynamics after a structural shock. Kilian and Murphy (2012) propose to add size restrictions to the elements of matrix $\mathcal{A}_k$, such that the elasticity of certain variables remains within the range of those observed in the data. This extra information may help to fine-tune the set of accepted matrices $\mathcal{A}_k$, as the number of models that generate economically implausible responses decreases.[9] We will come back to this point in the next section.

We follow closely the procedure of Rubio-Ramírez *et al.* (2010) to obtain sign-identified matrices $\mathcal{A}_k$, which is a generalization of Uhlig (2005)'s procedure. Our algorithm is the following.

Algorithm 1 *Consider H draws from the joint distribution of parameters $\Gamma_j(\Theta_j)$, for $j \in \{s, e\}$, of the reduced-form VAR.*[10] *Then, follow the next steps:*

1. *For each draw $h \in \{1, ..., H\}$ of $\Gamma_j(\Theta_j)$, find a random $n \times n$ matrix $\mathcal{R}_k$ from the standard normal distribution, and compute the orthonormal matrix $\mathcal{Q}_k$ from the QR-decomposition of $\mathcal{R}_k$, so that $\mathcal{Q}_k' \mathcal{Q}_k = \mathbf{I}$.*

2. *The candidate matrix is given by $\mathcal{A}_k = \underline{\text{chol}}(\hat{\mathbf{\Omega}}_{\boldsymbol{\varepsilon}}) \mathcal{Q}_k'$, where $\underline{\text{chol}}(\hat{\mathbf{\Omega}}_{\boldsymbol{\varepsilon}})$ is the lower-triangular Cholesky decomposition of matrix $\hat{\mathbf{\Omega}}_{\boldsymbol{\varepsilon}}$. Compute impulse responses using $\mathcal{A}_k$, and check whether these responses satisfy the imposed sign restrictions. If they do, keep $\mathcal{A}_k$; if they do not, drop $\mathcal{A}_k$ and find a new matrix $\mathcal{R}_k$.*

3. *For draw h, repeat steps 1 and 2 until finding a total of $\{\mathcal{A}_k\}_{k=1}^{K}$ matrices, for $K \geq 1$.*

4. *From the latter set, seek a matrix $\mathcal{A}_h$ so that*

$$\mathcal{A}_h = \underset{\mathcal{A}_k \in \{\mathcal{A}_k\}_{k=1}^{K}}{\arg\min} \left(vec\left(\mathcal{A}_{med}\right) - vec\left(\mathcal{A}_k\right)\right)^2,$$

[9] In addition, Fry and Pagan (2011) suggest finding the model with the impulse responses closest to the median responses as a representative model. Baumeister and Hamilton (2014) suggest using prior information about the elasticity of responses of certain variables in order to measure the plausibility of the candidate sign-restricted models.

[10] An estimate of $\Gamma_j(\Theta_j)$ can be obtained through Gibbs sampling for bayesian VARs, or bootstrapping for classical inference. In our empirical application, we follow the latter.

where $\mathcal{A}_{med}$ is a matrix composed of the median value of each entry of all matrices in $\{\mathcal{A}_k\}_{k=1}^{K}$. Keep $\mathcal{A}_h$ and drop any other $\mathcal{A}_k \neq \mathcal{A}_h$.

The result of the algorithm is a set $\{\mathcal{A}_h\}_{h=1}^{H}$ of sign-identified matrices, each of which is the closest to the median of its respective draw h. The original algorithm of Rubio-Ramírez *et al.* (2010) contain only steps 1 to 3. Their procedure features both virtues and caveats. Among the virtues, Arias *et al.* (2014) show that each sign-identified matrix $\mathcal{A}_k$ is a draw from the posterior distribution of structural parameters conditional on the sign restrictions, and thus the uncertainty of the procedure is properly taken into account. Further, the computing timing decreases substantially as compared to Uhlig (2005)'s procedure. A caveat, as pointed out by Baumeister and Hamilton (2014), is that the inherent composition of matrix $\mathcal{Q}_k$ yields sign-identified impulse responses that follow truncated Cauchy distributions. In other words, the econometrician may be imposing more prior information than what she might be aware of, as the restricted responses might not only be greater or smaller than zero, but also inside a finite interval. To see this graphically, Figure A.1 in the technical appendix shows the distribution of impact responses of output, inflation, and the nominal interest rate to a monetary policy shock that resorted from a random draw h of the Monte Carlo exercise of Section 3. The truncation of these distributions in the upper panel is visible, and we have verified that it does not vanish as we increase the sample size (as predicted by Baumeister and Hamilton, 2014). Step 4 in Algorithm 1 builds distributions for the posterior responses that are close to normal, centered around the median responses of each draw h, thus greatly reducing truncation (see the bottom panel of Figure A.1 of the technical appendix). Further, Section 3.2 shows that Algorithm 1 can successfully recover the true impulse responses of the data generating process, provided that we include prior information about the interest-rate elasticity of inflation in the sign restrictions procedure.

A common practice of sign restrictions is that, to increase accuracy, it is recommended to identify several shocks even if the researcher cares only about a single type of them (see Canova and Paustian, 2011). Also, a variance decomposition can only be estimated as long as there are as many identified shocks as variables in the SVAR (see Fry and Pagan, 2011).

In the context of SOEs, Ho and Yeh (2010) apply sign restrictions to measure monetary policy for Taiwan; Jääskelä and Jennings (2011) compare sign restrictions with exclusion restrictions in a simulated sample from an estimated DSGE model for Australia; while Bjørnland and Halvorsen (2014) use a hybrid strategy of exclusion and sign restrictions to identify the effects of monetary policy and exchange rate shocks on Australia, Canada, New

Zealand, Norway, Sweden and the U.K.

3 A controlled Monte Carlo experiment

In this section, we investigate the conditions that allow a SVAR for a small open economy to identify the effects of an exogenous shock to monetary policy on output, inflation, and the nominal interest rate. We do so through a controlled experiment, in which we simulate data from two hypothetical small open economies, which differ only on the amount of information agents have when taking decisions. In one economy agents hold lagged information and react with one period delay to the monetary policy shock; in the other economy, agents have complete information and respond to the monetary policy shock within the same period. Exclusion restrictions imply a timing for the impulse responses to a monetary policy shock that corresponds to the first economy, while sign restrictions imply a timing coherent with the second economy. We analyse the performance of each set of identifying assumptions under two scenarios: when their implied timing is correct (*i.e.*, it corresponds to that of the data generating process, or DGP, for short), and when their implied timing is incorrect.[11] We also investigate how adding a set of exogenous variables changes the performance of each set of identifying assumptions.

3.1 A hypothetical small open economy

Consider the following reduced-form relations describing the dynamics of output, y_t, inflation, π_t, and the nominal interest rate, i_t, of a small open economy,

$$\begin{aligned} y_t &= \left(1-\gamma_y\right) \mathrm{E}_{t-k}\left\{y_{t+1}\right\} + \gamma_y y_{t-1} - \sigma^{-1}\mathrm{E}_{t-k}\left\{i_t - \pi_{t+1}\right\} + \alpha y_t^{\star} + \varepsilon_{y,t}, \\ \pi_t - \gamma_\pi \pi_{t-1} &= \beta \mathrm{E}_{t-k}\left\{\pi_{t+1} - \gamma_\pi \pi_t\right\} + \kappa y_t + \varepsilon_{\pi,t}, \\ i_t &= \rho i_{t-1} + (1-\rho)\left(\eta_\pi \pi_t + \eta_y y_t\right) + \varepsilon_{i,t}, \end{aligned}$$

where $y_t^{\star}$ is foreign output. The model economy consists of a New-Keynesian IS curve, a Phillips curve, a Taylor rule, and faces aggregate demand shocks $\varepsilon_{y,t}$, cost-push shocks $\varepsilon_{\pi,t}$, and monetary policy shocks $\varepsilon_{i,t}$. The foreign-output term in the IS curve implies that when foreign income increases, so does home income too, since exports demand is higher.

[11] Jääskelä and Jennings (2011) perform a similar exercise to ours, but their complete-information DGP invalidates by construction exclusion restrictions. In our case, we give both identifying approaches equal chances by letting the timing of the DGP vary.

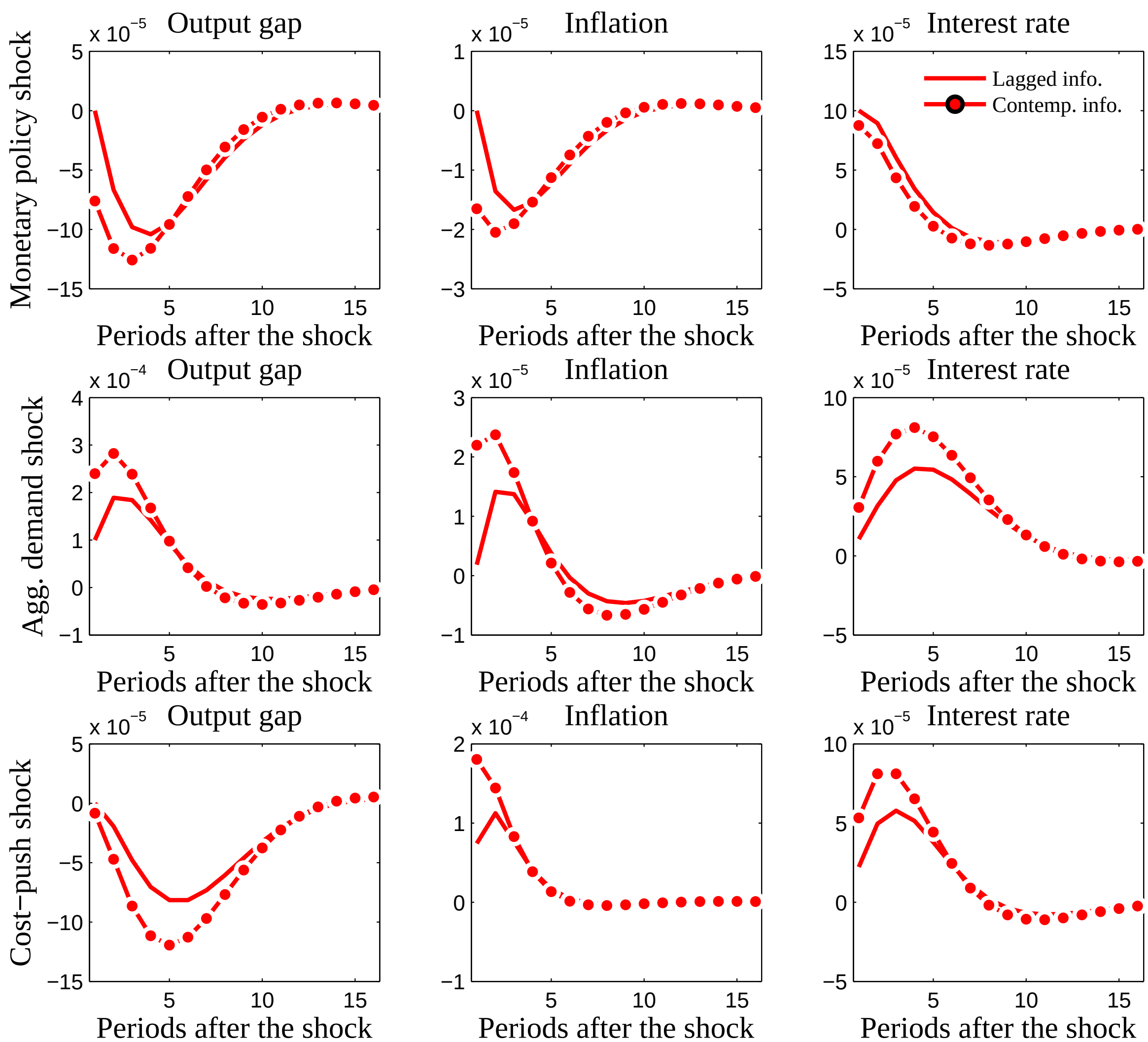
Although the model is deliberately stylized, it captures the essence of the small open economy portrayed in Galí and Monacelli (2005). In contrast with their framework, we have added intrinsic inertia in output and inflation ($\gamma_x > 0$, for $x \in \{y, \pi\}$), a feature of the data that usually emerges in empirical works, and we make explicit the influence of foreign output on home output. We use the following calibration: $\beta = .995$, $\sigma = 6$, $\kappa = .025$, $\rho = .8$, $\eta_\pi = 1.5$, $\eta_y = .5$, $\gamma_y = .5$, $\gamma_\pi = .35$, and $\alpha = .8$. All of these values, except for the last one, are within the ballpark of those used in the New Keynesian literature (see Canova and Paustian, 2011). For α, we chose a relatively high value in order to increase the correlation between foreign and home output, which accounts for the observed strong business cycle co-movement between trading partners (see Blonigen *et al.*, 2014). For the simulated samples, the correlation between y_t and y_t^* varies around .80 and .95. Interestingly, for the Mexican case, later studied in the paper, the correlation between home and U.S. output, both in gap measures, is around 0.88. We calibrate the shocks as mildly persistent processes, with unitary variances.[12]

Notice that the expectation operator E_{t-k}, conditional on information gathered k periods ago, implies different responses of home variables to shocks as k varies. For instance, assume that in time t there is a positive monetary policy shock innovation, $\varepsilon_{i,t} > 0$. If $k = 0$, agents realize that the nominal interest rate has risen and cut their demand, putting downward pressure on prices within the same period. In contrast, if $k = 1$, agents in time t are not aware that the nominal interest rate has changed, and do not adjust their demand, leaving prices unaffected. In this case, the adjustment only starts in period $t + 1$ and continues thereafter. The parameter k indicates two different states of nature that imply different responses of home variables to all shocks in the economy. These responses are shown in Figure 1, where we label the state $k = 1$ as the *lagged information* economy, and the state $k = 0$ as the *contemporaneous information* economy. Notice that, after a monetary policy shock (row 1 in Figure 1), the impact responses of output and inflation in the lagged information economy are zero, and thereafter smaller in size than their counterparts in the contemporaneous information economy.

Exclusion restrictions imply delayed responses of output and inflation to a monetary policy shock that corresponds to the timing of the lagged information economy, as Figure 1

[12]For all shocks, we use AR(1) processes. For $\varepsilon_{y,t}$ and $\varepsilon_{\pi,t}$, the persistent parameter equals .5, while for $\varepsilon_{i,t}$ it equals .2. The latter implies that neither the demand shock or the cost-push shock are very persistent, while the monetary policy shock denotes a short-lived deviation from the monetary policy rule. This last feature has being found in empirical applications.

Figure 1: Responses to structural shocks in two hypothetical SOEs



Note: The figure shows the responses of the output gap, home inflation, and the nominal interest rate of two small open economy models to shocks to monetary policy (row 1), to the aggregate demand (row 2), and to the marginal cost of home producers (row 3). The two models are essentially the same, but differ on the information available to agents when taking decisions. In the lagged information economy (plain line) agents hold outdated information about macroeconomic shocks, while in the contemporaneous information economy (line with circles) agents hold updated information.

shows. Similarly, sign restrictions allow for contemporaneous responses of output and inflation, and match the timing of the contemporaneous information economy. The exercises in the next subsection discuss the virtues and caveats of using both identifying assumptions when the DGP is conditioned on either $k = 0$ or $k = 1$.

Finally, to close the model, we assume that the foreign economy is isomorphic to the home economy, with the difference that home output does not affect foreign output. That is, we assume that the foreign economy is block-exogenous to the home economy

$$\begin{aligned} y_t^\star &= (1-\gamma)\,\mathrm{E}_{t-k}\left\{y_{t+1}^\star\right\} + \gamma y_{t-1}^\star - \sigma^{-1}\mathrm{E}_{t-k}\left\{i_t^\star - \pi_{t+1}^\star\right\} + \varepsilon_{y,t}^\star, \\ \pi_t^\star - \gamma_\pi \pi_{t-1}^\star &= \beta \mathrm{E}_{t-k}\left\{\pi_{t+1}^\star - \gamma_\pi \pi_t^\star\right\} + \kappa y_t^\star + \varepsilon_{\pi,t}^\star, \\ i_t^\star &= \rho i_{t-1}^\star + (1-\rho^\star)\left(\eta_\pi \pi_t^\star + \eta_y y_t^\star\right) + \varepsilon_{i,t}^\star, \end{aligned}$$

where the super index * denotes a foreign variable. We consider a complete structure of the foreign economy because we investigate the properties of adding different subsets of foreign variables into the VARs in the next subsection.

3.2 The experiments

We performed our Monte Carlo experiments as follows: we ran 1,000 simulated samples, with 200 observations each, for each state of nature described above: the lagged information economy, and the contemporaneous information economy.[13] For each sample, we ran a simple VAR with only home variables (*i.e.*, with $Y_t = \{y_t, \pi_t, i_t\}$), and a VAR with all variables, home and foreign, in the lines of section 2 (*i.e.*, with $Y_t = \{y_t, \pi_t, i_t\}$ and $Z_t = \{y_t^\star, \pi_t^\star, i_t^\star\}$). We include 4 lags in the VARs to ensure that the reduced-form VAR models capture the dynamics of the DGP, although the results are qualitatively similar when we consider 2 or 3 lags in the VARs. Then, we apply exclusion restrictions and sign restrictions separately in each VAR. For exclusion restrictions, the variables are ordered as y_t, π_t, and i_t, *i.e.*, we assume that output and inflation do not respond to the monetary policy shock in the impact period, while the nominal interest rate does. For sign restrictions, we identify a monetary policy shock, a transitory supply-side shock, and an aggregate demand shock. Each of these shocks is mirrored in the model by $\varepsilon_{i,t}$, $\varepsilon_{\pi,t}$, and $\varepsilon_{y,t}$, respectively. The signs we use for identification are inspired by Figure 1 and are fairly robust across different models (see Canova and Paustian,

[13]The initial values of the system variables equal the steady-state equilibrium, which is normalized to zero. We used a burning period of 100 observations, which is enough to eliminate the effects of the initial values on the simulated aggregated dynamics.

2011). That is, after a contractionary monetary policy shock, output and inflation fall while the nominal interest rate increases; in turn, after a positive aggregate demand shock, all the three endogenous variables increase; and finally, after a positive cost-push shock, output falls while inflation and the nominal interest rate increase. These sign restrictions are summarized in Table 1.

Table 1: Sign restrictions for the hypothetical SOEs

Shock	y_t	π_t	i_t
Monetary policy shock	$-$	$-$	$+$
Aggregate demand shock	$+$	$+$	$+$
Transitory supply-side shock	$-$	$+$	$+$

Note: The sign refers to the impact response of the variable of interest due to a structural shock. A '+' (or '-') indicates that $\partial x_t / \partial \upsilon_{j,t} > 0$ (or < 0), where x_t is the variable of interest and $\upsilon_{j,t}$ is the structural shock studied.

We now investigate whether we can recover the true impulse responses to a monetary policy shock using the two sets of identifying assumptions analyzed in this paper. To organize the analysis, we split our discussion in the following questions:

1. When the timing of the assumed identifying restrictions corresponds to the timing of the DGP, are the true impulse responses identified? What is the effect of adding exogenous variables in the performance of each set of identifying assumptions?

2. When the timing of the assumed identifying restrictions does not correspond to the timing of the DGP, which identification approach has the worst performance? Which one is more robust when the timing of the DGP is unknown?

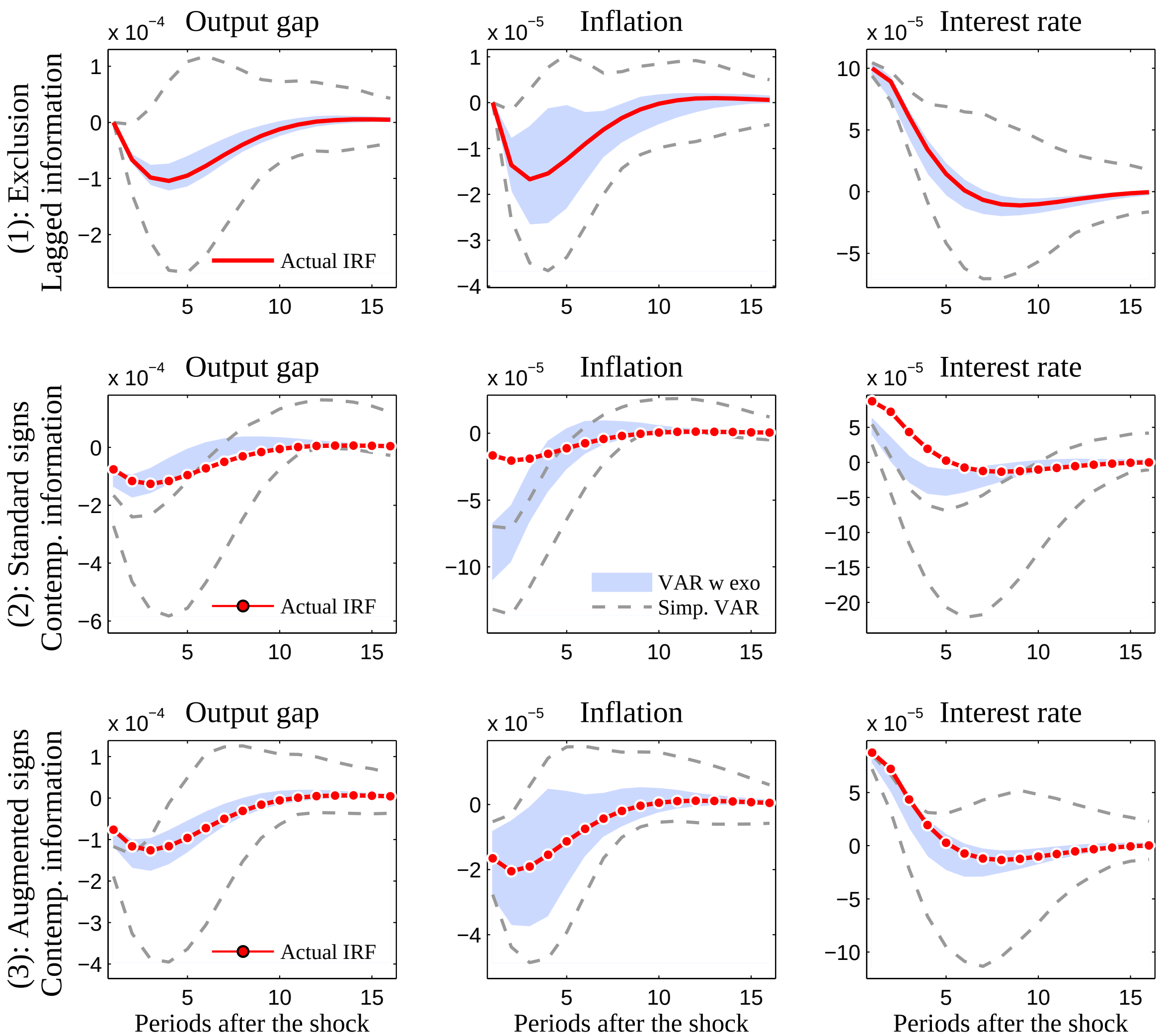
We now provide answers to these questions.

3.2.1 When the timing is correct

Let us focus first on exclusion restrictions. The first row of Figure 2 displays the actual impulse responses to a monetary policy shock issued by the lagged information economy (red plain line), along with the 68-th percent confidence interval of the simple VAR (dashed lines) and the VAR with exogenous variables (light blue area) issued by exclusion restrictions.

Notice that the VAR with exogenous variables performs remarkably well in this case. In fact, adding the block-exogenous (foreign) variables to the VAR not only substantially shrinks uncertainty, but it is also crucial to fully recover the transmission of the shock under exclusion restrictions. In contrast, the simple VAR fails to deliver responses for output and inflation significantly different from zero along the whole horizon.

Figure 2: When the timing of the identifying assumptions is correct



Note: The plain line and the line with circles represent the actual DGP of the economy, corresponding to the lagged information environment or the contemporaneous information environment, respectively. The dashed lines show the 68-th uncertainty bands for the impulse responses of the VAR without foreign variables, while the blue light areas display the said bands for the VAR with foreign variables.

Now we turn to sign restrictions. The second row of Figure 2 shows the true responses of a DGP with contemporaneous information (red line with circles), along with the 68-th model-uncertainty bands from the SVARs with standard sign restrictions. Recall that these bands show a subset of responses generated with matrices $\mathcal{A}_k$, so that $\varepsilon_t = \mathcal{A}_k \upsilon_t$ and $\mathcal{A}_k$ fulfills the imposed signs. To select these matrices, we followed Algorithm 1, and selected a $K = 50$ per draw. As it was the case with exclusion restrictions, the VAR with exogenous variables displays more precise bands than the simple VAR, especially for output. However, it is worrisome that the sign-identified responses of inflation are larger in absolute value than those of the actual DGP, especially on the impact period. In the case of the nominal interest rate, the sign-identified responses are actually smaller as compared to the DGP. Kilian and Murphy (2012) notice that there are cases in which standard sign restrictions imply implausible responses of certain variables of interest. The reason is that we chose the sign of the impact responses, but we do not limit their magnitude. Thus, without any prior information other than the required signs, models that imply large responses are considered equally likely as those that imply small responses.[14] For the case of the oil market, Kilian and Murphy (2012) put a restriction on the price elasticity of oil supply so that it rules out sign-identified matrices $\mathcal{A}_k$ that generate economically implausible responses of the oil supply to oil price shocks. As a result, the set of plausible sign-restricted SVAR models shrinks substantially, and that helps to better represent the dynamics of the oil market, as argued by Kilian and Murphy.

In our particular case, let us consider a modified sign restrictions approach augmented with an elasticity bound on the impact response of inflation. For practical matters, the elasticity bound should be determined using information from observed data. Consider, thus, the following regression

$$\Delta\pi_t = \beta_0 + \beta_i \Delta i_t + \beta_{ii} D_t \Delta i_t + \omega_t, \tag{11}$$

where ω_t is an ARMA(p,q) process with white noise innovations, and $D_t = 1$ if $\Delta i_t \times \Delta\pi_t < 0$, and zero, otherwise. The dummy variable registers changes in inflation and the interest rate of opposite signs, which is likely to happen after a monetary policy shock. The interest-rate elasticity of inflation, conditional on changes of opposite signs, is thus given by $\beta^- = \beta_i + \beta_{ii}$. In our simulated samples, $\beta^- < 0$. Therefore, as a rule-of-thumb, we impose an elasticity bound on the impact response of inflation after a contractionary monetary policy shock of the

[14]Kilian and Murphy (2012) argue that the cost of remaining agnostic about the precise values of the structural parameters is that the data are potentially consistent with a wide range of structural models that are equally admissible, since they all satisfy the identifying restrictions. Without further assumptions, there is no way of knowing which of these models is most likely.

form

$$\beta^{-}_{lower}\Delta i_t \leq \Delta\pi_t < 0, \tag{12}$$

where the subindex $lower$ indicates the 95-th percent lower bound of coefficient β^{-}, or the most negative value on its confidence interval. This inequality restriction is a soft one, since we give a range of responses to inflation that seem plausible given the observed data. In addition, we take the lower bound of β^{-} since the regression equation 11 may present a bias due to endogeneity (the short-term interest rate may react to current changes in inflation) and the omission of other variables that affect inflation; therefore, β^{-} might not be precisely pinned down by its point estimate $\hat{\beta}^{-}$. For simplicity, we opted to run regression (11) on a large simulated sample of 10,000 observations for each DGP, and we then imposed the corresponding β^{-}_{lower}-bound to each sub sample.[15] The results of this exercise are depicted in the last row of Figure 2. The improvement in terms of accuracy of the sign-identified responses with an elasticity bound is remarkable, not only for inflation but also for the nominal interest rate. With the proposed correction, the true responses of the DGP lie well inside the 68-th model-uncertainty bands. Notice as well that the simple VAR with augmented sign restrictions plausibly identifies the shock, a result that contrasts with the case of exclusion restrictions.

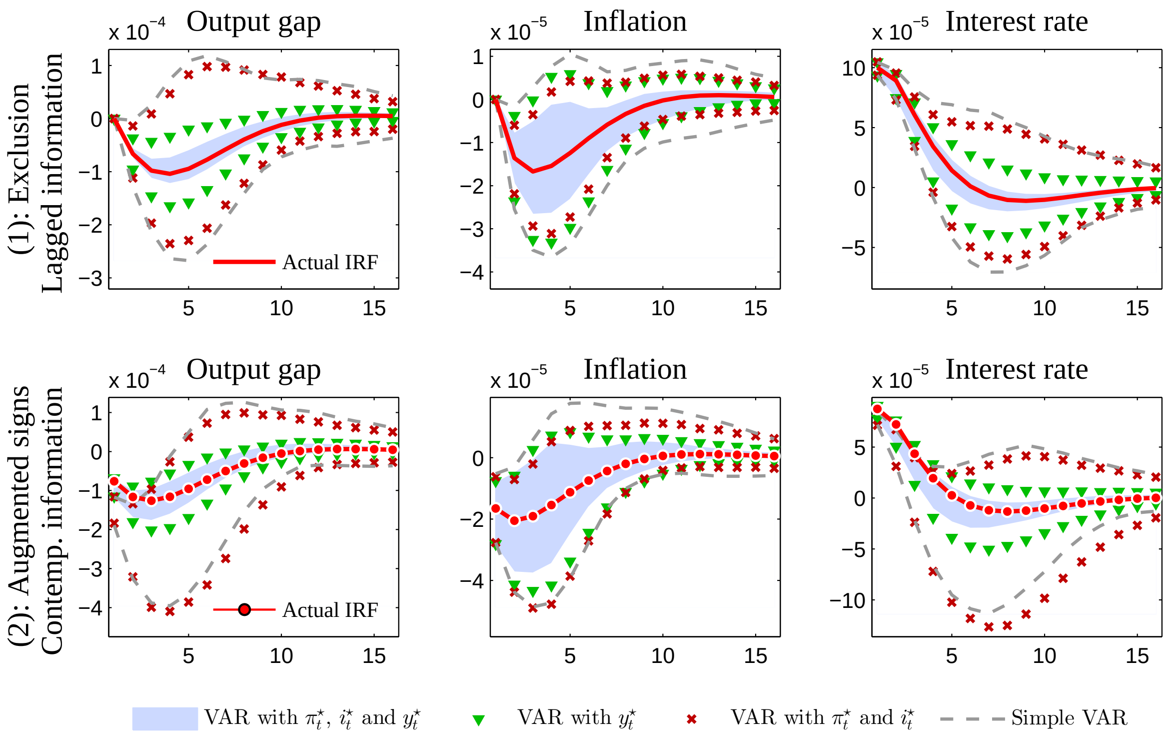
The elasticity-bound constraint of the augmented sign restrictions procedure generates important changes to the posterior distribution of impulse responses of the sign-identified SVARs. On the technical appendix, Figure A.1, we compare the distribution of the impact response of output, inflation, and the nominal interest rate to the monetary policy shock issued by the standard and augmented sign restrictions. In the top panel, the distributions come from an individual random draw from the posterior distribution of structural parameters, while the bottom panel shows the posterior distribution of closest-to-the-median models, issued by Algorithm 1. The bound constraint on the SVAR-based interest-rate elasticity of inflation tilts rightwards the distributions of all variables, not only inflation. Also, the size constraint induces a more precise inference about the responses of the nominal interest rate, which becomes clearly positive and closer to the actual DGP, as Figure 2 shows.

Effect of adding exogenous variables. In Section B of the technical appendix, we use a simple example to show that, when the correlation between foreign and home innovations is

[15]In our regressions, we found that an ARMA(2,2) in ω_t ensures that the innovations of regression (11) are white noise. The bounds imposed in the augmented sign restrictions are $\beta^{-}_{lower} = .5$ when the DGP is the contemporaneous information economy, and $\beta^{-}_{lower} = .2$ when the DGP is the lagged information economy.

different from zero, the estimated variance-covariance matrix $\hat{\Omega}^s_\varepsilon$ of the simple VAR is biased. The latter implies an important handicap for the exclusion restrictions approach, since its identifying power relies only on the information contained in this matrix. In contrast, sign restrictions add extra information (precisely, the theoretically robust signs) to the candidate matrices $\mathcal{A}_k$; this extra information helps identifying the shock, despite the bias in $\hat{\Omega}^s_\varepsilon$.

Figure 3: Effect of adding different foreign variables in the VAR



Note: The plain line and the line with circles represent the actual DGP of the economy, corresponding to the lagged information environment and the contemporaneous information environment, respectively. The dashed lines show the 68-th uncertainty bands for the impulse responses of the VAR without foreign variables, while the blue light areas display the said bands for a VAR with all foreign variables, the green triangles for a VAR with only foreign output, and the crosses for a VAR with foreign inflation and the foreign nominal interest rate.

In the context of our hypothetical small open economies, Figure 3 shows that the accuracy of the estimated impulse responses improves the most when we add foreign output instead of foreign inflation and/or the foreign nominal interest rate. The reason is that foreign output is highly correlated with home output (with a coefficient of about 80 percent), and so it contains the most relevant information about the correlation between foreign and home reduced-form innovations. In contrast, foreign inflation and the foreign nominal interest rate contain less useful information regarding the correlation between foreign and home reduced-form innovations.

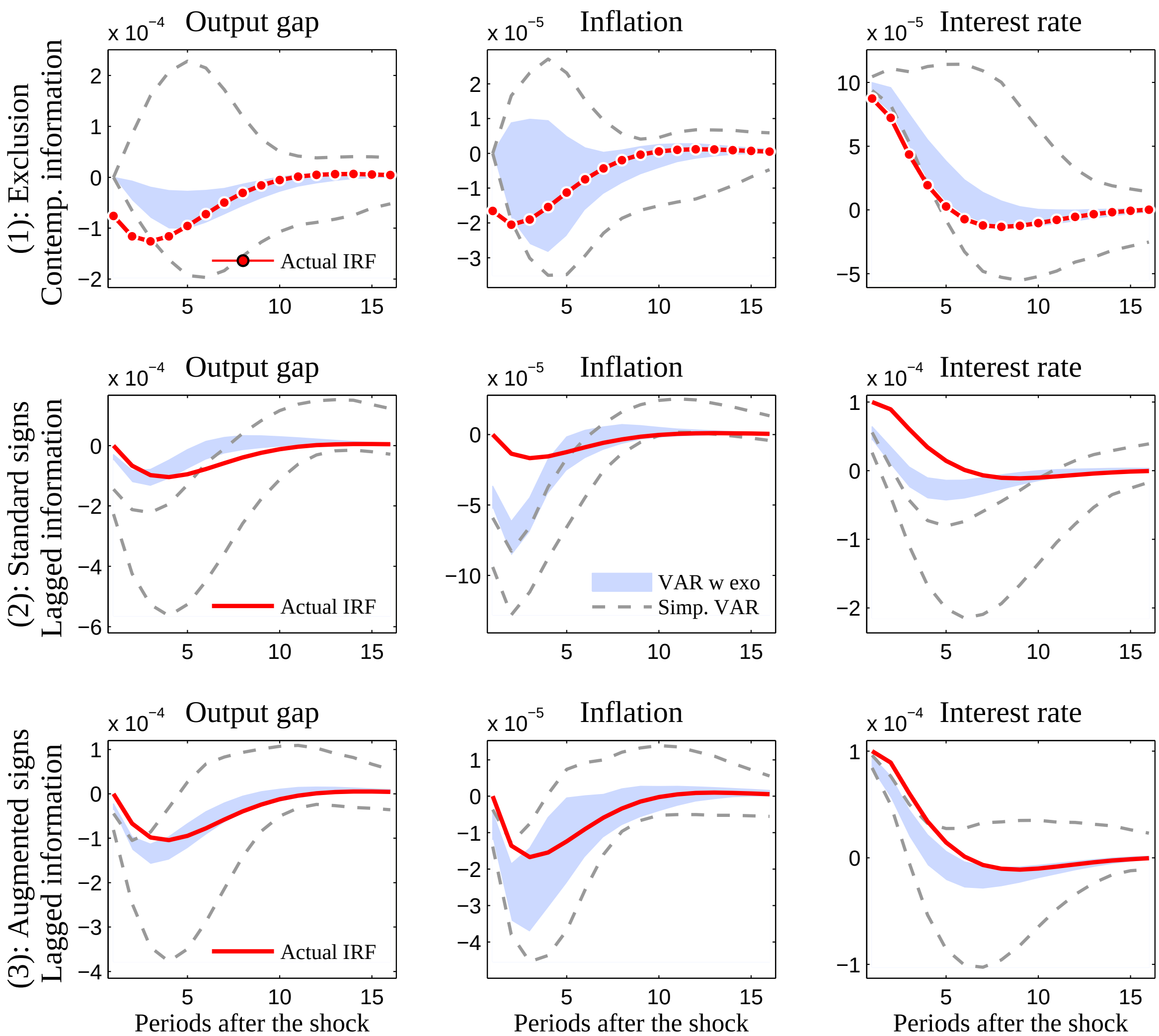
3.2.2 When the timing is incorrect

Now we turn to the case in which the timing of the DGP differs from the implied timing of the identifying restrictions. We describe first the results from exclusion restrictions. The first row of Figure 4 shows that, when the DGP corresponds to the contemporaneous information economy, exclusion restrictions not only miss the impact responses of output and inflation, but also artificially delay the peak responses of these variables for a few periods. In addition, exclusion restrictions in this environment do not rule out the presence of a price puzzle, even though the actual DGP contains none. Jääskelä and Jennings (2011) find similar results in their analysis. Finally, the performance of the simple VAR substantially worsens when the timing mismatches that of the DGP, particularly for output.

The second and third rows of Figure 4 show the impulse responses of the DGP with lagged information, and the model-uncertainty bands for sign-identified responses in such an environment. Once more, standard sign restrictions deliver a poor fit with respect to the true impulse responses. In contrast, the elasticity-bounded or augmented sign restrictions in row 3 display narrower and reasonably close-to-target responses, especially the VAR with exogenous variables. Notice as well that the peak responses of output and inflation are not delayed, and a price puzzle is correctly ruled out.

Next, we test which set of identifying restrictions is farther away from the actual DGP when we vary the timing of the true impulse responses. We focus on the SVARs with exogenous variables identified through exclusion restrictions and augmented sign restrictions. The top panel of Figure 5 displays, in row 1, the DGP with lagged information and, in row 2, the DGP with contemporaneous information. The light blue area represents the uncertainty bands for exclusion-identified responses and the dotted lines denote the said bands for augmented-sign-identified responses. As the figure shows, the more accurate procedure is the one that matches the timing of the DGP in place. But how badly do the identifying restrictions perform when they mismatch the timing of the DGP? This question is tackled by the second panel of Figure 5, which shows on the left picture the distribution of the total root mean squared error (RMSE) of each set of identifying assumptions across the 1,000 simulated samples for each of the two DGPs considered. The picture confirms that, when the identifying restrictions match the timing of the DGP, they attain a lower RMSE, on average. However, notice that the distribution of augmented sign restrictions is actually quite similar regardless of the DGP in place, while that of exclusion restrictions differs substantially. The latter means that, when the timing of the DGP is unknown, the expected RMSE of exclusion

Figure 4: When the timing of the identifying assumptions is incorrect



Note: The plain line and the line with circles represent the actual DGP of the economy, corresponding to the lagged information environment or the contemporaneous information environment, respectively. The dashed lines show the 68-th uncertainty bands for the impulse responses of the VAR without foreign variables, while the blue light areas display the said bands for the VAR with foreign variables.

restrictions increases faster with the probability of misspecifying the timing of the DGP than the expected RMSE of augmented sign restrictions. In other words, augmented sign restrictions are overall robust in terms of accuracy. This claim is shown graphically on the bottom right picture of Figure 5.

To summarize, the controlled Monte Carlo experiments illustrate three points. First, the two set of identifying assumptions can potentially recover the true impulse responses to a monetary policy shock when they match the timing of the actual DGP. In this context, the accuracy of sign restrictions strengthens substantially when we include an elasticity bound on the responses of variables that otherwise would seem implausibly large, as was the case for home inflation. Second, adding exogenous variables is crucial for exclusion restrictions to identify the shock, while not so for sign restrictions. In general, however, adding foreign variables reduces the uncertainty bands in both approaches. And third, the augmented sign restrictions approach is more robust when the timing of the actual DGP is unknown.

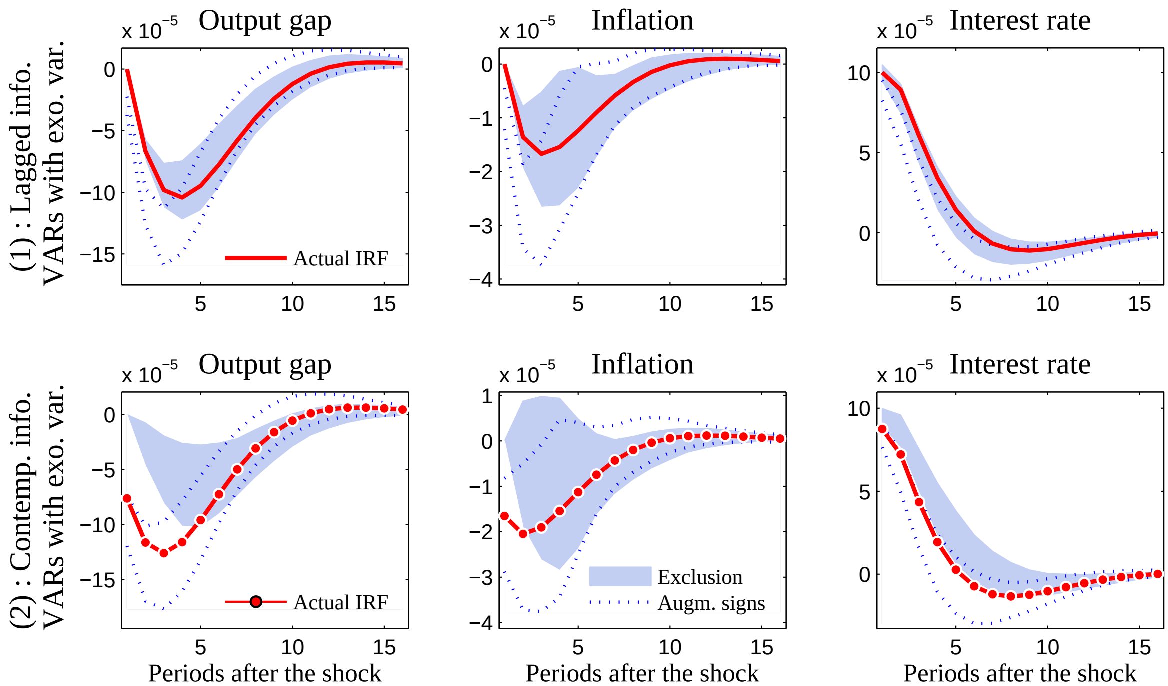
4 Application with Mexican data

In this section, we apply the identifying assumptions discussed above to study the effects of a monetary policy shock in Mexico.[16] We compare thus exclusion restrictions (in their Cholesky-recursive form), with sign restrictions with and without a bound on the interest-rate elasticity of inflation. We show that several of the features introduced in the last section are present in the estimations with Mexican data. For instance, we show that exclusion restrictions register the smallest and most delayed peak responses of inflation, and do not discard the presence of a price puzzle. In contrast, sign restrictions reduce the chances of a price puzzle substantially, and reflect, in general, more stable responses than exclusion restrictions, regardless of whether we include or not the block-exogenous variables to the VAR. Finally, augmented sign restrictions discard implausibly large responses of inflation on the impact period.

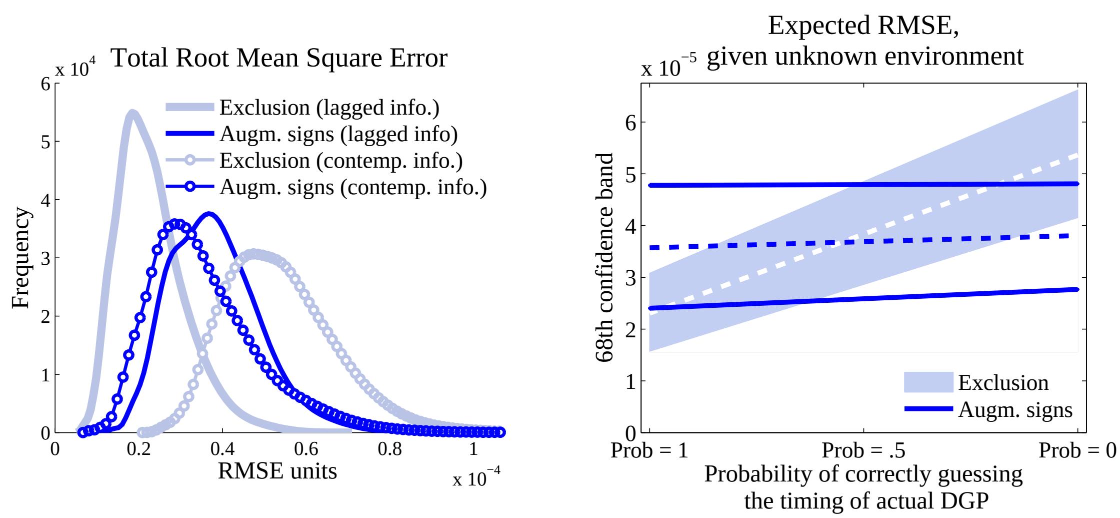
[16] Mexico is a fine example of a small open economy. Its degree of openness (i.e., the ratio between [export + imports]/GDP) scores an average of 44 percent in the sample period studied. The main trading partner of Mexico is the U.S., where over 80 percent of the Mexican exports are sold. Further, Mexico is about 9 times smaller in terms of annual GDP than its North-American neighbour. Given these characteristics, it seems reasonable to assume that the U.S. business cycle affects that of Mexico, but not vice versa. Finally, the correlation between the Mexican output gap and the U.S. output gap is of about 86 percent for the sample period considered.

Figure 5: Robustness of identifying assumptions

(TOP PANEL)



(BOTTOM PANEL)



Note: In the top panel, the plain line and the line with circles represent the actual DGP of the economy, corresponding to the lagged information environment or the contemporaneous information environment, respectively. The dashed lines show the 68-th uncertainty bands for exclusion-identified SVAR, while the blue light areas display the said bands for the sign-identified SVAR. The bottom-left panel presents the distributions of the root mean square error achieved by each SVAR under either the lagged information DGP, or the contemporaneous information DGP. The bottom-right panel gives a 68-th confidence band for the expected root mean squared error of each SVAR, given the probability that the timing of the SVAR matches that of the actual DGP.

4.1 Data and estimation of the reduced-form VARs

The home data we use to estimate both the simple VAR and the VAR with exogenous variables are a measure of the home output gap, core CPI inflation, producer price inflation, the real exchange rate, the short-term interest rate, and money growth. We decided to include producer price inflation, which measures the prices of intermediate goods, to mitigate the so called "price puzzle" usually present in monetary SVARs when using exclusion restrictions (see Sims and Zha, 2006; Christiano *et al.*, 2005). We included money growth to provide the system with more information about the monetary transmission mechanism. Section C of the technical appendix describes in detail each of these variables and their properties. Our sample period spans from January 2001 to March 2014, at a monthly frequency. We start our sample in 2001 because Chiquiar *et al.* (2010) show that Mexican inflation became significantly less persistent from that year onwards, once the Mexican central bank (*Banxico*) adopted an inflation-targeting regime. The change in policy strengthened the convergence of inflation towards the central bank's target of 3 percent, which was officially set for the year 2003 and thereafter.[17] In view of these facts, we decided to remove changes in trend inflation induced by the convergence episodes. We opted for a simple strategy to compute trend inflation, denoted by $\bar{\pi}_t$, as we extracted the low frequency component of inflation through an HP filter.[18] To maintain a long-term equilibrium consistency with all nominal variables, we also remove trend inflation from producer prices, the nominal interest rate, and money growth.[19] The set of detrended home variables is portrayed in Figure 6, and summarized in vector Y_t as

$$Y_t = \begin{bmatrix} \text{Output gap} \\ \text{Inflation gap} \\ \text{Producer price inflation gap} \\ \text{Real exchange rate depreciation} \\ \text{Nominal interest rate gap} \\ \text{Money growth gap} \end{bmatrix} = \begin{bmatrix} y_t - \bar{y}_t \\ \pi_t - \bar{\pi}_t \\ \pi_t^p - \bar{\pi}_t \\ \Delta q_t \\ i_t - \bar{\pi}_t \\ \Delta m_t - \bar{\pi}_t \end{bmatrix}. \tag{13}$$

[17]According to *Banco de México's Quarterly Inflation Report for June-September 2000*, the inflation goal for 2001 was set to 6.5 percent; for 2002, it was 4.5 percent; and finally, for 2003, the Bank introduced its medium-term target of 3 percent.

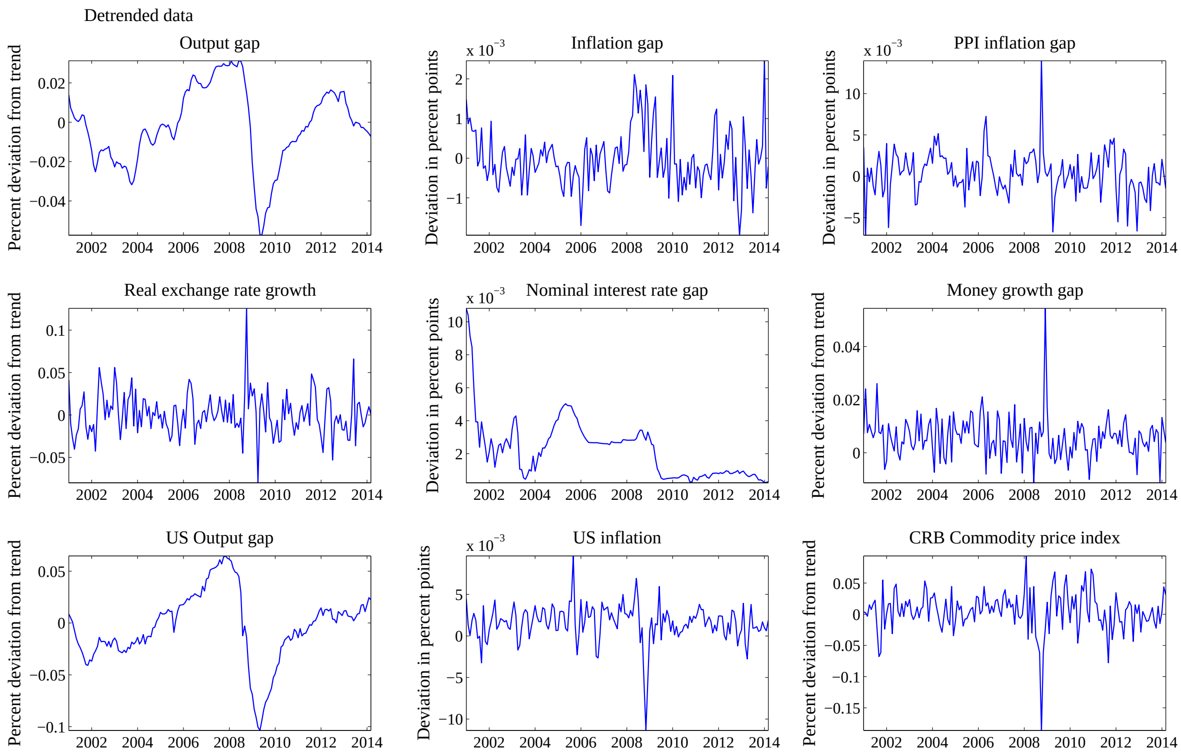
[18]We set λ equal to $129,600$, as proposed by Ravn and Uhlig (2001) for monthly data. As an alternative, we also explored the Stock and Watson (2007) unobserved-variable method to extract trend inflation. The results do not change much using the two methodologies, so we chose the simplest one for the ease of exposition.

[19]In a standard New Keynesian model with trend inflation, as in Ireland (2007), Cogley *et al.* (2010), or Christiano *et al.* (2014), all nominal variables, such as prices, nominal interest rates, and wages, grow in the long-term at their equilibrium level, which is proportional to trend inflation.

For the block-exogenous variables, we include two U.S. variables and one world variable, given by a measure of the U.S. output gap, the PCE inflation rate, and the CRB spot-price inflation, which accounts for fluctuations in the price of world-traded commodities.[20] We decided not to remove trend inflation from PCE and CRB inflation rates, since the trend in both variables is flat for the sample period studied. Further details on these variables can be found in Section C of the technical appendix. The set of foreign variables is displayed as well in Figure 6, and is summarized in vector Z_t as:

$$Z_t = \begin{bmatrix} \text{US output gap} \\ \text{US inflation} \\ \text{CRB inflation rate} \end{bmatrix} = \begin{bmatrix} y_t^{us} - \bar{y}_t^{us} \\ \pi_t^{us} \\ \pi_t^{crb} \end{bmatrix}.$$

Figure 6: Detrended data used in estimations



Note: All domestic variables were extracted from INEGI and Banco de México, except for GDP, that was computed as in Elizondo (2012). For U.S. variables, we used the FRED2 database, while we gathered the CRB index from Bloomberg. For some variables we extracted a low frequency component. For the output measures, we used the HP filter for each variable. For domestic core inflation, producer price index inflation, the nominal interest rate, and money growth, we used the trend of core inflation as computed by the HP filter.

[20]Our results are robust to adding the fed funds rate to the VAR. We decided to exclude this variable here because the fed funds rate has been near its zero-lower-bound since the end of 2008, and has therefore become uninformative since that date.

We estimate both the simple VAR and the VAR with exogenous variables through maximum likelihood, where we use 3 lags for each VAR.[21] We compute the joint distribution of parameters of the VARs, $\Gamma_j(\Theta_j)$ for $j \in \{s, e\}$, using bootstrapping as in Runkle (1987), or Dupaigne and Fève (2009). This approach is suitable for small samples.[22] We denote as $\hat{\Gamma}_j(\hat{\Theta}_j)$ the bootstrapped estimate of $\Gamma_j(\Theta_j)$. We also checked that our estimated parameters $\hat{\Theta}_j$ lead to globally stationary VARs models for the sample period studied (the details are in Section C of the technical appendix). Finally, notice that a VAR with detrended variables is a good approximation of a DGP like the one described in Section 3.1. The reason is that, in the long term, all detrended variables and those stated in gap terms should be stationary.[23] In addition, Fry and Pagan (2011) argue that a VAR with stationary variables is the appropriate summative model to identify transitory shocks such as a monetary policy shock, an aggregate demand shock, or a cost-push shock.[24]

4.2 Results with exclusion restrictions

We ordered home variables just as shown in equation (4.1), which implies that, under the recursive assumption, neither the home output gap, CPI inflation, producer price inflation, or the real exchange rate contemporaneously react to the monetary policy shock. This ordering assumption is recurrent in the literature and it is founded on the principle that economic activity variables take time to adjust to nominal disturbances.[25] In contrast, the nominal interest rate and money growth react on impact to the shock. Figure 7 shows the 68-th percent confidence bands of the impulse responses of home variables to a contractionary monetary policy shock, identified using the estimated matrix $\hat{\Omega}_{\varepsilon}^{s}$ for the simple VAR (dashed lines) and

[21]The SIC and AIC information criteria suggest one and two optimal lags, respectively. Hatemi and Hacker (2009) propose to use a LR test to set the optimal lags when the above criteria disagree. In this context, the optimal lag order thrown by the LR test is three.

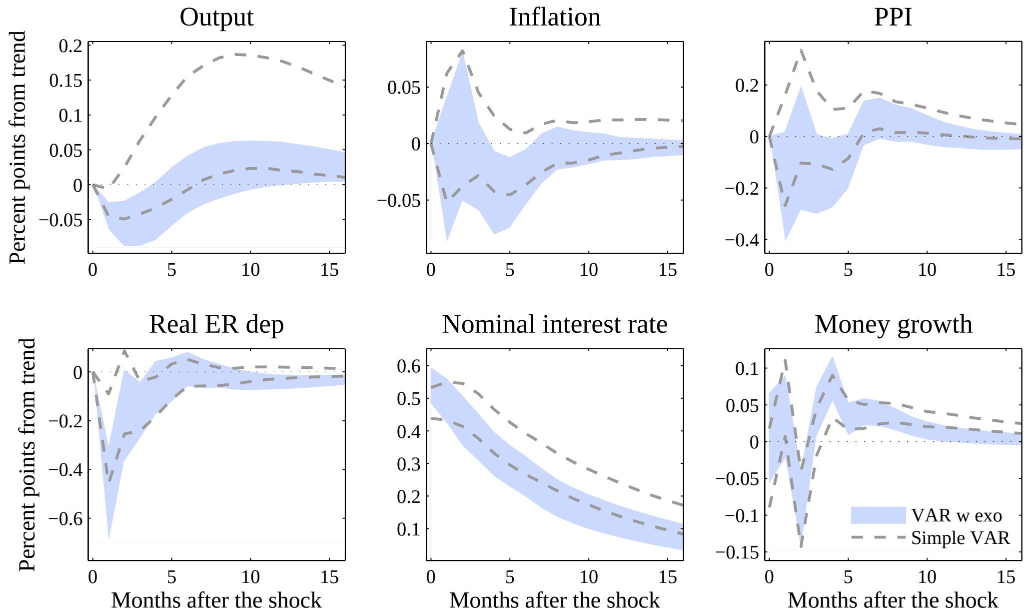
[22]In Section D of the technical appendix, we estimate the simple VAR with bayesian methods and show that the differences with bootstrapping are minimal.

[23]See also Canova and De Nicoló (2002), pp. 1134, who argue that, given a small sample, tests for integration and cointegration may have low power. In such a case, it is reasonable to be guided by economic theory, which points to the variables that are likely to be stationary under standard assumptions.

[24]Further, Fry and Pagan (2011) argue that, when one aims to identify both transitory and permanent shocks at the same time, the appropriate summative model is a VECM. The reason is that a permanent shock supposes a unit-root in the variables studied. As we do not aim to identify permanent shocks, a VAR with detrended variables is the appropriate summative model for our framework.

[25]For the real exchange rate, the ordering assumption is more debatable, since the former will tend to move with the nominal exchange rate, which reacts quickly to changes in monetary policy in economies with a floating regime. However, we have checked that changing the ordering of the real exchange rate, so that it responds to the monetary policy shock on impact, does not alter our main results.

Figure 7: Exclusion restrictions with Mexican data



Note: The dashed lines show the 68-th confidence interval bands for the impulse responses of the VAR without foreign variables, while the blue light areas display the said bands for the VAR with foreign variables. The order of the variables in the Cholesky decomposition is the same that the one that appears in the graphs.

$\hat{\Omega}^e_\varepsilon$ for the VAR with exogenous variables (light blue area). To compute these bands, we used 2,000 draws from the bootstrapped joint distribution of the corresponding VAR, *i.e.*, $\hat{\Gamma}_j(\hat{\Theta}_j)$ for $j \in \{s, e\}$. The effect on the real exchange rate, the nominal interest rate, and money growth in both VARs is quite similar. Namely, the exchange rate appreciates[26] with a period delay, while the short-term nominal interest rate increases at impact, and money growth oscillates around zero. However, for output and prices the differences are more important. Through the lens of the simple VAR, the home output gap even increases few months after the shock, while core CPI inflation responses are not significant and fluctuate around zero. The VAR with exogenous variables importantly shrinks the confidence band for the home output gap, which now features a significant but moderate fall in the aftermath of the shock. Similarly, core CPI inflation now significantly falls in a hump-shaped pattern, reaching its peak effect within 6 months after the shock. Interestingly, exclusion restrictions do not rule out the possibility of a price puzzle.

Similar to Section 3.2, we observed that adding the block-exogenous variables to the VAR is crucial for a plausible identification with recursive restrictions. The reason is that the

[26]We take the convention that a fall in the real exchange rate indicates an appreciation.

estimated matrix $\hat{\Omega}^s_\varepsilon$ is biased, which pollutes the only information used by the simple VAR to identify the shock (see Section B of the technical appendix). In contrast, the estimated matrix $\hat{\Omega}^e_\varepsilon = \hat{H} + \hat{\Omega}_{\varepsilon\eta}\left(\hat{\Omega}_\eta\right)^{-1}\hat{\Omega}_{\eta\varepsilon}$ includes a correcting term that accounts for the covariance between home and foreign innovations (*i.e.*, $\hat{\Omega}_{\eta\varepsilon}$), which improves accuracy and the ability of exclusion restrictions to provide a plausible identification.

4.3 Results with sign restrictions

Sign restrictions do not require any variable ordering to identify the monetary policy shock. We impose sign restrictions for the home variables' responses only in the impact period to three types of shocks: a monetary policy shock, an aggregate demand shock, and a transitory aggregate supply shock. Although we care just about the first shock, Canova and Paustian (2011) show that identifying more shocks increases the performance of sign restrictions. The imposed signs, which are robust across a great variety of models,[27] are summarised in Table 2.

Table 2: Sign restrictions for the Mexican VARs

Shock	y_t	π_t	π^p_t	Δq_t	i_t	Δm_t
Monetary policy shock	$-$	$-$	?	$-$	$+$	$-$
Aggregate demand shock	$+$	$+$	$+$	$-$	$+$	?
Transitory supply-side shock	$+$	$-$	$-$	$+$	?	?

Note: The sign refers to the impact response of the variable of interest due to a structural shock. A '+' (or '-') indicates that $\partial x_t/\partial \upsilon_{j,t} > 0$ (or < 0), where x_t is the variable of interest and $\upsilon_{j,t}$ is the structural shock studied.

Accordingly, after a contractionary monetary policy shock, home output gap, core CPI inflation, and money growth fall, the real exchange rate appreciates, and the nominal interest rate increases. We remain agnostic for producer price inflation and impose no restrictions; notice that the unrestricted response in this variable will give us important information about the likely reaction of core CPI prices. For a positive aggregate demand shock, output, core CPI and producer price inflations, and the nominal interest rate increase, the real exchange rate appreciates, and we impose no restrictions on money growth. Finally, for a positive aggregate supply shock, output increases and the real exchange rate depreciates, while core

[27]See Section 3.1 and Figure 1, and also Galí and Monacelli (2005), or Canova and Paustian (2011).

CPI and producer price inflations fall, and we remain agnostic on the responses of the nominal interest rate and money growth.

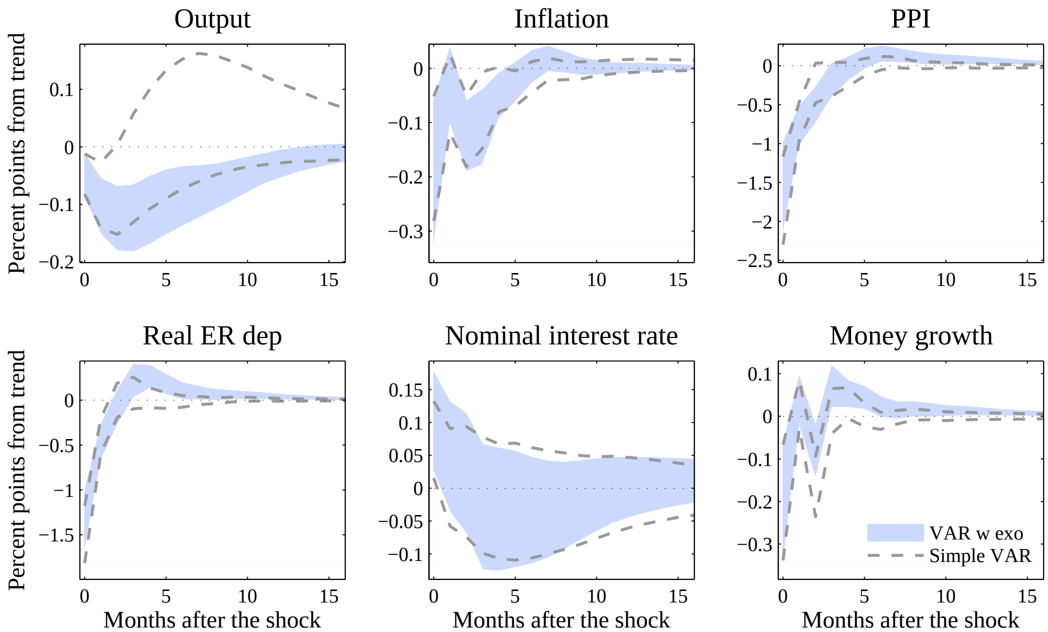
The above identifying assumptions suffice for an approach based on *standard sign restrictions*. However, we are also interested in ruling out implausible large responses of core CPI inflation on impact, so we add a size restriction as the one introduced in Section 3.2. For an identification based on these *augmented sign restrictions*, we ran a regression of changes in core CPI inflation on contemporaneous changes in the short-term nominal interest rate, as in equation (11). We assumed an ARMA(1,2) model for the error term of this regression to ensure that its innovations are close to white noise. We found a point estimate of $\hat{\beta}^{-} = -.035$, while the lower bound of its 95 percent confidence interval equals $\hat{\beta}^{-}_{lower} = -.7$ (which shows that the elasticity is not precisely estimated). Thus, for a contractionary monetary policy shock, our augmented sign restrictions require the signs of Table 2 plus an elasticity bound for the impact response of core CPI inflation of the form $\hat{\beta}^{-}_{lower}\Delta i_t \leq \Delta\pi_t \leq 0$.

Figure 8 displays the 68-th percent model-uncertainty bands for the simple VAR (dashed lines), and the VAR with exogenous variables (light blue area). The top panel displays the responses issued by standard sign restrictions and the bottom panel those produced by augmented sign restrictions. The intervals were computed for 2,000 draws from $\hat{\Gamma}_j(\hat{\Theta}_j)$ for $j \in \{s, e\}$. For each draw, we followed the Algorithm 1, and selected a $K = 100$. We effectively searched for 200,000 matrices $\mathcal{A}_k$ that fulfill the restrictions, but we kept only 2,000 that were closest to the median for each draw of $\hat{\Gamma}_j(\hat{\Theta}_j)$. Overall, it is striking that the responses issued from the VARs with and without foreign variables are remarkably similar, except for the home output gap. In this case, adding the set of block-exogenous variables to the VAR shrinks the model-uncertainty bands substantially. This result was expected since home and foreign output gaps have a correlation of .86 in the observed sample. Thus, foreign output contains important information to correct the bias in $\hat{\Omega}_{\varepsilon}$. Also notice that the monetary policy shock seems plausibly identified, as core CPI inflation significantly falls for a few periods (output does so with humped-shaped pattern in the VAR with exogenous variables), the real exchange rate strongly appreciates on impact, and producer prices, for which we did not impose any restrictions, fall on impact and return to their initial state after a few months.

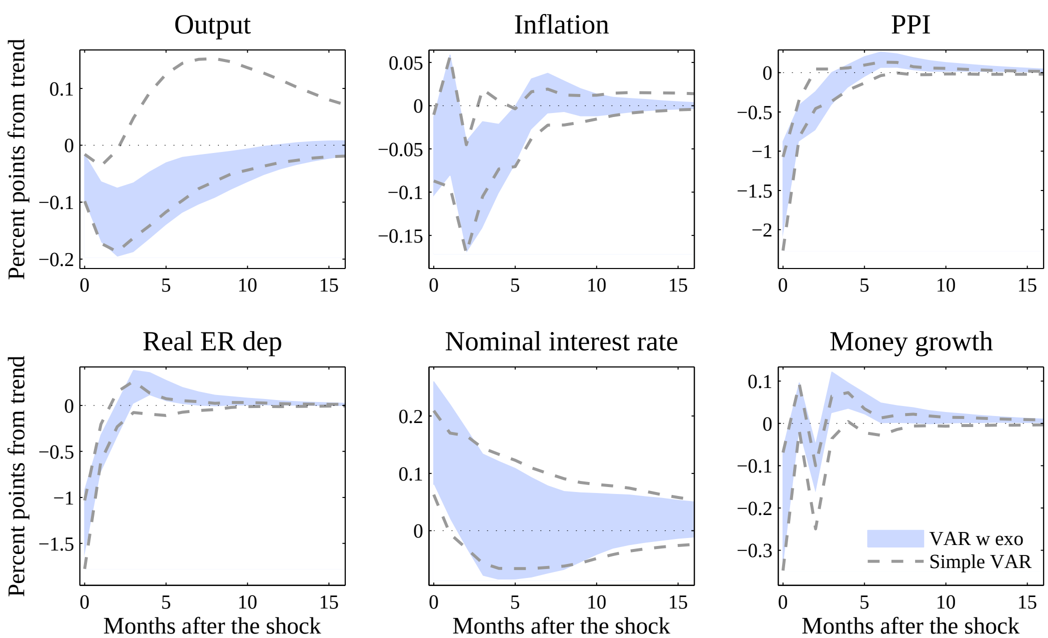
Notice that the peak responses of CPI inflation from standard sign restrictions differ from their counterparts with augmented sign restrictions. For the former, the impact response of CPI inflation corresponds to the peak response, a feature that seems hard to reconcile with the vast literature of monetary SVARs. In particular, the standard-sign-identified SVARs yield a

Figure 8: Sign restrictions with Mexican data

(STANDARD SIGN RESTRICTIONS)



(SIGN RESTRICTIONS WITH AN ELASTICITY BOUND)



Note: The dashed lines show the 68-th model-uncertainty bands for the impulse responses of the VAR without foreign variables, while the blue light areas display said bands for the VAR with foreign variables.

median interest-rate elasticity of inflation on impact (i.e., $\Delta\pi_t/\Delta i_t$) equal to -2.5 and -2 for $\hat{\Gamma}_s(\hat{\Theta}_s)$ and $\hat{\Gamma}_e(\hat{\Theta}_e)$, respectively. However, in the data, the lower bound of this elasticity was found to be equal to -.7, *i.e.*, around 3 times smaller in absolute value than the median of the standard-sign-identified SVARs. When we add the elasticity bound to the augmented sign restrictions, the median interest-rate elasticity of inflation falls to around -.35 in both types of VARs. As a consequence, the peak response of CPI inflation in the augmented-sign-identified SVARs happens within 6 months after the monetary policy shock, and not at impact.

4.4 Comparison across identifying assumptions

Figure 9 compares the 68-th percent uncertainty bands for the responses issued from different identifying assumptions applied only to the VAR with exogenous variables. In the top panel, we compare exclusion restrictions (light blue area) with augmented sign restrictions (dotted lines). After a couple of periods, all variables respond in the same direction under the two approaches. However, the quantitative differences are substantial. Notably, exclusion restrictions find smaller responses for output, CPI inflation, producer price inflation, and the real exchange rate as compared to augmented sign restrictions. In contrast, the responses of the short-rate interest rate are larger under exclusion restrictions, while money growth responses look similar under the two types of identifying restrictions.

Regarding CPI inflation, three remarks are in order. First, the dynamics of inflation under exclusion restrictions are somewhat more persistent than under augmented sign restrictions. Second, the peak response under exclusion restrictions achieves a maximum of 5 basis points after an increase of around 50 basis points in the nominal interest rate, both changes in annual terms; for augmented sign restrictions, the maximum peak response is three times larger, for an increase of just 15 to 20 basis points in the nominal interest rate. And third, under augmented sign restrictions, a price puzzle occurs with lower probability than with exclusion restrictions, and for shorter time too. Interestingly, two of these facts are reminiscent of the second row in Figure 5 of our Monte Carlo experiment in Section 3.2. The first and the third remarks call for the case in which the timing of the DGP corresponds to the timing of sign restrictions, *i.e.*, when inflation reacts contemporaneously to changes in the nominal interest rate. Also, under this case, the response of the nominal interest rate drawn from exclusion restrictions is larger than under augmented sign restrictions. How likely is it that CPI inflation reacts within the same period to changes in the monetary policy rate? Although this is a question that goes beyond the scope of this paper, since we do not have the micro data on prices at hand, the responses of producer prices gives us an indication. Recall that we gave no restrictions either of sign or of magnitude to this variable. Accordingly, under augmented sign restrictions, producer prices fall within the impact period of the monetary policy shock, and they do so with a high volatility. As the prices of raw inputs fall, it is likely that there will be a limited pass through to consumer prices as well. However, notice that augmented sign restrictions do not discard models in which CPI inflation does not answer on impact to the monetary shock.

The bottom panel of Figure 9 compares the responses of standard sign restrictions (light blue area) with those of augmented sign restrictions (again, the dotted lines). There are only two important differences between these responses. The obvious one is that the impact response of CPI inflation is muted under augmented sign restrictions; although, for the rest of the horizon, the model uncertainty bands are essentially identical, ruled by the dynamics of the data. The second difference happens with the nominal interest rate. Similar to the Monte Carlo experiment, the sign-identified responses of the short-term interest rate were pushed upwards when we ruled out implausibly large inflation responses.

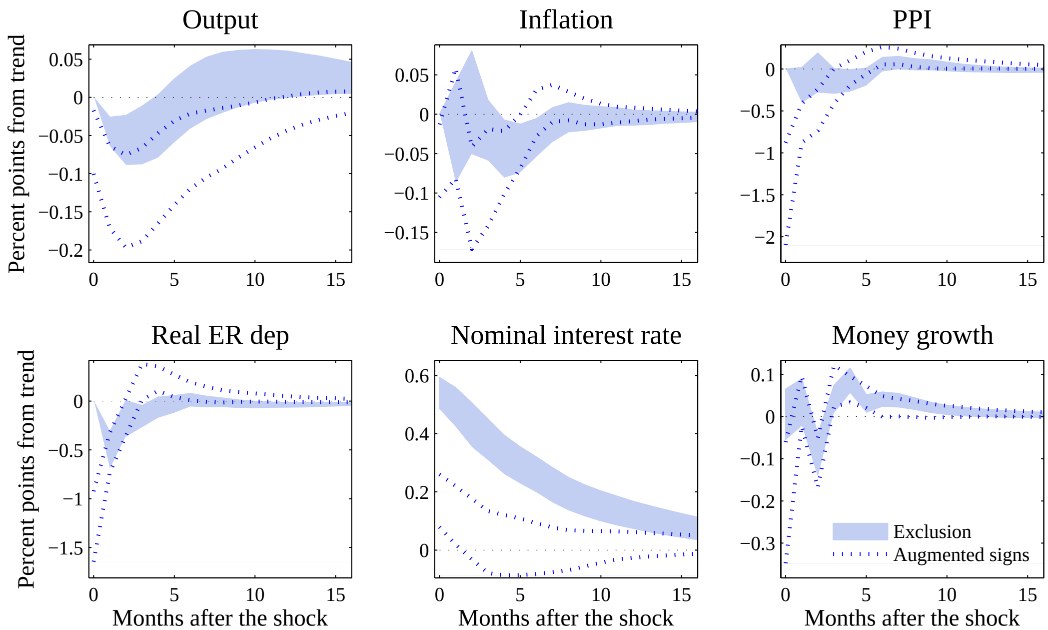
4.5 Other studies with Mexican data

Gaytán and González (2006) also measure monetary policy shocks in Mexico and study changes in the monetary transmission mechanism after *Banxico* formally adopted inflation targeting in 2001.[28] In particular, Gaytán and González use a regime-switching VAR model and identify 3 regimes in the sample period from January 1994 to January 2005: a *crisis* regime is associated to the periods around and after the Tequila crisis of 1994, and the Asian currency crisis of 1997; a *calm* regime corresponds to the relative smooth periods that followed the crises; and the third regime corresponds to inflation targeting. Then, for *calm* and *inflation-targeting* regimes, the authors compute the responses of the real exchange rate, the output gap, CPI inflation, inflation expectations, and the nominal interest rate to a monetary policy shock through recursive restrictions. Despite the differences in periods and variables, the responses found by Gaytán and González are qualitatively similar to those presented in our estimations with recursive restrictions. For instance, for the calm regime, the authors also find that the peak responses of inflation happens within the first 6 months after the monetary

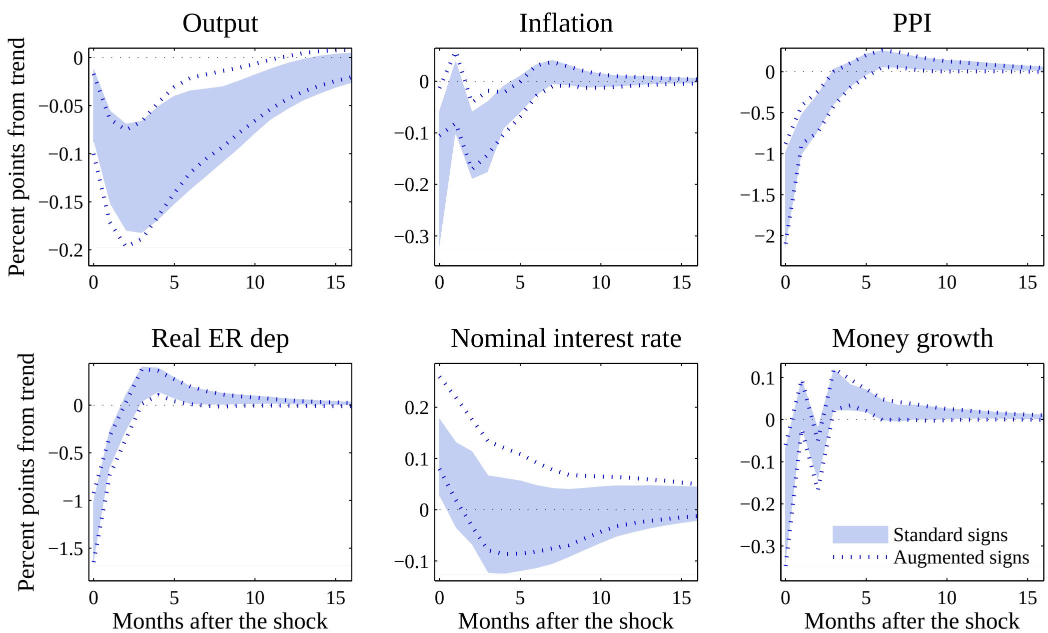
[28]Other studies also document changes in the monetary transmission mechanism after inflation targeting in Mexico. See for instance Ramos-Francia and Torres (2006), Capistrán and Ramos-Francia (2010), Chiquiar *et al.* (2010), García-Verdú (2006), and Cortés (2013).

Figure 9: Comparison of identifying assumptions on VAR with exogenous variables

(EXCLUSION VS SIGNS WITH AN ELASTICITY BOUND)



(STANDARD SIGNS VS SIGNS WITH AN ELASTICITY BOUND)



Note: In the top panel, the dashed lines show the 68-th confidence interval band for the impulse responses of the augmented-sign-identified SVAR, while the blue light areas display the said bands for the exclusion-identified SVAR. In the bottom panel, the dashed lines show the 68-th confidence interval band for the impulse responses of the augmented-sign-identified SVAR, while the blue light areas display the said bands for the standard-sign-identified SVAR.

policy shock.

In a related study, Sidaoui and Ramos-Francia (2008) document changes in inflation dynamics using the credit channel and the expectations channel of the monetary transmission mechanism. In particular, they use a VAR model and a small-scale macroeconomic model, and show that after 2001 changes in prices are no longer permanent after credit shocks. Similarly, they show that a cost push shock exerts smaller second-round effects in the Mexican economy after 2001. The results of Sidaoui and Ramos-Francia, although not strictly comparable to ours, are complementary to our measurement of the monetary transmission mechanism in Mexico in late years.

5 Conclusions

We have compared the robustness of two types of identifying restrictions for a monetary policy shock applied to SOEs: exclusion restrictions and sign restrictions. Specifically, we focused on Cholesky-type recursive restrictions, standard sign restrictions, and elasticity-bounded or augmented sign restrictions. We set out a Monte Carlo exercise with simulated data from a hypothetical SOE, in which we varied the timing of responses of the DGP, so that it matched either the implicit timing of recursive restrictions or that of sign restrictions. Then, we ran VARs with and without foreign variables, and identified SVARs that either matched or mismatched the timing of the actual DGP.

The lessons from our Monte Carlo experiments are the following. Exclusion restrictions perform fine when their timing matches that of the DGP and all relevant foreign variables are included in the VAR. However, exclusion restrictions perform poorly when they mismatch the timing of the DGP, and they fail to identify the monetary policy shock when the VAR omits foreign variables. In such a case, the exclusion-identified SVAR yields odd responses for output and it does not discard a price puzzle for inflation, even if the actual DGP contains none. On the other hand, standard sign restrictions systematically overshoot the responses of inflation, and undershoot those of the nominal interest rate, as compared to the DGP. To fix this caveat, and in the spirit of Kilian and Murphy (2012), we add a size restriction on the first response of inflation after the shock, which we build after a simple regression that estimates the contemporaneous interest-rate elasticity of inflation. As a result, the augmented sign restrictions substantially improve accuracy, and prove robust to both the timing of the responses of the DGP, and the set of foreign variables included in the VAR. Last, the SVARs with foreign variables always display thinner uncertainty bands for their impulse responses than those issued by SVARs without foreign variables.

Finally, we also identify SVARs using Mexican data with exclusion, standard sign, and augmented sign restrictions to document similarities with the Monte Carlo exercise. Remarkably, most of our results hold in the empirical application too.

For the sake of clarity, we have left out important questions from our analysis. For instance, we have not studied how well the above identifying restrictions capture the effects of foreign shocks on home variables, or how we can efficiently estimate VARs with block exogeneity when the monetary policy regime changes over time, either at home or abroad. We believe these questions are important for inflation-targeting SOEs, but as of now they are beyond the scope of this paper. Nonetheless, we plan to study them in future research.

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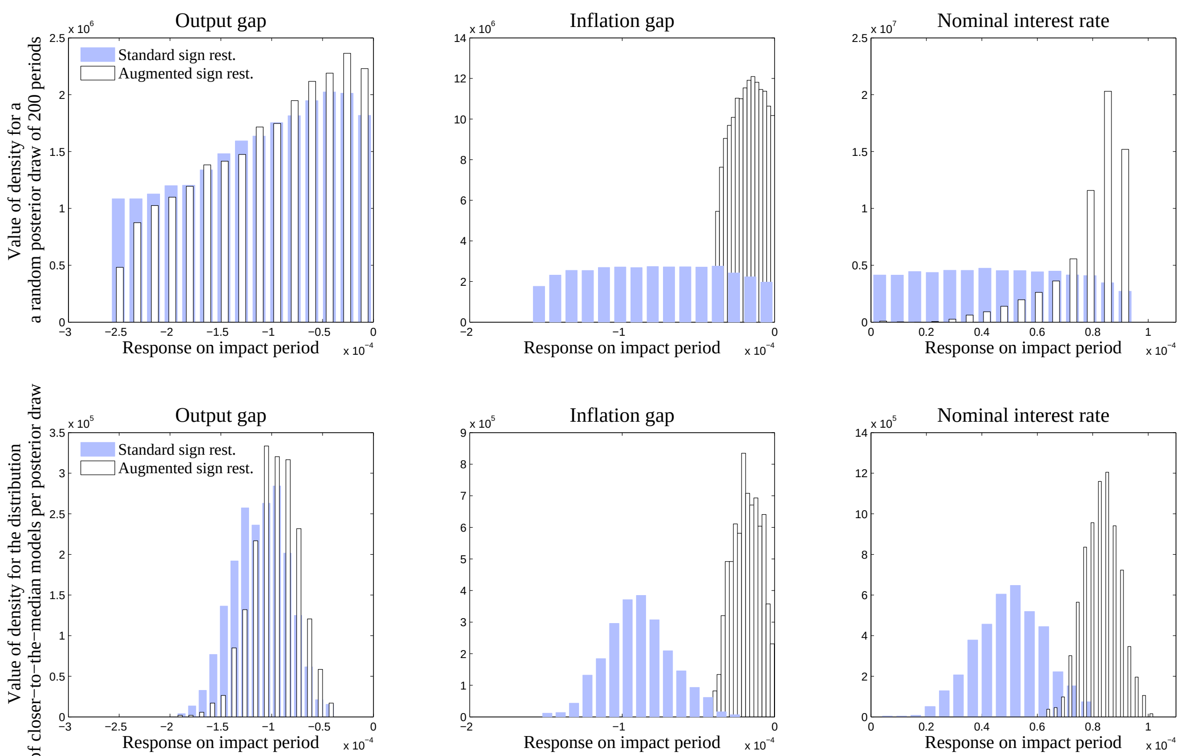
Technical appendix

A The Baumeister-Hamilton critique

Baumeister and Hamilton (2014) point out that the QR procedure to generate sign-identified models creates impulse responses that follow truncated Cauchy distributions. The reason is that the orthonormal rotation matrix $\mathcal{Q}$, used to compute the candidate model $\mathcal{A} = \underline{\text{chol}}(\hat{\Omega})\mathcal{Q}$, involves crossed-row restrictions that must be verified. That implies that while certain elements of $\mathcal{A}$ could take any values between 0 and $+\infty$ (or $-\infty$), some other elements would be inside an finite interval. In other words, the responses of some endogenous variables will be restricted to be not only positive or negative, but also inside the interval $(\underline{a}, \bar{a})$. To show this point precisely, the first row of Figure A.1 shows the responses of output, inflation, and the nominal interest rate to the monetary policy shock in the impact period, resorted from a random draw h from the Monte Carlo exercise of Section 3. In the figure, we show the responses from standard sign restrictions and augmented sign restrictions computed from the VAR with exogenous variables. For this example, we computed 10,000 sign-restricted models $\mathcal{A}$ to have a complete picture of the posterior distributions of impulse responses. Such distributions are clearly truncated, regardless of the sign-restrictions procedure used. Further, the truncation does not vanish even if we increase the number of observations by 50 times in the Monte Carlo exercise (*e.g.*, from 200 observations, as in the original exercise, to 10,000). The second row of Figure A.1 shows the distributions of responses resulting from Algorithm 1, which are composed of the closest-to-the-median responses per draw, for all the 1,000 draws considered in the Monte Carlo exercise. Although Algorithm 1 does not alleviate the finite-interval problem, it greatly moderates truncation in the distributions, which are now close to normal. Further, Section 3.2 shows that the median of these distributions could potentially be quite close to the true impulse responses, provided that relevant prior information is given to the sign restrictions procedure.

Figure A.1: Truncated distributions for a random draw and distributions resulting from Algorithm 1



Note: The top row shows the distributions of responses in the impact period that result from identifying 10,000 sign-restricted models $\mathcal{A}$, for a random draw of the Monte Carlo exercise of Section 3. The bottom row shows that distributions of impact responses that result from Algorithm 1 for the 1,000 draws of the Monte Carlo exercise In this latter case, we only identified 50 sign-restricted models per draw.

B Bias in the estimated simple VAR

Consider a data generating process (DGP) with only one exogenous variable z_t and two endogenous variables contained in vector $y_t = [y_{1t}\ y_{2t}]'$:

$$\begin{aligned} z_t &= Dz_{t-1} + \eta_t, & (14) \\ y_t &= Ay_{t-1} + Bz_{t-1} + \varepsilon_t, & (15) \\ \begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} &= \mathcal{A} \times \begin{pmatrix} \nu_t^* \\ \nu_t \end{pmatrix}, \text{ where } \mathcal{A} = \begin{pmatrix} 1 & 0 & 0 \\ \omega_1 & 1 & 1 \\ \omega_2 & 1 & -1 \end{pmatrix}, & (16) \end{aligned}$$

where D is a constant, A is a 2×2 persistence matrix, $B = [b_1\ b_2]'$ is a 2×1 vector, η_t and $\varepsilon_t = [\varepsilon_{1t}\ \varepsilon_{2t}]'$ are reduced-form innovations. In turn, ν_t^* and $\nu_t = [\nu_{1t}\ \nu_{2t}]'$ are the structural errors of the SVAR model with zero mean and with a variance-covariance matrix given by the identity matrix, i.e., $V(\upsilon_t) = \mathbf{I}_{3\times 3}$, where $\upsilon_t = [\nu_t^*\ \nu_t]'$. Notice that the structural matrix $\mathcal{A}$ implies that, when $\omega_j \neq 0$ for j = 1, 2, the structural exogenous innovation ν_t^* affects each element of ε_t, but neither of the structural shocks in ν_t affects η_t, i.e.,

$$\begin{aligned} \eta_t &= \nu_t^*, \\ \varepsilon_{1t} &= \omega_1 \nu_t^* + \nu_{1t} + \nu_{2t}, \\ \varepsilon_{2t} &= \omega_2 \nu_t^* + \nu_{1t} - \nu_{2t}. \end{aligned}$$

Further, since there is no serial correlation among the structural errors, all of the reduced-form innovations are white noise. The variance-covariance matrix of the reduced-form error vector $\xi_t = [\eta_t\ \varepsilon_t]'$ is given by $\mathrm{E}\{\xi_t \xi_t'\} = \Omega$, where[29]

$$\begin{aligned} \Omega &= \mathcal{A}\mathcal{A}' \\ &= \begin{pmatrix} 1 & \omega_1 & \omega_2 \\ \omega_1 & \omega_1^2 + 2 & \omega_1\omega_2 \\ \omega_2 & \omega_1\omega_2 & \omega_2^2 + 2 \end{pmatrix}. \end{aligned}$$

Notice that the variance of the structural impulse responses, *i.e.* $\mathrm{Var}\left(\partial y_{t+h} / \partial \upsilon_t'\right)$, is proportional to the variance of the structural errors, which is defined as

$$\mathrm{Var}(\upsilon_t) = \mathcal{A}^{-1} \Omega \left(\mathcal{A}'\right)^{-1} = I_2.$$

[29]This simple structure implies that the second moments of the reduced-form innovations are given by $\mathrm{E}\{\eta_t^2\} = 1$, $\mathrm{E}\{\eta_t \varepsilon_{jt}\} = \omega_j$, $\mathrm{E}\{\varepsilon_{jt}^2\} = \omega_j^2 + 2$, and $\mathrm{E}\{\varepsilon_{1t}\varepsilon_{2t}\} = \omega_1\omega_2$.

An econometrician can find unbiased estimates for all the parameters (slopes and variances) of the true DGP *if and only if* she estimates a VAR with exogenous variables with exactly the same structure and the same variables included in system $(14)-(15)$. However, assume for the time being that the econometrician aims to estimate a simple VAR of the form

$$y_t = A^s y_{t-1} + \varepsilon_t^s. \tag{17}$$

The econometrician ignores, thus, the block-exogenous structural shock ν_t^*, and proposes the following identifying structure

$$\varepsilon_t^s = \mathcal{A}^s \nu_t^s, \text{ with } \mathcal{A}^s = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \text{ and } \nu_t^s = \begin{pmatrix} \nu_{1t}^s \\ \nu_{2t}^s \end{pmatrix}. \tag{18}$$

Given the true DGP, the persistence matrix A^s maps into A, and the error term $\varepsilon_t^s = Bz_{t-1} + \varepsilon_t$, in particular $\varepsilon_{jt}^s = b_j z_{t-1} + \varepsilon_{jt}$ for $j \in \{1, 2\}$. The latter implies that the variance-covariance matrix of ε_t is given by:[30]

$$\begin{aligned} \mathrm{E}\left\{\varepsilon_t^s \varepsilon_t^{s\prime}\right\} &= \Omega^s \\ &= \begin{pmatrix} \frac{b_1^2}{1-d^2} + \omega_1^2 + 2 & \frac{b_1 b_2}{1-d^2} + \omega_1\omega_2 \\ \frac{b_1 b_2}{1-d^2} + \omega_1\omega_2 & \frac{b_2^2}{1-d^2} + \omega_2^2 + 2 \end{pmatrix}. \end{aligned}$$

Notice that the identifying matrix $\mathcal{A}^s$ has the same signs as the bottom-right block of matrix $\mathcal{A}$, and so the structural impulse responses computed from the VAR with exogenous variables (i.e. $\partial y_{t+h}/\partial \nu_t'$) and those computed from simple VAR (i.e., $\partial y_{t+h}/\partial \nu_t^{s\prime}$) will have the same signs, at least on impact. However, the variance of these responses may differ importantly due to the mis-specification of the simple VAR. To see this, notice that the identification scheme (18) implies that the variance of the structural shocks ν_t^s is given by

$$\begin{aligned} \mathrm{Var}\left(\nu_t^s\right) &= \left(\mathcal{A}^s\right)^{-1} \Omega^s \left(\mathcal{A}^{s\prime}\right)^{-1}, \text{ which yields} \\ &= \frac{1}{4}\begin{pmatrix} \mathrm{Var}\left(\nu_{1t}^s\right) & \frac{b_1^2 - b_2^2}{1-d^2} + \omega_1^2 - \omega_2^2 \\ \frac{b_1^2 - b_2^2}{1-d^2} + \omega_1^2 - \omega_2^2 & \mathrm{Var}\left(\nu_{2t}^s\right) \end{pmatrix}, \end{aligned}$$

where

$$\mathrm{Var}\left(\nu_{1t}^s\right) = \sum_{j=\{1,2\}} \left(\frac{b_j^2}{1-d^2} + \omega_j^2 + 2\right) + 2\left(\frac{b_1 b_2}{1-d^2} + \omega_1\omega_2\right), \text{ and} \tag{19}$$

$$\mathrm{Var}\left(\nu_{2t}^s\right) = \sum_{j=\{1,2\}} \left(\frac{b_j^2}{1-d^2} + \omega_j^2 + 2\right) - 2\left(\frac{b_1 b_2}{1-d^2} + \omega_1\omega_2\right). \tag{20}$$

Three remarks are in order.

[30]Notice that $\mathrm{Var}\left(z_t\right) = \frac{1}{1-d^2}$.

Remark 1. *The simple VAR will deliver unbiased estimates of the persistence matrix A if $\tilde{\varepsilon}_t$ is a white noise process.* In our simple example, this will be the case whenever z_t is also white noise, i.e. when $d \to 0$. It is straightforward to show that, when $|d| > 0$, then

$$\phi_t = d\,\phi_{t-1} + \eta_t^s$$

where $\phi_t = \varepsilon_{jt}^s - \varepsilon_{jt}$ and $\eta_t^s = b_j \eta_{t-1}$. In this case, the regressors and the error term of the simple VAR will not be orthogonal with the endogenous variables ($\mathrm{E}\left\{\varepsilon_t^{s\prime} y_{t-1}\right\} \neq 0$), and, thus, the estimator $\hat{A}^s$ will not converge to its true value A.

Remark 2. *Even in the case where ε_t^s is a white noise, its second moments are different from those of ε_t.* In fact, by comparing the bottom-right block of matrix Ω with the elements of Ω^s it is evident that there are biases due to the implicit presence of the exogenous variable z_t in ε_t^s. In particular, depending on the values taken by b_1 and b_2, the covariances between the reduced-form innovations in Ω^s can change their sign with respect to their Ω counterparts. This is important because the bias in Ω^s can cause negative outcomes for an identification strategy based on **exclusion restrictions**. This approach uses the Cholesky lower-triangular decomposition of Ω^s to produce impulse responses. Let $\mathfrak{A}^s = \mathrm{chol}\left(\Omega_\varepsilon^s\right)$ denote such a decomposition. Since Ω^s and Ω are different, there is no guarantee that the impulse responses computed through $\mathfrak{A}^s$ will be equal to those generated through $\mathfrak{A} = \mathrm{chol}\left(\Omega\right)$. Therefore, impulse responses of endogenous variables generated through $\mathfrak{A}^s$ can be completely uninformative.

In our estimations with Mexican data, some elements of Ω^s differ from those of Ω by a factor of 5 to 1, while we only observe one change of sign in the covariance of domestic output and inflation. These distortions explain the stark difference in the Cholesky-based impulses responses.

In contrast, **sign restrictions** reduce the distortions generated by the misspecified simple VAR by modifying matrix $\mathfrak{A}^s$. As explained in Section 4, this methodology proposes to identify a matrix of the form $\mathcal{A}_k^s = \mathfrak{A}^s(\mathcal{Q}_k^s)'$ where $\mathcal{Q}_k^s$ is a random rotation matrix drawn in the k-th iteration, so that $(\mathcal{Q}_k^s)'\mathcal{Q}_k^s$ equals the identity and $\mathcal{A}_k^s$ delivers the imposed signs in the impulse responses. In the same vein, applying this methodology to a VAR with exogenous variables delivers an identifying matrix $\mathcal{A}_k = \mathfrak{A}(\mathcal{Q}_k)'$ so that the signs of $\mathcal{A}_k$ and $\mathcal{A}_k^s$ are the same. This correcting property of sign restrictions will be more successful when either the parameter values b_j are small, or the correlation between η_t and ε_t, governed by the parameters ω_j, is also small. In our estimations with Mexican data, the largest correlation found between η_t and ε_t is about -.30, and it relates the CRB inflation with the real exchange rate.

Remark 3. *If the correlation between the η_t and ε_t is small, then the confidence bands of the structural impulse responses across VARs will be similar*. In the case of sign restrictions, the variance of the structural impulse responses in the simple VAR can be larger than in the VAR with exogenous variables. This point is proven by equations (19) and (20), that state the variance of the structural errors ν_{jt}^{s} in the simple SVAR. These variances depend on the parameters ω_j, while in the true DGP the variance of ν_{jt} is equal to 1 by construction. In case ω_j and b_j are small, we should observe that the impulse responses computed through sign restrictions in the two VARs have similar uncertainty bands.

To see this point clearly, we use the system (14)-(16) to perform a Monte Carlo simulation, in which we set all of the parameter values so that the sign and zero restrictions indicated in (16) are satisfied and the cross correlations between exogenous and endogenous reduced-form errors satisfy the following values:[31] $\rho\left(\eta_t, \varepsilon_{1t}\right) = -.2$, $\rho\left(\eta_t, \varepsilon_{2t}\right) = -.2$, and $\rho\left(\varepsilon_{1t}, \varepsilon_{2t}\right) = .9$, where the function $\rho\left(\cdot\right)$ denotes the correlation between any two variables. The experiment consists in comparing the structural impulse responses of variable y_{1t} to a shock in the structural error ν_{2t} and ν_{2t}^{s}. We simulate 1,000 samples, each one with 153 observations (as in our sample set, although we used 200 periods as a burning sample) and estimate both a VAR with exogenous variables with the same structure as in $(19)-(20)$, and a simple VAR as in (17). We then apply the sign restrictions methodology to the two estimations. The results of this experiment are depicted in Figure B.2, where the first row shows the results for the VAR with exogenous variables and the second row for the simple VAR, where two cases are presented. On the left, we present the case where η_t and ε_t are correlated. On the right, we present the case where they are not correlated. The plain line is the true impulse response elicited from the DGP of our example, while the dashed lines denote the median and the 68-th percent model-uncertainty band for the SVARs impulse responses.

Notably, we observe that the median responses of the two SVARs are quite similar and do

[31] To calibrate our example, we ran a VAR with exogenous variables using the observations of the U.S. output gap as the exogenous variable, and the Mexican output and inflation gaps as endogenous variables. We then proposed a matrix $\mathcal{A}$ so that $\mathcal{A}\mathcal{A}' = \hat{\Omega}$, and sign and zero conditions imposed by equation (16) are fulfilled. However, the signs for ω_j were set free. The resulting values are the following: $d = 0$, $b_1 = .13$, $b_2 = 0$, $A = \begin{pmatrix} .8 & -.4 \\ .01 & .28 \end{pmatrix}$, and $\Omega = \begin{pmatrix} 4.55 & .53 & -.04 \\ .53 & .77 & -.02 \\ -.04 & -.02 & .04 \end{pmatrix} \times 10^{-3}$.

a very good job in recovering the true responses of the DGP. Second, the uncertainty band for the impulse responses of the SVAR with exogenous variables is quite tight, and it is actually the same regardless of what the value of the correlation between η_t and ε_t is. Third, notice that for the case in which $\rho(\eta_t, \varepsilon_{jt}) \neq 0$, the uncertainty band of the impulse responses for the simple SVAR is quite large. In contrast, and as predicted by our example above, when we set this correlation to zero, the uncertainty band of the simple SVAR decreases substantially and converges towards the one of the SVAR with exogenous variables.

Summing up. Exclusion restrictions use the estimated variance-covariance matrix $\hat{\Omega}$ and the ordering of variables as the crucial information to identify a structural shock. However, when matrix $\hat{\Omega}$ is biased due to a mis-specification or the omission of relevant exogenous variables in the VAR, then the impulse responses gathered from the distorted matrix $\hat{\Omega}$ might be totally uninformative. A biased $\hat{\Omega}$ is obtained in a simple VAR when the reduced-form innovations are affected by the block-exogenous innovations. In contrast, the VAR with exogenous variables computes an error variance-covariance matrix $\hat{\Omega}$ that is unbiased. But to obtain it, one must choose the appropriate set of block-exogenous variables that appear in the true DGP of the system, which by definition is not observable.

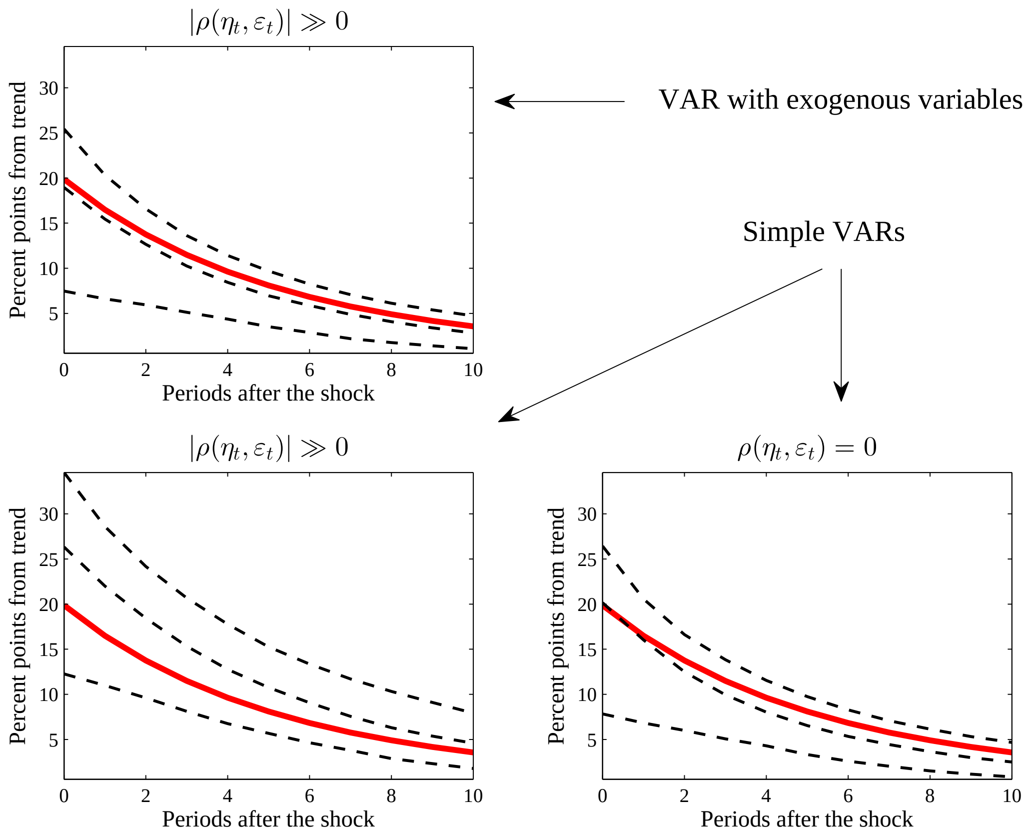
In contrast, sign restrictions use, in addition to matrix $\hat{\Omega}$, a set of auxiliary orthonormal matrices $\mathcal{Q}$ that help achieving impulse responses coherent with the predictions of a reasonable model of the DGP. Further, when the simple VAR features a biased matrix $\hat{\Omega}$, the instrumental matrices $\mathcal{Q}$ can help to correct this bias. The correction is more likely to be successful when the contemporaneous correlation between foreign and domestic innovations is small. Further, our Monte Carlo exercise shows that sign restrictions identify quite well the true impulse response of the DGP as long as all endogenous variables are allowed to respond to the structural shock.[32]

C Details about the Mexican data used

All home variables were obtained from either *INEGI* (the Mexican National Bureau of Statistics) or *Banco de México*, at a monthly frequency. To obtain the output gap, first we applied

[32]In our Monte Carlo exercise, exclusion restrictions fail by default because it is assumed that the identifying matrix $\mathcal{A}$ in the DGP is not lower triangular. As a consequence, exclusion restrictions would impose a zero restriction in the response of y_{1t} to a shock in v_{2t} when there is actually none.

Figure B.2: Bias in the simple VAR, a Monte Carlo simulation



Note: The top graph represents the IRF in the real world in the Monte Carlo exercise. The graph on the left panel below corresponds to the IRF if the correlation between exogenous and endogenous variables is big, while the graph on the right panel below is the IRF in case this correlation is zero or near zero.

the methodology provided by Elizondo (2012) to estimate a monthly series for the logarithm of GDP using as input INEGI's IGAE index,[33] which measures activity in Mexico through surveys. Elizondo (2012) interpolates the monthly GDP series through an algorithm based on the Kalman filter, and shows that the estimated series fits the observed quarterly GDP quite well.[34] To finally obtain the output gap, we subtract from the logarithm of monthly per-capita GDP its long-term trend as computed by an HP filter, adjusted to the monthly frequency.

Consumer and producer prices, money growth, and the output gap were seasonally adjusted. The quarterly GDP and monthly IGAE series were originally adjusted by INEGI. For consumer inflation and money growth we used the Tramo method, while for producer price inflation, a more volatile variable, we used Additive Census X12. Our results are not sensitive to the method employed to seasonally adjust the above series. Finally, the nominal interest rate and the real exchange rate were not seasonally adjusted. Figure 6 shows the detrended variables used in the empirical application. For inflation and producer price inflation, we used the month-by-month percentage change of the Core Consumer Price Index (*inflación subyacente*) and the National Index for Producer Prices excluding oil, respectively. We use the core consumer price inflation because the headline inflation measure is more volatile and subject to temporary shocks, like bad weather conditions on agriculture, that are beyond the control of the Central Bank. Notice that we have detrended observed inflation by a measure of trend inflation. The difference between these two variables denotes the inflation gap, as defined by Cogley *et al.* (2010). We opted for a simple strategy to compute trend inflation, denoted by $\bar{\pi}_t$, as we extracted the low frequency component of inflation through an HP filter. We use a λ equals to $129,600$, as proposed by Ravn and Uhlig (2001).[35]

For the real exchange rate depreciation, we used the exchange rate parity with respect to Mexico's 21 main trading partners. This exchange rate is computed as: $q = (p^*/p) \times (e/e^*)$; e is the MXN/USD nominal exchange rate , e^* is the exchange rate of world currency per U.S. dollar, p is the consumer prices index in Mexico, p^* is weighted average of consumer prices of 21 countries, and q is the real exchange rate index.[36]

[33]The IGAE acronym stands for *Índice Global de Actividad Económica*.

[34]In a robustness exercise, we use a measure the IGAE index and the Industrial Production index as alternative measures of activity. All the results were qualitatively unaffected.

[35]As an alternative, we used the Stock and Watson (2007) unobserved-variable method to extract trend inflation. The results do no change much using the two methodologies, so we chose the simplest one for ease of exposition.

[36]Source: International Financial Statistics, IMF and Banco de México.

The short-run nominal interest rate is measured with the interest rate for Treasuries Certificates (*Cetes*) for 28 days. This rate is in annual terms. In our analysis, we transform this interest rate in monthly terms. As money growth we use M2 as the monetary aggregate.

For consistency with the long-term equilibrium of the economy, we assume that the low frequency component of all nominal variables (i.e., π_t, π_t^p, i_t, $\Delta m2_t$) is proportional to trend inflation, and so we use this variable to detrend the rest of nominal quantities. We can see this fact in a standard New Keynesian model with target inflation, as in Ireland (2007), Cogley *et al.* (2010) or Christiano *et al.* (2014), it is the case that at the steady state producer price inflation (i.e., the one of intermediate goods) equals consumer price inflation (i.e., the one of final goods), which in turn must equal target inflation. In addition, the nominal interest rate should be equal to the real interest rate plus target inflation. And, finally, in a model with money-in-the-utility function, as in Ireland (2007), inflation should be equal to money growth in excess, that is, after discounted for economic growth. Figure 6, shows that trend inflation does a pretty good job in detrending the nominal variables.

U.S. variables are obtained from the FREDII database, while the commodity price index is obtained from Bloomberg. For U.S. output we used an HP-filtered measure of the Industrial Production Index; for U.S. inflation and the commodity price inflation we compute the month-by-month percent change in the Personal Consumption Expenditure index, and the CRB commodity price index, respectively. Notice that we do not deflate U.S. inflation using a measure for trend inflation as the latter is practically invariant for the time period considered (see Figure 6). As Ireland (2007) and Cogley *et al.* (2010) show, U.S. trend inflation peaked in 1979, while it stabilized around 2 percent annually after 1995. For this reason, we also chose not to deflate commodity-price inflation because of a similar reason.

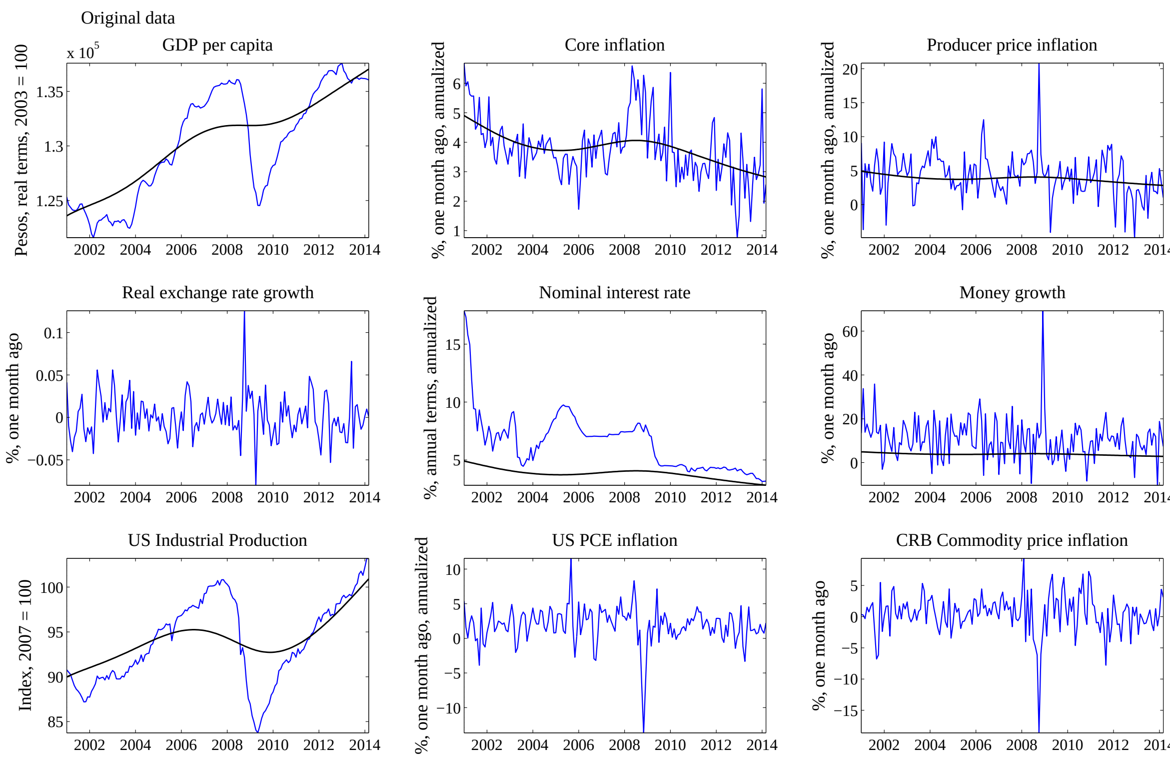
We verified that our VARs are stationary, as all roots lie within the unit circle (see Figure C.4). In case roots are inside the unit-circle, then the VAR model is stationary. In contrast, if at least one of the VAR roots is equal or larger than one, then the model is not stationary. In such a case, certain results (such as the impulse-response standard errors) will not be valid. In particular, the AR Roots test reports the inverse roots of the characteristic AR polynomial (see Luetkepohl and Saikkonen, 2000), which in the case of the simple VAR has $n \times p$ roots, and for the VAR with exogenous variables we have $(n + m) \times p$.[37]

Finally, we perform the Ljung-Box Q test with 3 lags on each series of residuals of the

[37] Additionally, we have made two joint unit-root tests (Levin, Lin & Chu and Breitung) in which it is assumed that there is a common unit root. The tests were rejected, indicating that the variables included in our VARs are stationary.

VAR equations, with the null hypothesis of no autocorrelation. The only equation for which the null is rejected is the one for the output gap. To do not reject the null, we need to include 12 lags in the VAR, which would mean to over-estimate several parameters. However, we have a relatively small sample (153 observations) for the number of parameters to be estimated in such case (around 500). In addition, with this number of lags, the monetary policy shock is not identified. We have to make a trade-off between identifying the shock or do not rejecting the null. Further, we consider the LR test to choose the optimal lag in our VARs models (see Canova, 2007; Chapter 4).

Figure C.3: Original data

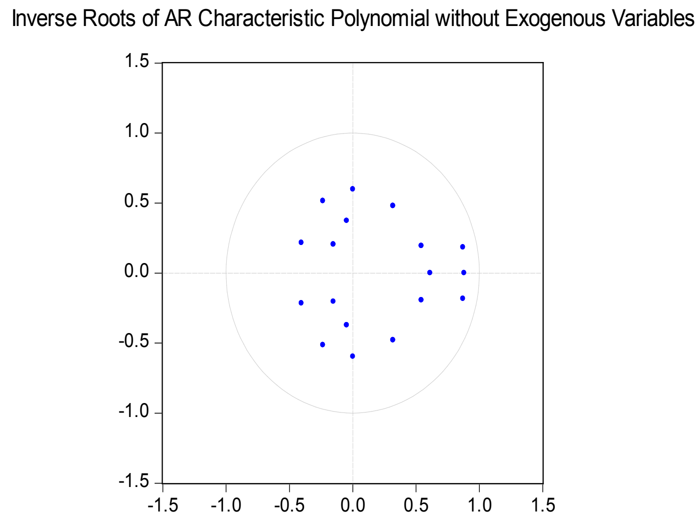


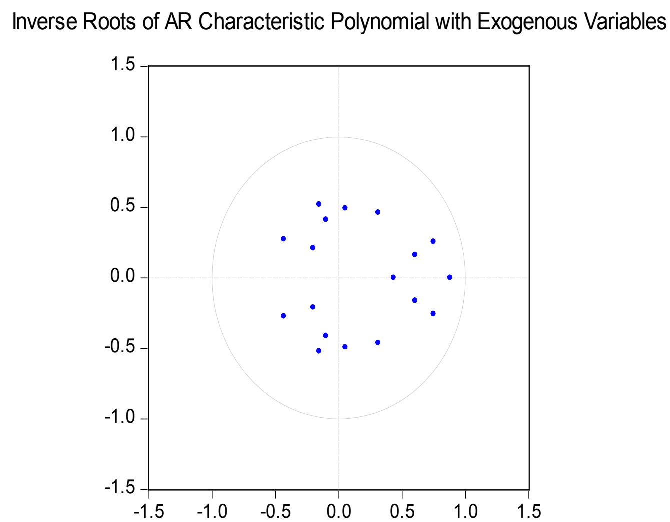
Note: All domestic variables were extracted from INEGI and Banco de México, except for GDP, that was computed as in Elizondo (2014). For U.S. variables, we used the FRED2 database, while we gathered the CRB index from Bloomberg. For some variables we extracted a low frequency component. For the output measures, we used the HP filter for each variable. For domestic core inflation, producer price index inflation, the nominal interest rate, and money growth, we used the trend of core inflation as computed by the HP filter. In each graph, the dotted line marks the beginning of the benchmark sample period, namely 2001M1-2013M9.

D Bayesian VARs vs bootstrapping VARs with Mexican data

This section briefly shows that estimating the VAR either by bayesian methods or bootstrapping yields very similar results. The exercise is performed only for the VAR without ex-

Figure C.4: Inverse roots of AR characteristic polynomial

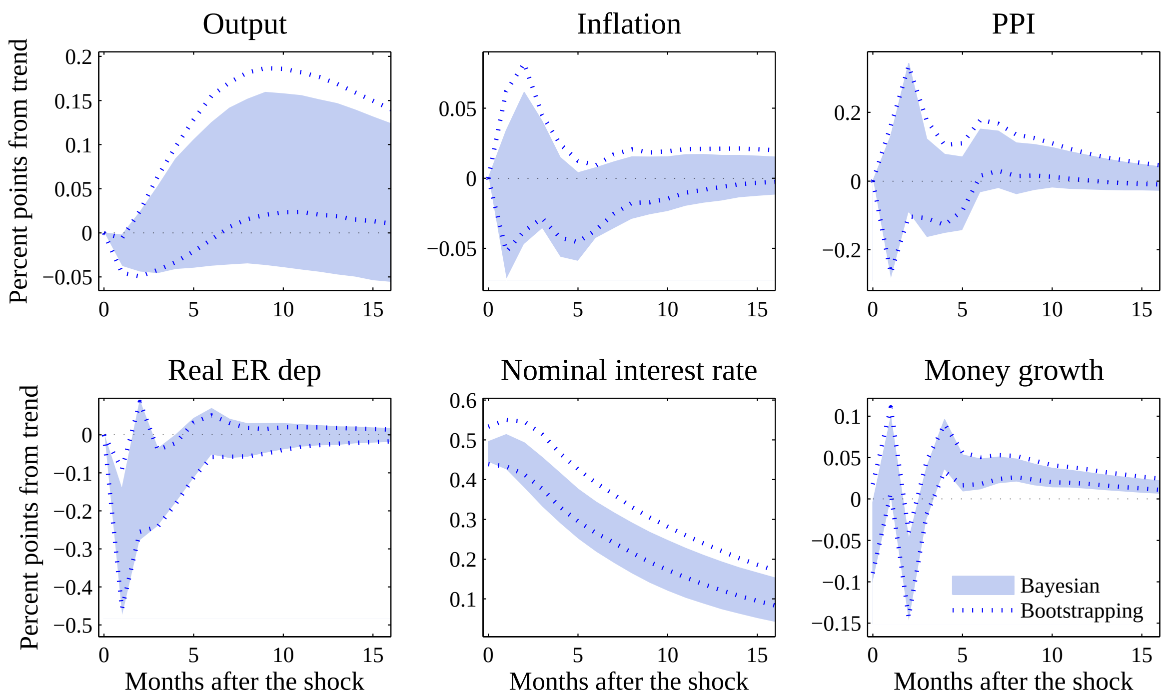




Note: In case roots are inside the unit-circle, then the VAR model is stationary. In contrast, if at least one of the VAR roots is equal or larger than one, then the model is not stationary. In such a case, certain results (such as the impulse-response standard errors) will not be valid. In particular, our VARs are stationary, as all roots lie within the unit circle.

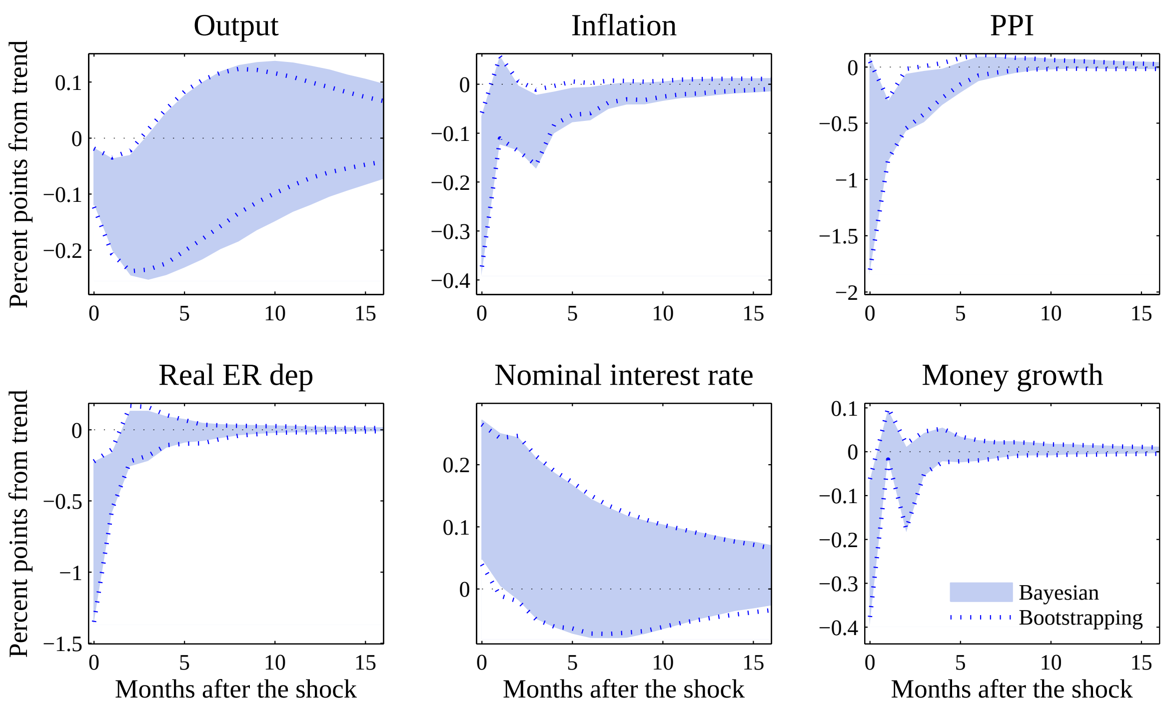
ogenous variables. For the bayesian estimation, we used Gibbs sampling to find the joint distributions of parameters and we consider a conventional inverse Wishart-Gaussian prior on the reduced-form parameters and uniform prior on the rotation matrices to obtain the posterior distribution of the impulse-response functions once the structural shock identification strategy was applied. For the bootstrapping estimations, we followed the classic methodology of Runkle (1987). Figures D.5 and D.6 present the comparison between estimating methods for the exclusion restrictions and standard sign restrictions, respectively.

Figure D.5: Exclusion-identified SVARs with Mexican data: bayesian vs bootstrapping estimations



Note: The dashed lines show the 68-th uncertainty bands for the impulse responses of the VAR without foreign variables estimated through bootstrapping, while the blue light areas display the same VAR estimated with bayesian techniques using a conventional inverse Wishart-Gaussian prior on the reduce-form parameters.

Figure D.6: Sign-identified SVARs with Mexican data: bayesian vs bootstrapping estimations



Note: The dashed lines show the 68-th uncertainty bands for the impulse responses of the VAR without foreign variables estimated through bootstrapping, while the blue light areas display the same VAR estimated with bayesian techniques using a conventional inverse Wishart-Gaussian prior on the reduce-form parameters.