

# Quantification of systemic risk from overlapping portfolios in the financial system

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[www.complex-systems.meduniwien.ac.at](http://www.complex-systems.meduniwien.ac.at)

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with Sebastian Poledna  
and Serafín Martínez-Jamarillo

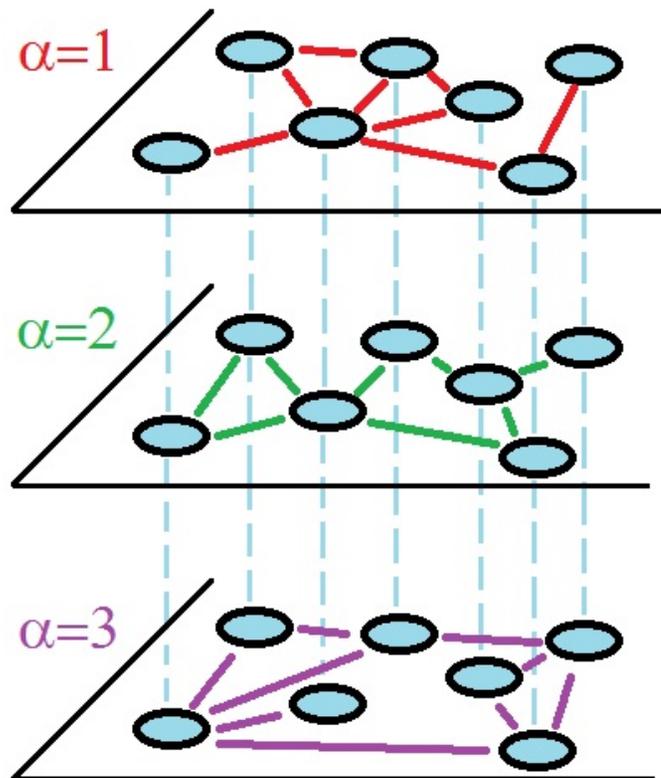
# Systemic risk

- risk that significant fraction of financial network defaults
  - systemic risk **is not the same** as default risk
  - systemic risk **is not the same** as economic risk
  - banks care about credit-default risk
  - banks have no means to manage systemic risk
- role of regulator: **manage systemic risk**
- incentivise banks to think of SR

# Two origins of systemic risk

- **synchronisation of behaviour:** herding, fire sales, margin calls, various amplification effects – may involve networks
- **networks of contracts:** this is what the financial system is

# Systemic risk is created on multiplex networks



layer 1: lending–borrowing loans

layer 2: derivative networks

layer 3: collateral networks

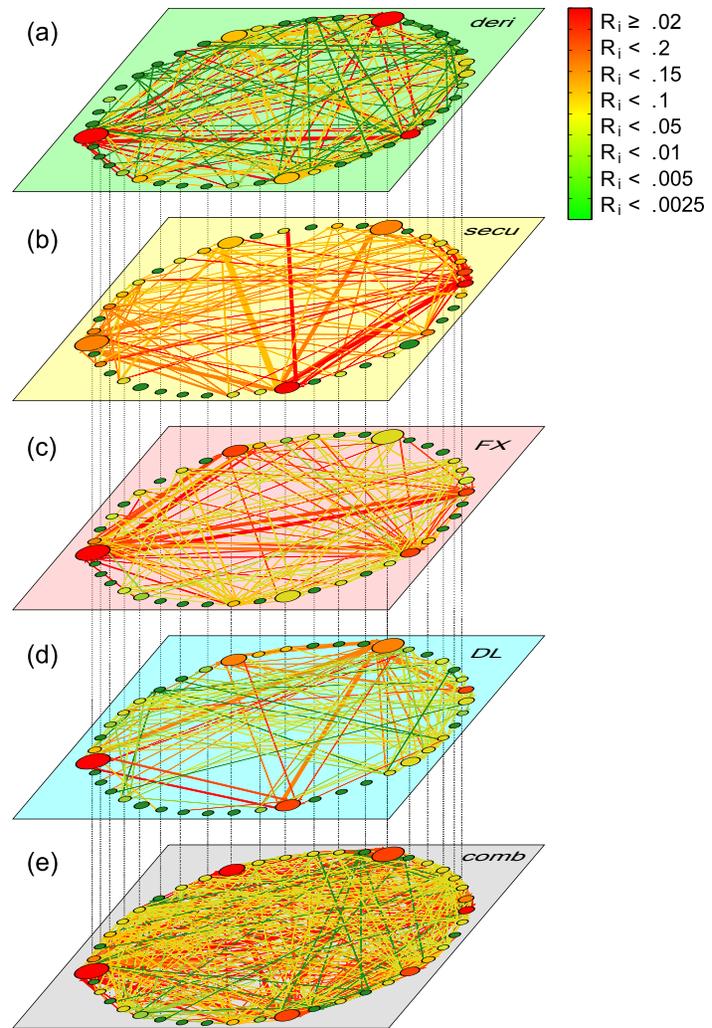
layer 4: securities networks

layer 5: cross-holdings

layer 6: overlapping portfolios

layer 7: liquidity: over-night loans

layer 8: FX transactions

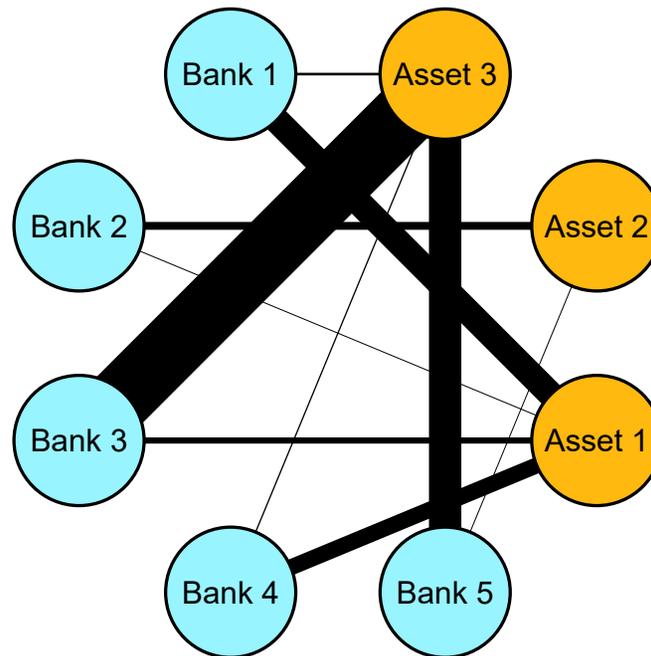


# Different exposure types

- interbank lending: deposits and loans
- security cross-holdings: bank  $i$  holds securities of bank  $j$
- derivatives
- foreign exchange (settlement risk)
- overlapping portfolios → indirect exposure

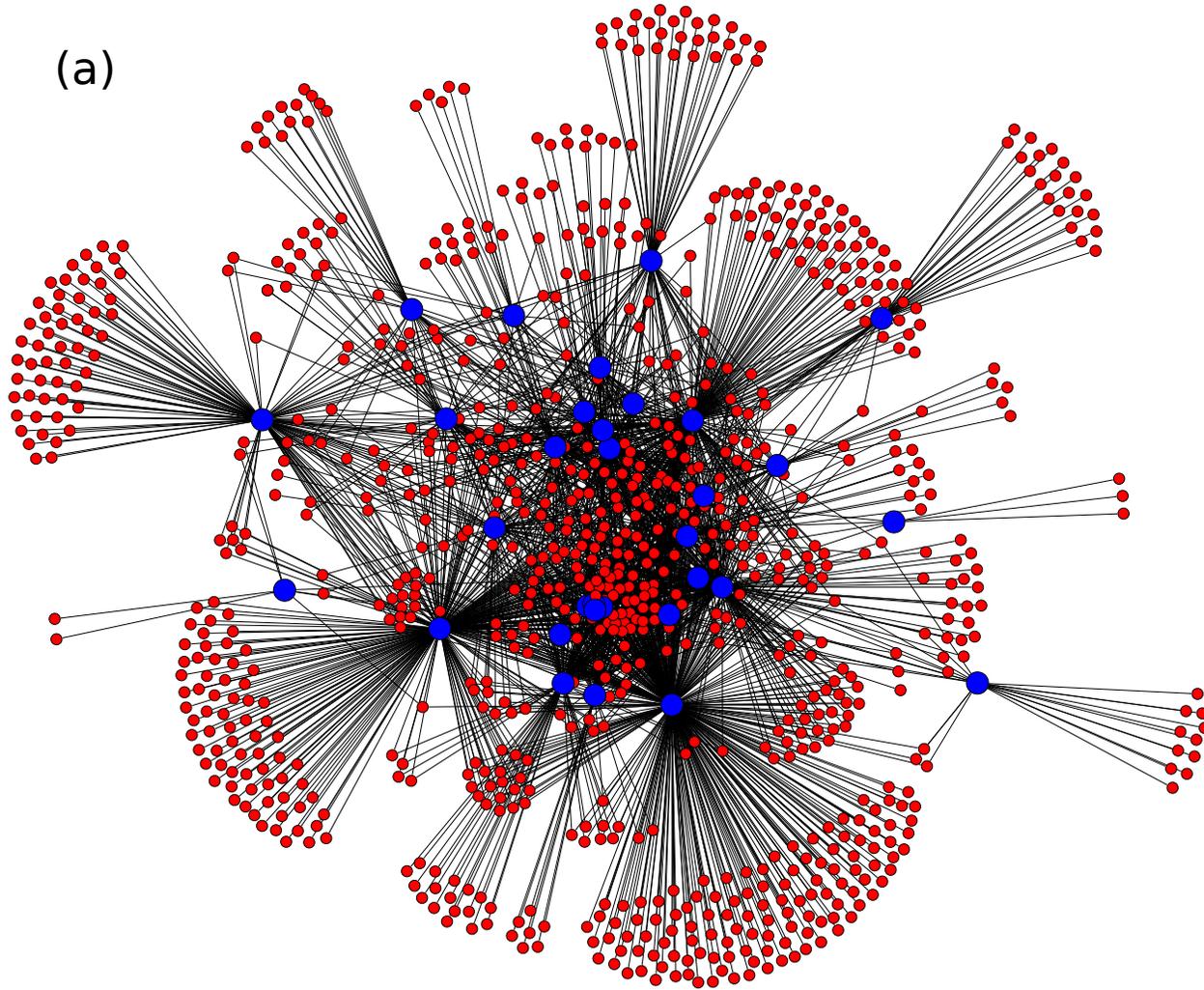
for Mexican data: exposures are known on daily level

# Exposure from overlapping portfolios



# Overlapping portfolios in Mexican banks

(a)



# Market depth and linear price impact

- market depth  $D_k = c \frac{\langle \text{vol}_k \rangle_{\text{day}}}{\sigma_k}$
- total portfolio value of bank  $i$ ,  $S_i = \sum_k S_{ki} p_k$

If bank  $i$  sells  $S_{ki}$  of asset  $k$ , price is depressed by  $\frac{S_{ki}}{D_k}$

If bank  $j$  owns  $S_{kj}$  of asset  $k \rightarrow$  face loss of  $S_{kj} \frac{S_{ki}}{D_k}$

$$\rightarrow X_{ij}^{\text{OP}} = \sum_{k=1}^K S_{kj} S_{ki} \frac{1}{D_k}$$

# Quantification of SR

# Systemic risk – quantification

**Wanted:** systemic risk-value for every financial institution

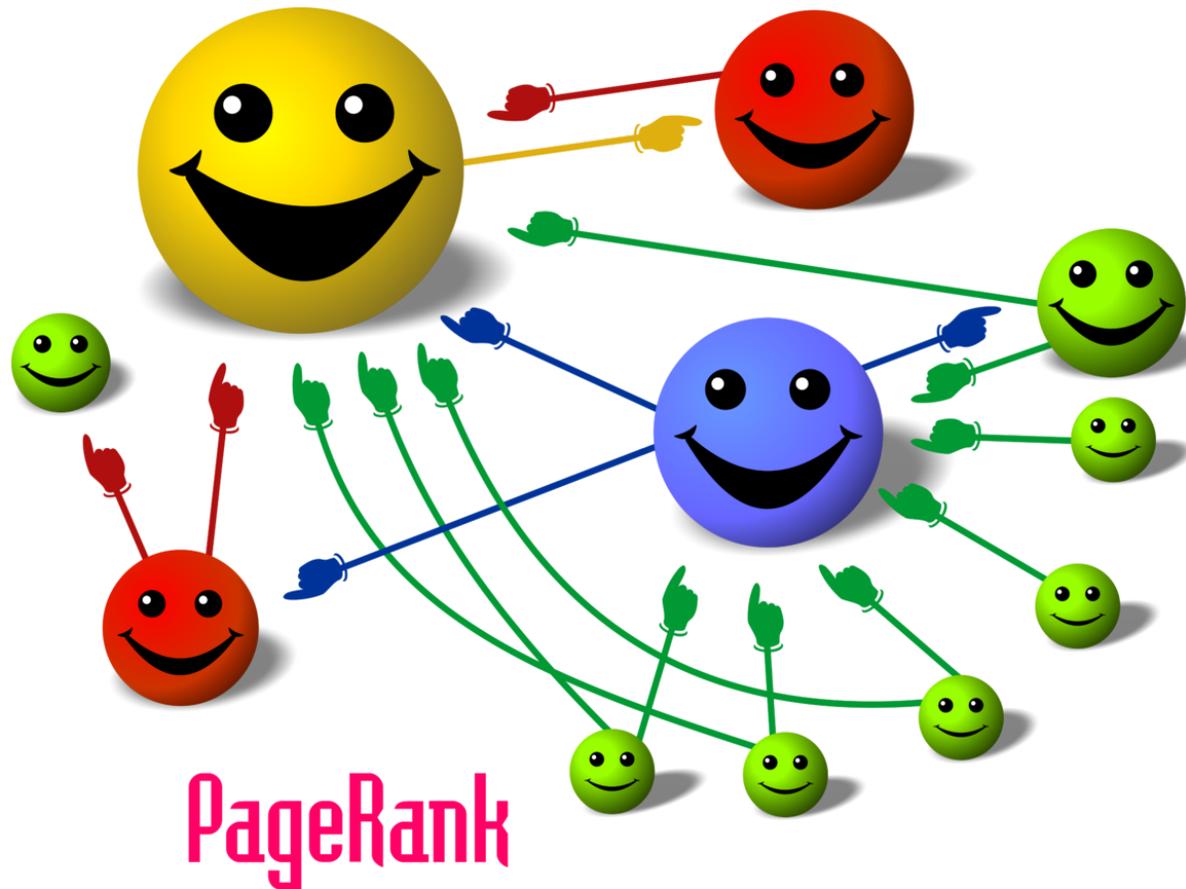
given: entire network

Google has similar problem: value for importance of web-pages

→ page is important if many important pages point to it

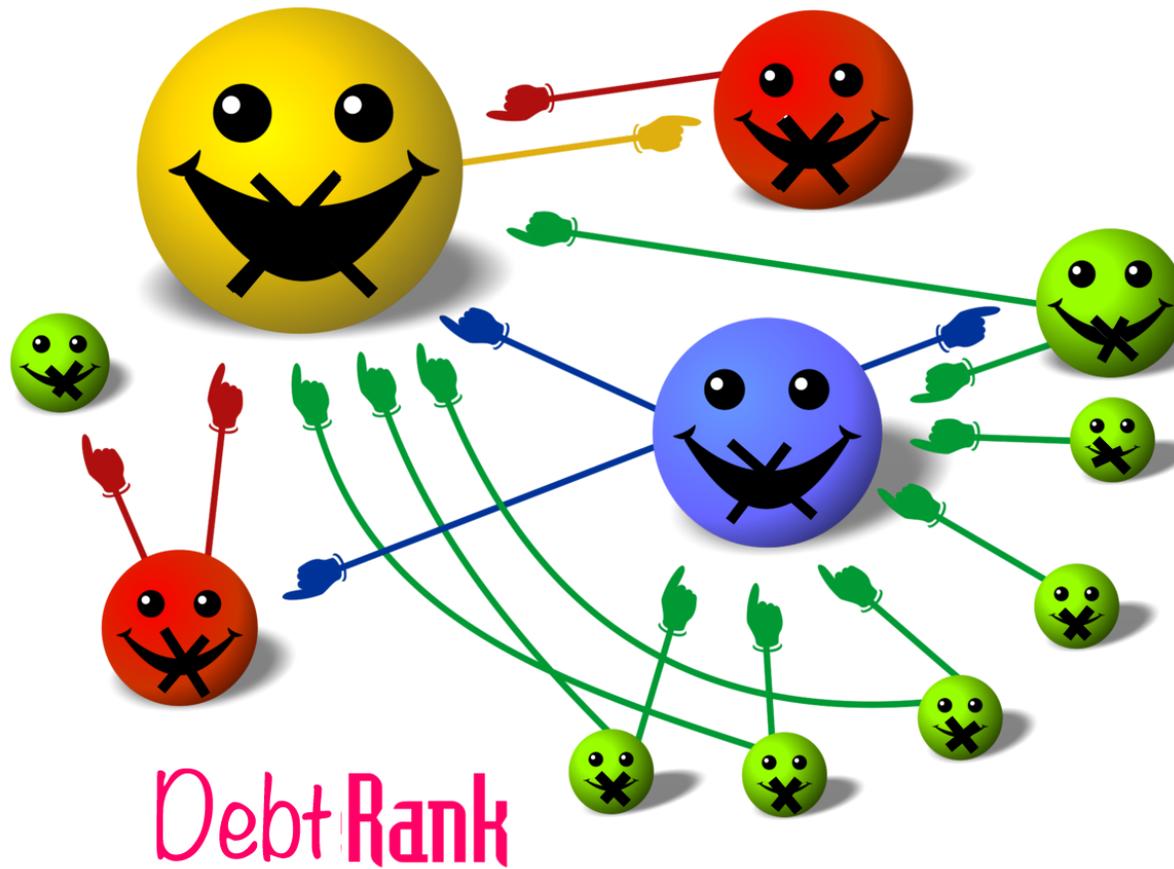
→ number for importance → PageRank

page is **important** if many **important** pages point to it



source Wikipedia cc-license

institution **system.** risky if **system.** risky institutions lend to it



# Systemic risk factor – DebtRank $R$

... is a “different Google” – adapted to context of systemic risk  
(S. Battiston et al. 2012)

**superior to:** eigenvector centrality, page-rank, Katz rank ...

## Why?

- **economic value** in network that is affected by node's default
- capitalization/leverage of banks taken into account
- cycles taken into account: no multiple defaults

# DebtRank

- recursive method
- corrects Katz rank for loops in the exposure network
- if  $i$  defaults and can not repay loans,  $j$  loses  $L_{ij}$ . If  $j$  has not enough capital to cover that loss  $\rightarrow j$  defaults
- impact of bank  $i$  on neighbors  $I_i = \sum_j W_{ij} v_j$   
with  $W_{ij} = \min \left[ 1, \frac{L_{ij}}{C_j} \right]$ , outstanding loans  $L_i = \sum_j L_{ji}$ , and  $v_i = L_i / \sum_j L_j$
- impact on nodes at distance two and higher  $\rightarrow$  recursive

$$I_i = \sum_j W_{ij} v_j + \beta \sum_j W_{ij} I_j,$$

If the network  $W_{ij}$  contains cycles the impact can exceed one  
→ DebtRank (S. Battiston et al. (2012))

- nodes have two state variables,  $h_i(t) \in [0, 1]$  and  $s_i(t) \in \{Undistress, Distress, Inactive\}$

- Dynamics:  $h_i(t) = \min \left[ 1, h_i(t-1) + \sum_{j|s_j(t-1)=D} W_{ji} h_j(t-1) \right]$

$$s_i(t) = \begin{cases} D & \text{if } h_i(t) > 0; s_i(t-1) \neq I \\ I & \text{if } s_i(t-1) = D \\ s_i(t-1) & \text{otherwise} \end{cases}$$

- DebtRank of set  $S_f$  (set of nodes in distress), is

$$R_S = \sum_j h_j(t)v_j - \sum_j h_j(1)v_j$$

Measures distress in the system, excluding initial distress. If  $S_f$  is a single node, DebtRank measures its systemic impact on the network.

- DebtRank of  $S_f$  containing only the single node  $i$  is

$$R_i = \sum_j h_j(t)v_j - h_i(1)v_i$$

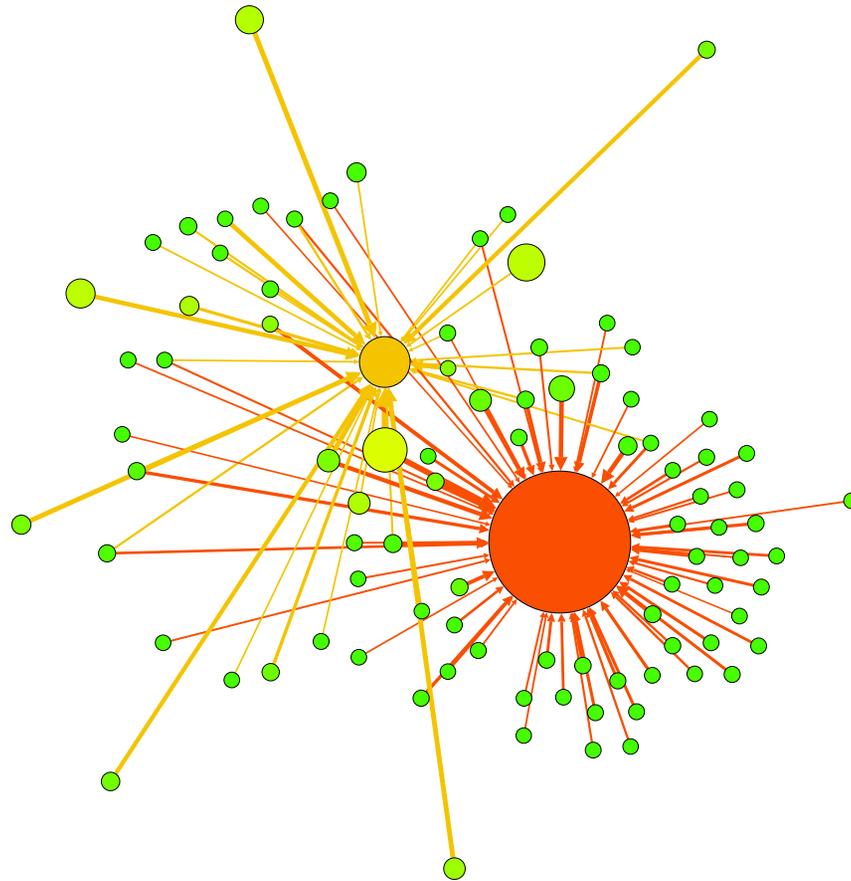
# Systemic risk of nodes

**Input:** Network of contracts between banks

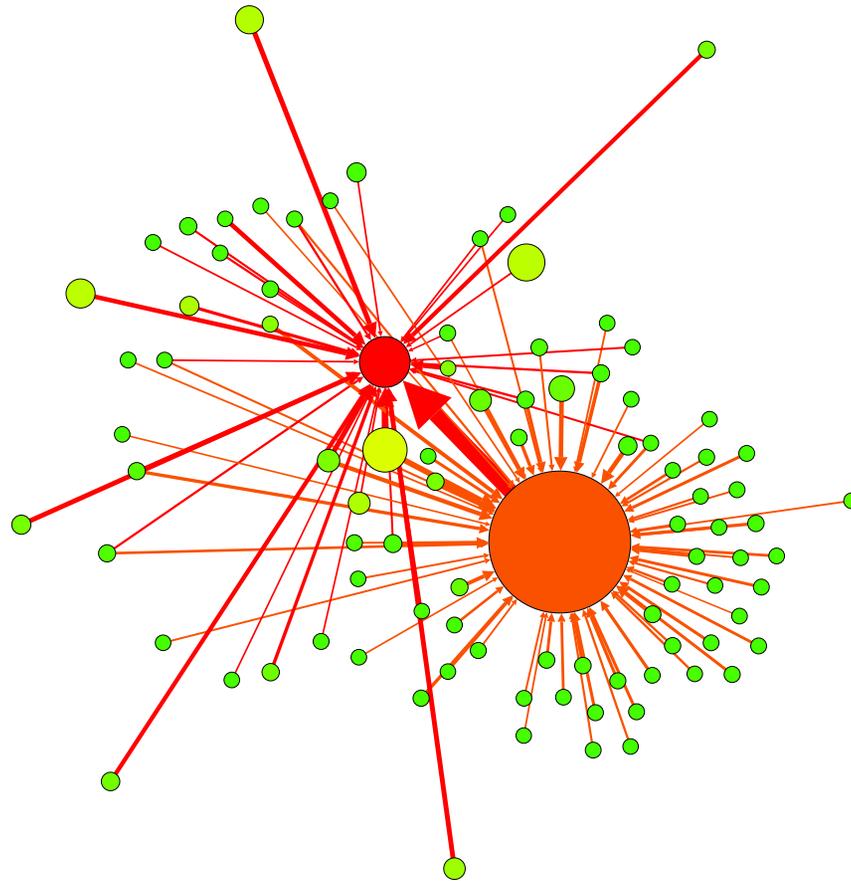
Compute = DebtRank; think of a complicated first eigenvector

**Output:** all banks  $i$  get damage value  $R_i$  (% of total damage)

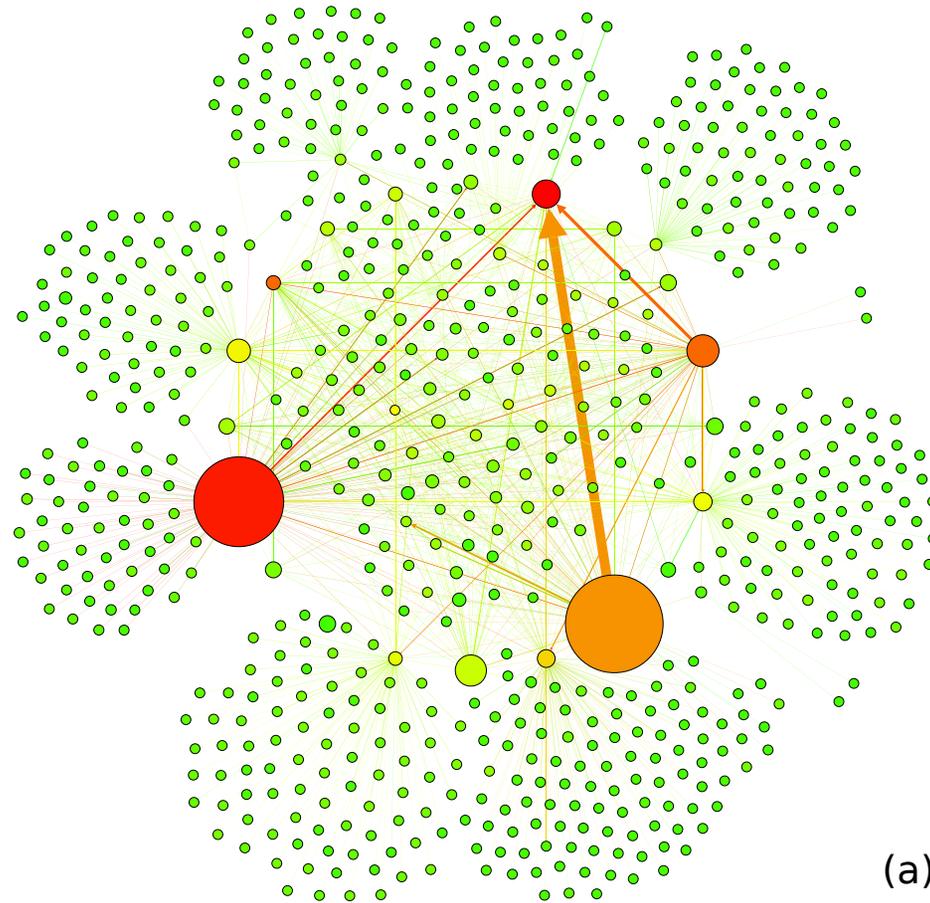
# Systemic risk spreads by borrowing



# Systemic risk spreads by borrowing



# DebtRank Austria Sept 2009

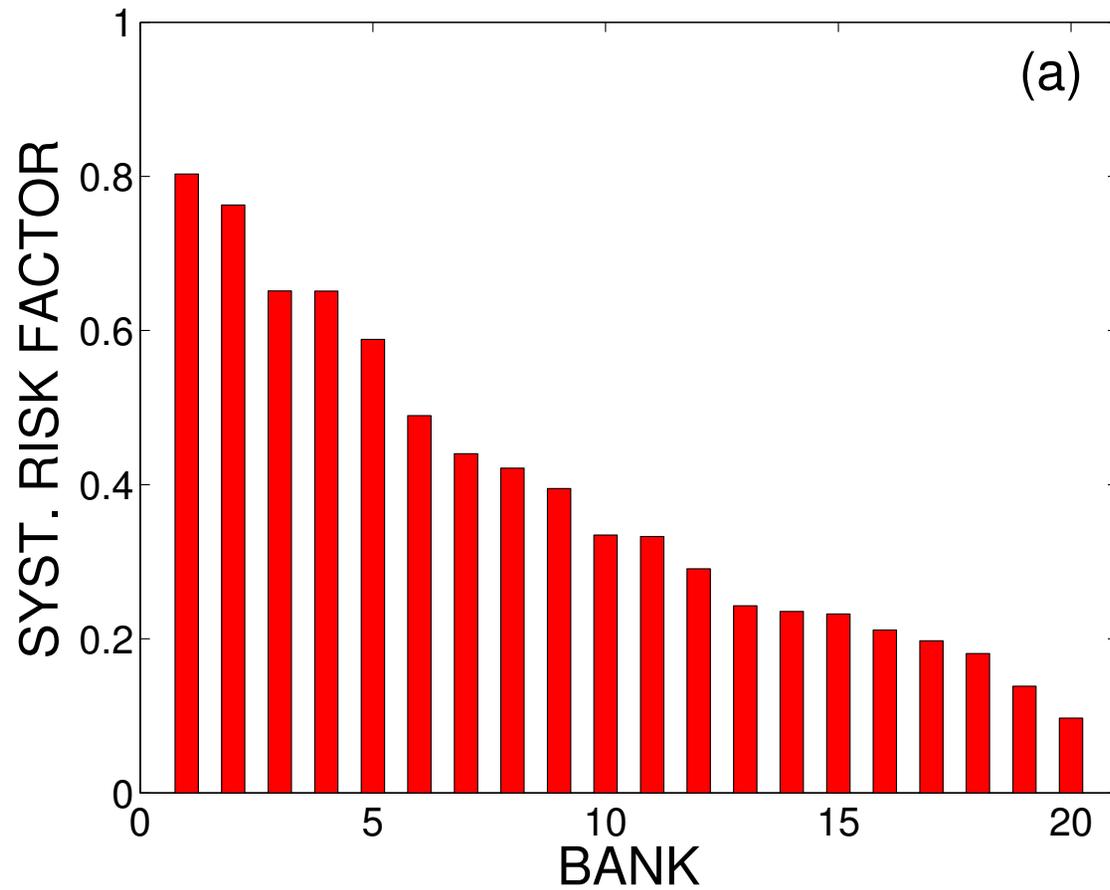


note: size is **not proportional** to systemic risk

note: **core-periphery** structure

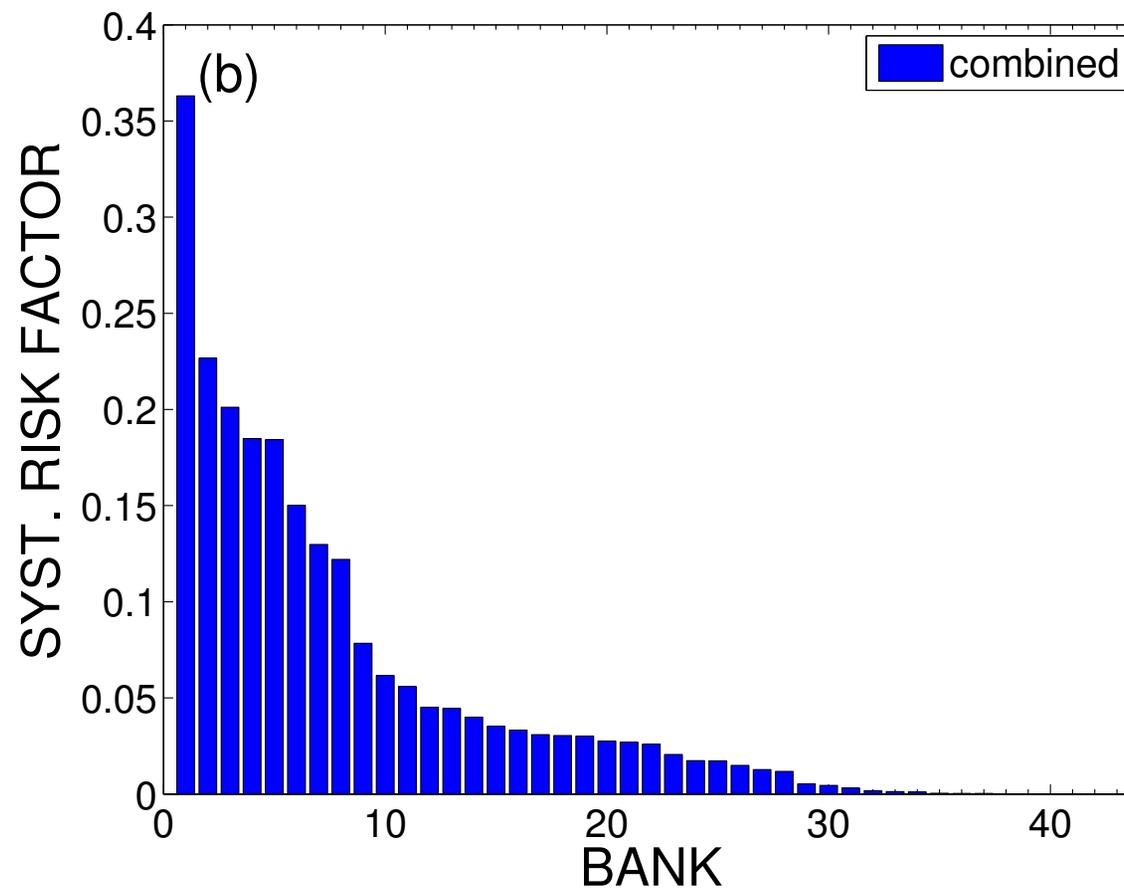
# Systemic risk profile

## Austria



# Systemic risk profile

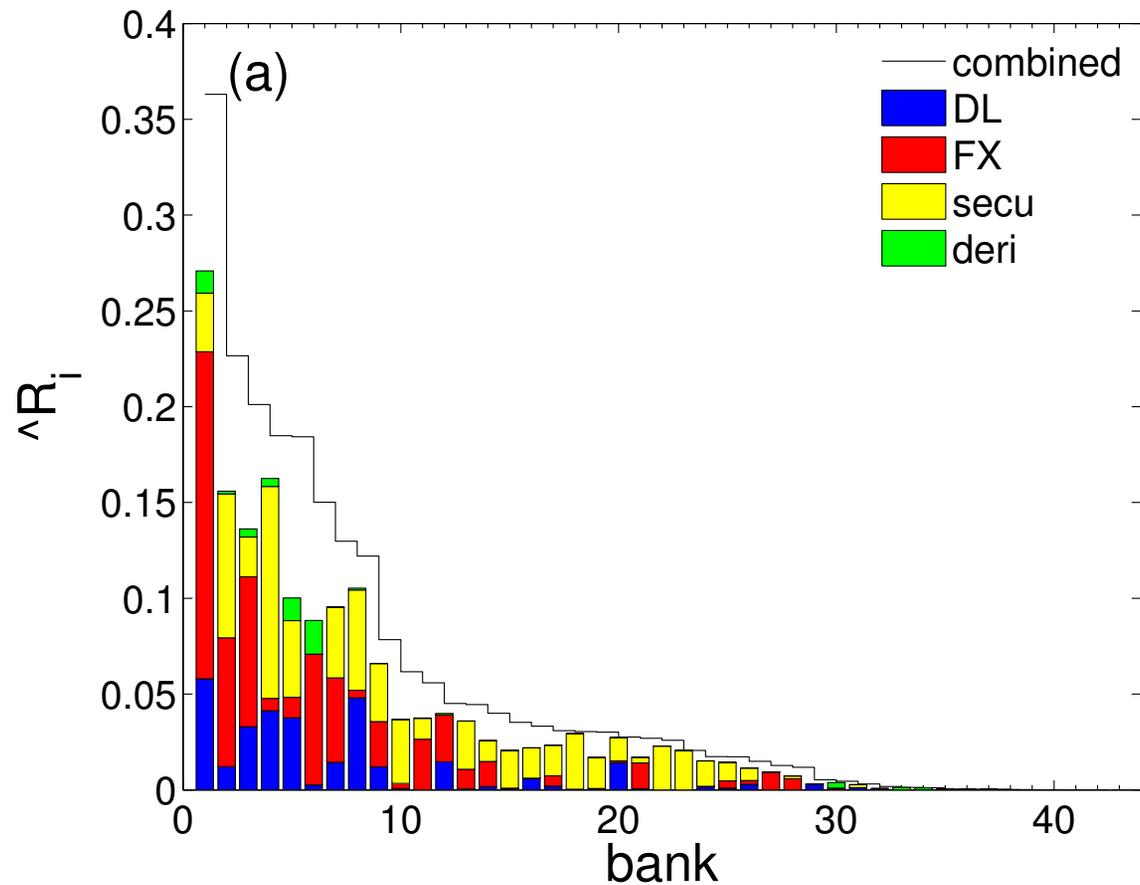
Mexico\*



\*with Serafín Martínez-Jaramillo and team at Banco de Mexico

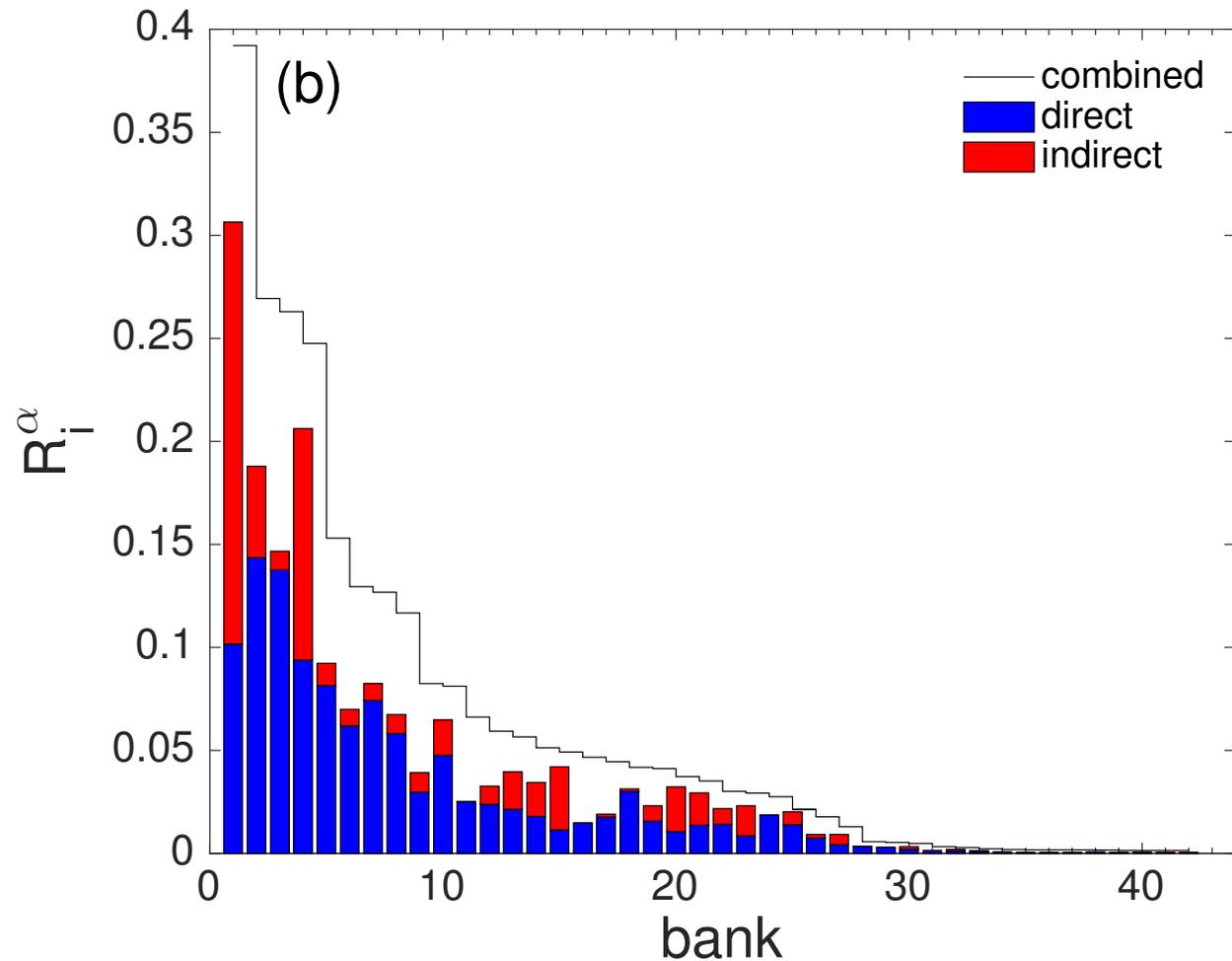
# Systemic risk profile from direct exposures

## Mexico\*

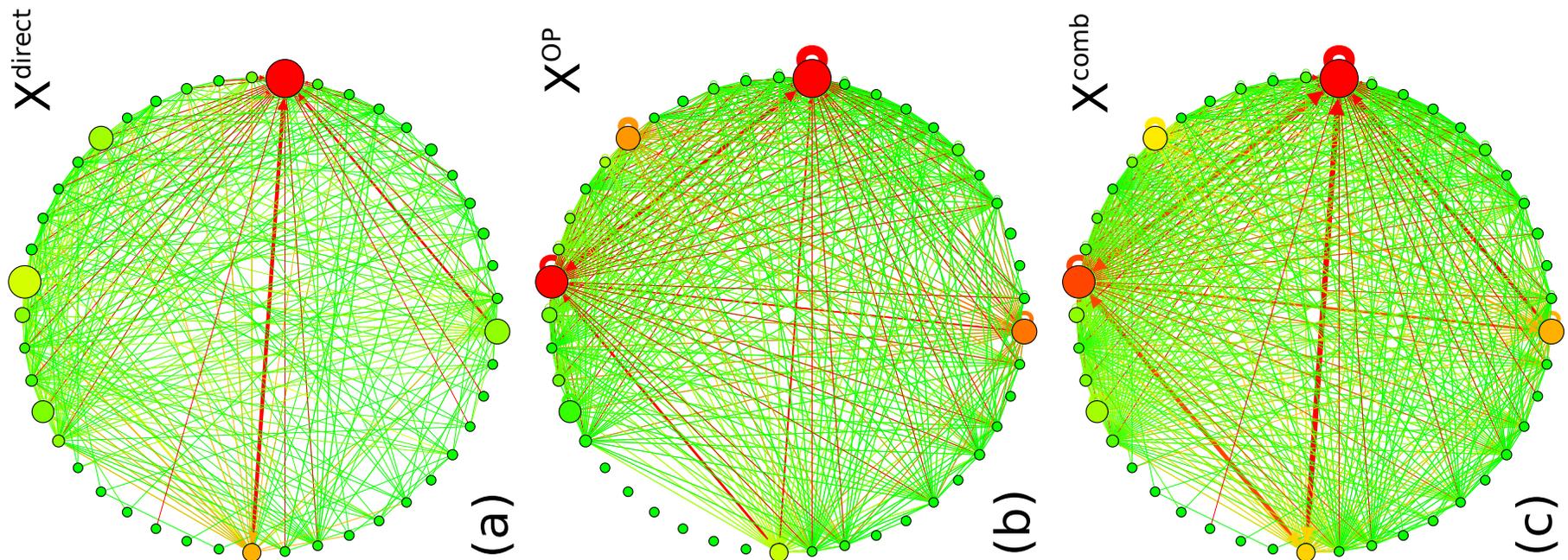


\*with Serafín Martínez-Jaramillo and team at Banco de Mexico

# Systemic risk profile from overlapping portfolios



# Exposures from direct exposures and overlapping portfolios



total exposure overlapping pf  $\sum_{i,j} X_{ij}^{\text{OP}} \sim 1 \times 10^{12}$  Mex\$  
 total direct exposures  $\sum_{i,j} X_{ij}^{\text{direct}} \sim 3.3 \times 10^{11}$  Mex\$

# How big is the next financial crisis?

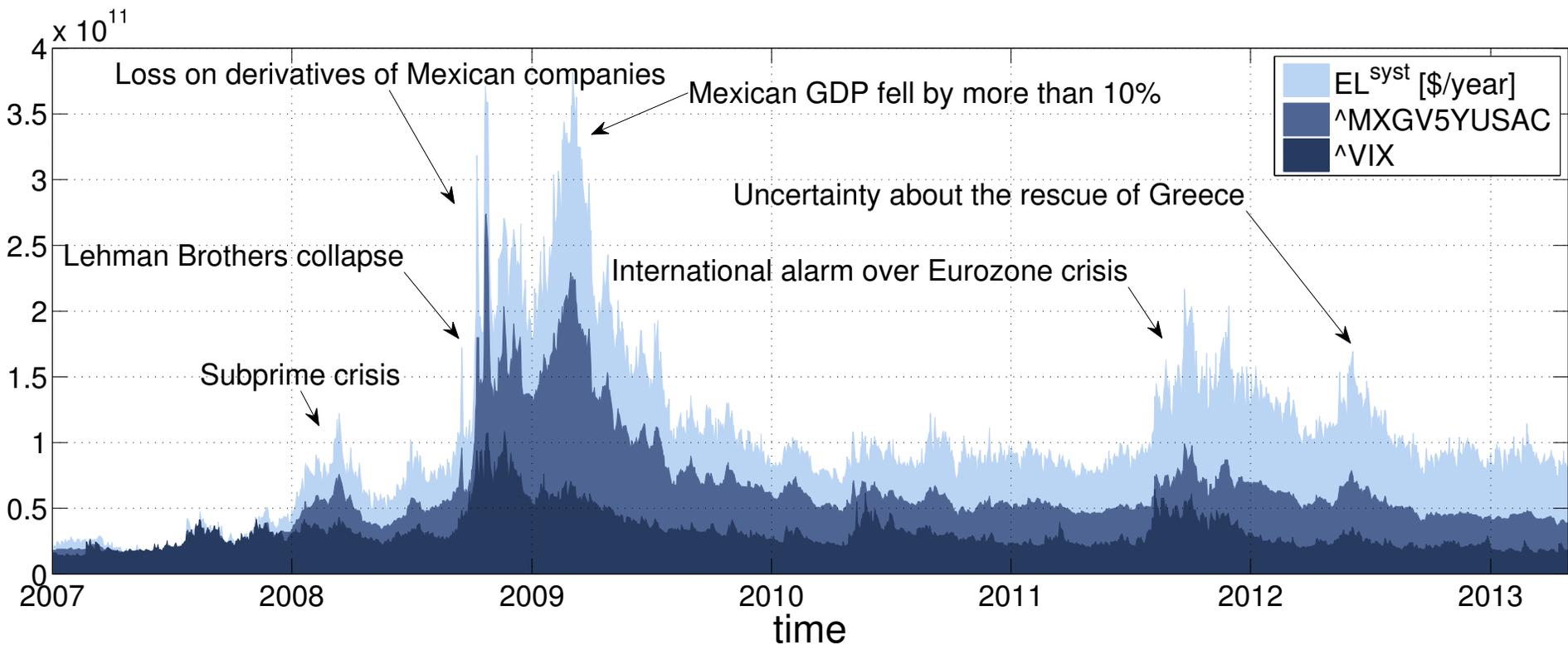
# Expected systemic loss [Euro / Year]

$$\text{Expected systemic loss} = \sum_i p_{\text{default}}(i) \cdot \text{DebtRank}(i)$$

$$\text{Expected loss}(i) = \sum_j p_{\text{default}}(j) \cdot \text{Loss-given-default}(j) \cdot \text{Exposure}(i,j)$$

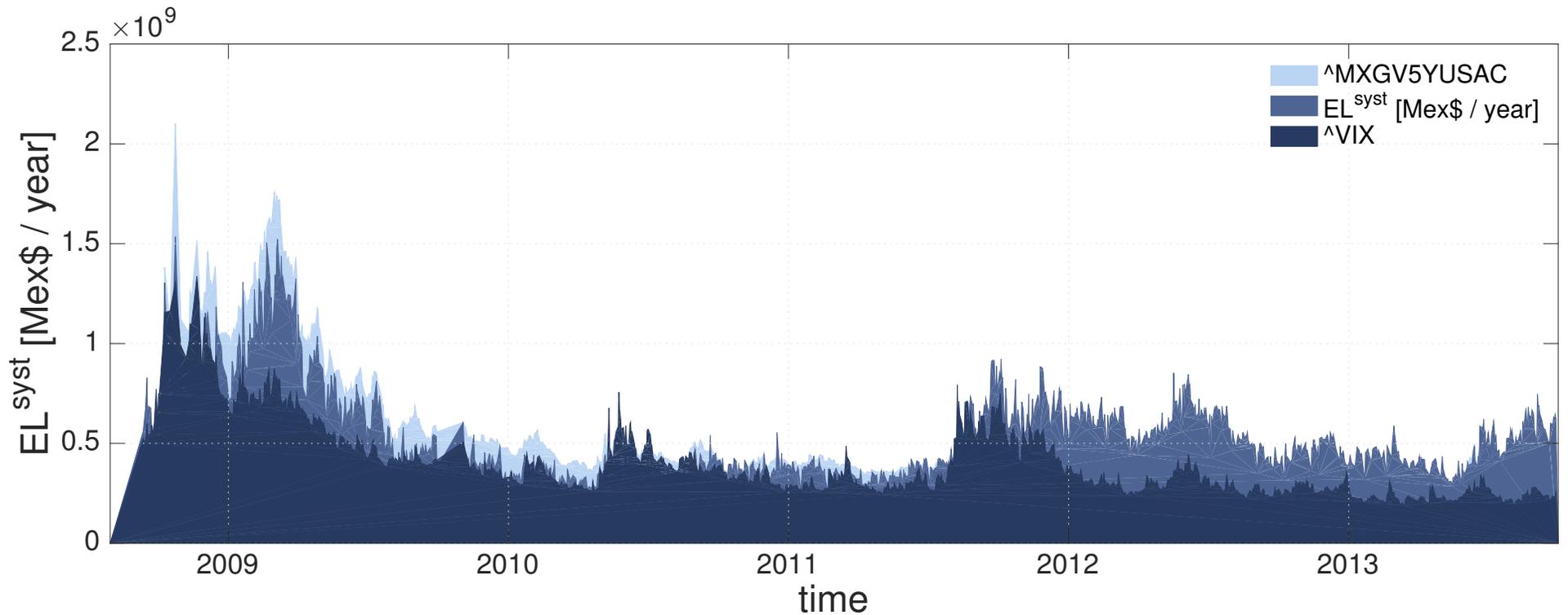
$$\begin{aligned}
\text{EL}^{\text{sys}t} &= V \sum_{S \in \mathcal{P}(B)} \prod_{i \in S} p_i \prod_{j \in B \setminus S} (1 - p_j) (R_S) \\
&\approx V \sum_{S \in \mathcal{P}(B)} \prod_{i \in S} p_i \prod_{j \in B \setminus S} (1 - p_j) \left( \sum_{i \in S} R_i \right) \\
&= V \sum_{i=1}^b \underbrace{\left( \sum_{J \in \mathcal{P}(B \setminus \{i\})} \prod_{j \in J} p_j \prod_{k \in B \setminus (J \cup \{i\})} (1 - p_k) \right)}_{=1} p_i R_i \\
&= V \sum_{i=1}^b p_i R_i
\end{aligned}$$

# Expected systemic loss index for Mexico\*



\*with Serafin Martinez-Jaramillo and team at Banco de Mexico, 2014

# Expected systemic loss from overlapping pfs



# Expected systemic loss index

- expected losses per year within country in case of severe default and NO bailout
  - rational decision on bailouts
- allows to compare countries
- allows to compare situation of country over time
  - are policy measures taking action in Spain? in Greece?
- note: **importance to details !!!**

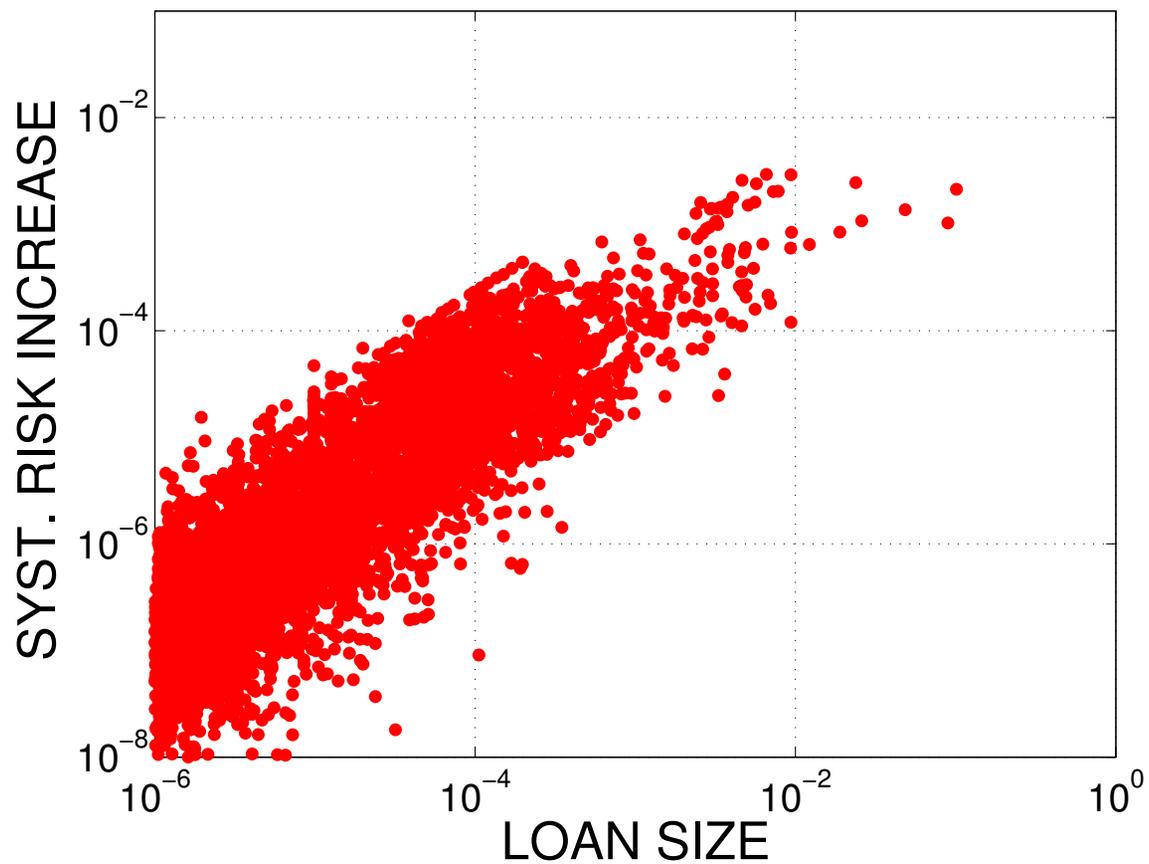
# Observation

Systemic risk of a node changes with **every** transaction

# Marginal expected systemic loss index

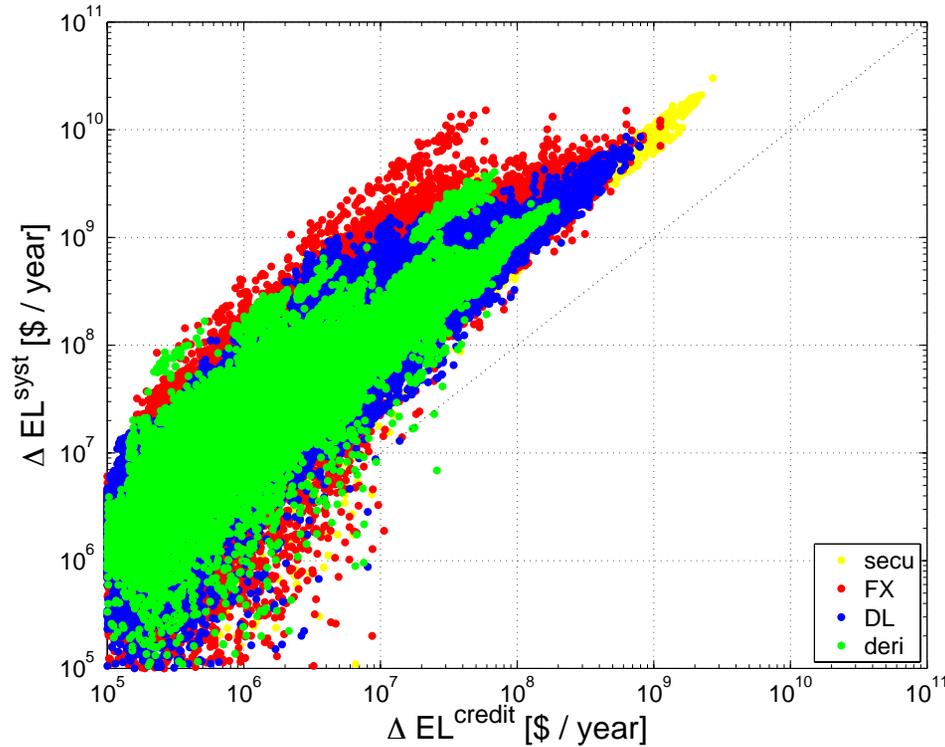
$$\Delta EL^{\text{systr}}(\Delta X_{kl}) = \sum_{i=1}^B p_i [V(X_{ij} + \Delta X_{kl}) R_i(X_{ij} + \Delta X_{kl}, C_i) - V(X_{ij}) R_i(X_{ij}, C_i)]$$

# Austria all interbank loans



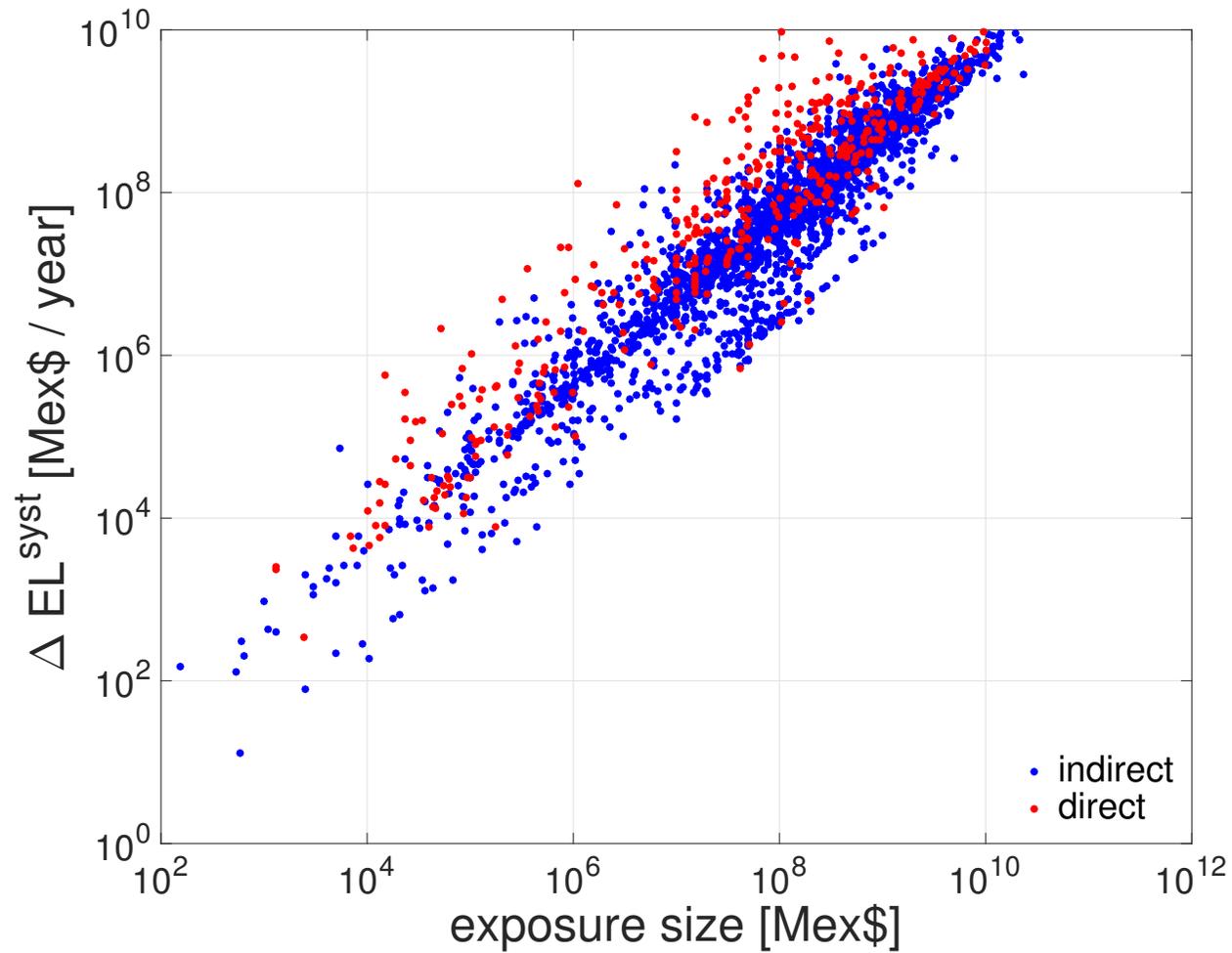
note orders of magnitude !

# Mexican data



$\Delta EL^{syst} > \Delta EL^{credit} \rightarrow$  defaults **do not only affect lenders**  
but involves third parties

# Marginal systemic risk from overlapping pfs



systemic risk is an externality

# Management of systemic risk

- systemic risk is a network property
- manage systemic risk: **re-structure financial networks**  
such that cascading failure becomes unlikely / impossible

systemic risk management  
=  
re-structure networks

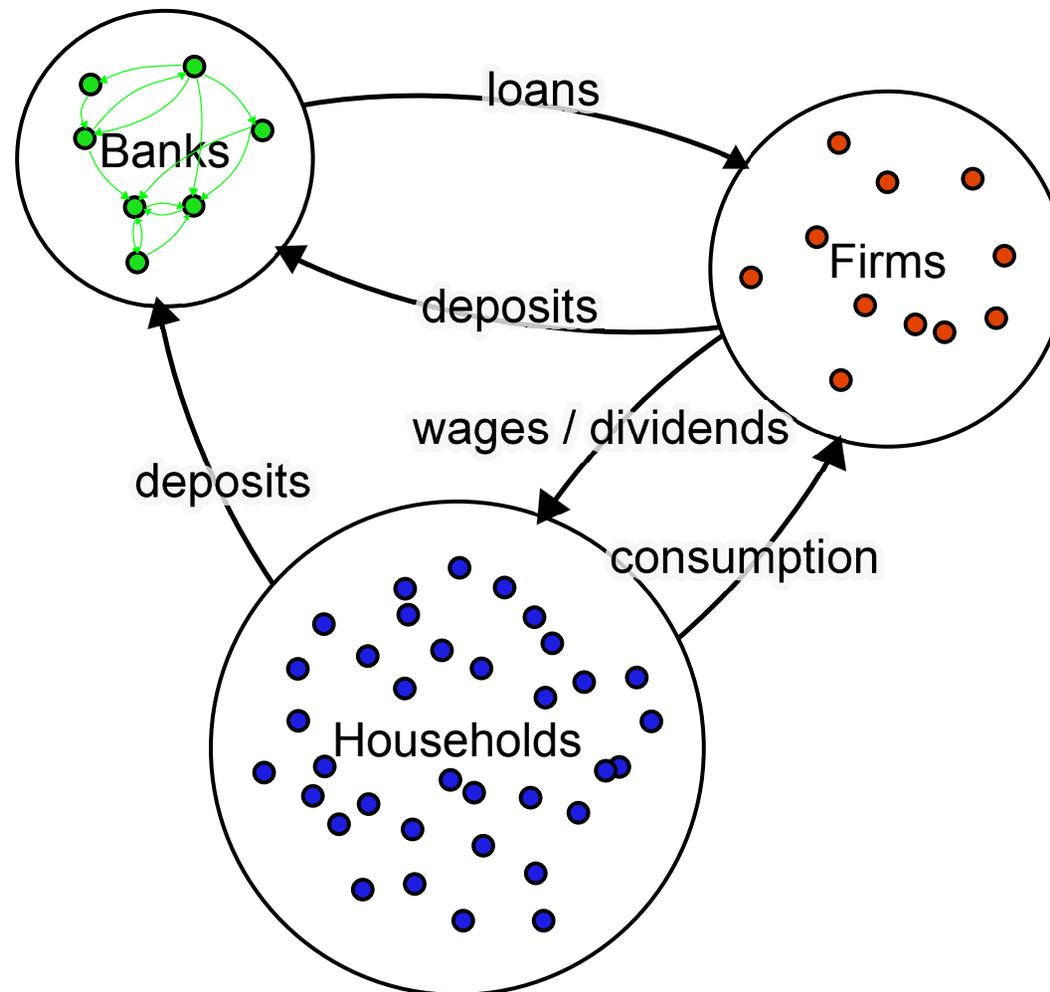
# Systemic risk elimination

- systemic risk spreads by borrowing from risky agents
- how risky is a transaction? → increase of expected syst. loss
- ergo: restrict transactions with high systemic risk

→ **tax those transactions** that increase systemic risk

- size of tax  $\propto$  expected systemic loss of transaction

# To test efficacy of tax: Crisis Macro-Financial Simulator (schematic)



# The agents

- **firms:** ask bank for loans: random size, maturity  $\tau$ ,  $r^{\text{f-loan}}$ 
  - firms sell products to households: realise profit/loss
  - if surplus → deposit it bank accounts, for  $r^{\text{f-deposit}}$
  - firms are bankrupt if insolvent, or capital is below threshold
  - if firm is bankrupt, bank writes off outstanding loans
- **banks** try to provide firm-loans. If they do not have enough
  - approach other banks for interbank loan at interest rate  $r^{\text{ib}}$
  - bankrupt if insolvent or equity capital below zero
  - bankruptcy may trigger other bank defaults
- **households** single aggregated agent: receives cash from firms (through firm-loans) and re-distributes it randomly in banks (household deposits,  $r^{\text{h}}$ ), and among other firms (consumption)

# For comparison: implement Tobin-like tax

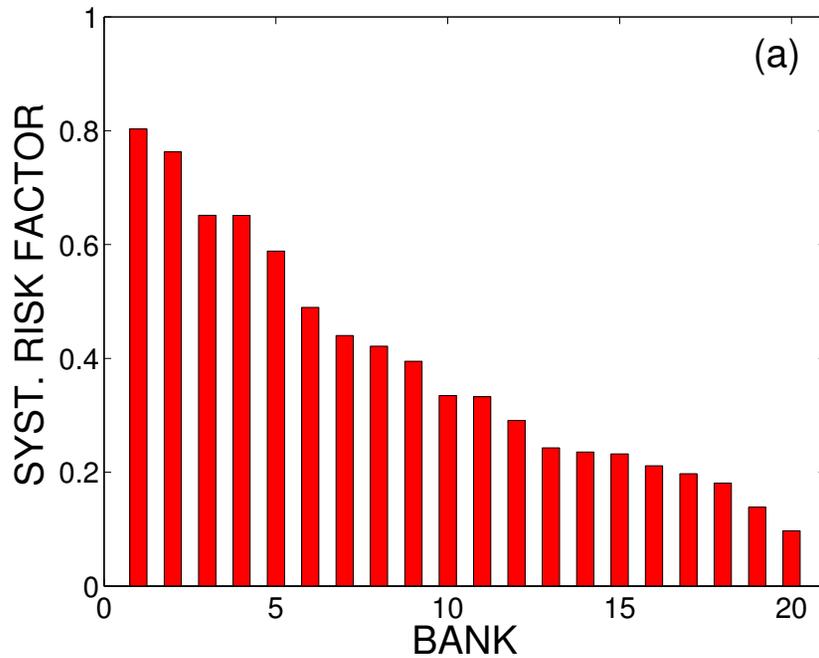
- tax all transactions regardless of their risk contribution
- 0.2% of transaction ( $\sim 5\%$  of interest rate)

# Comparison of three schemes

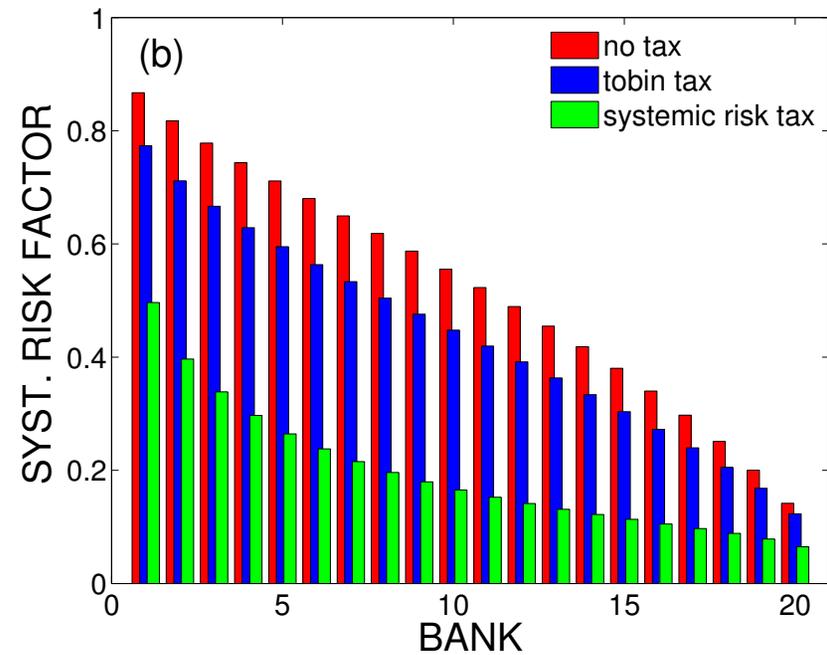
- No systemic risk management
- Systemic Risk Tax (SRT)
- Tobin-like tax

# Model results: Systemic risk profile

## Austria

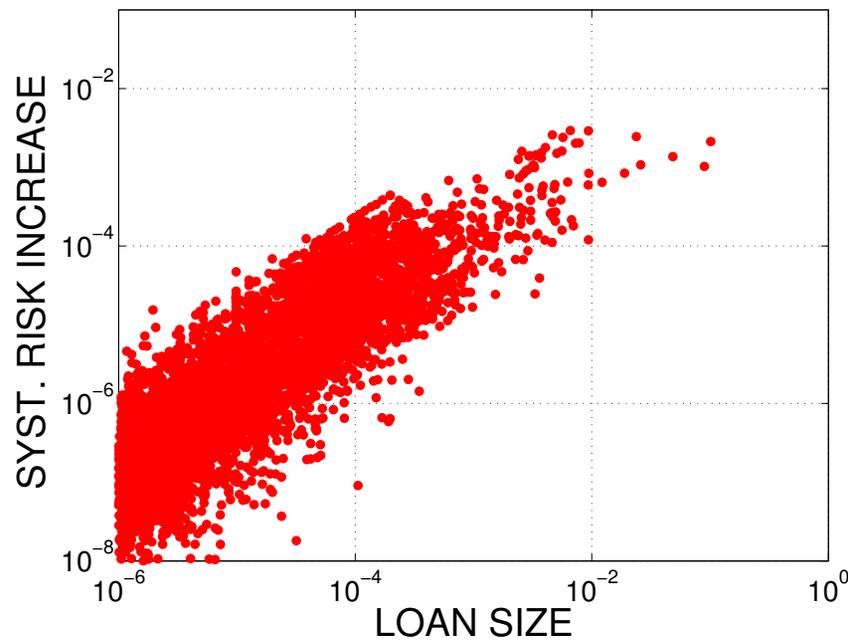


## Model

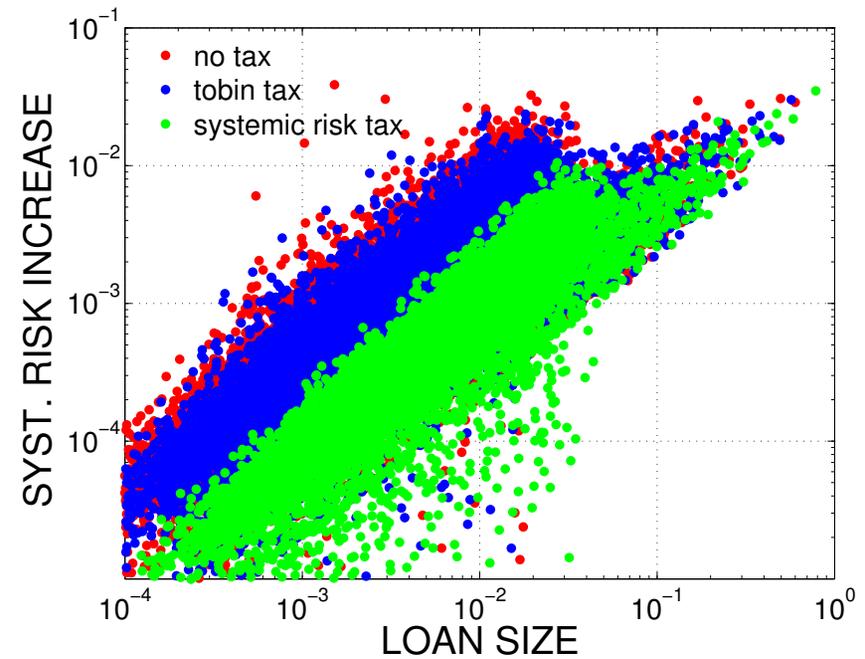


# Model results: Systemic risk of individual loans

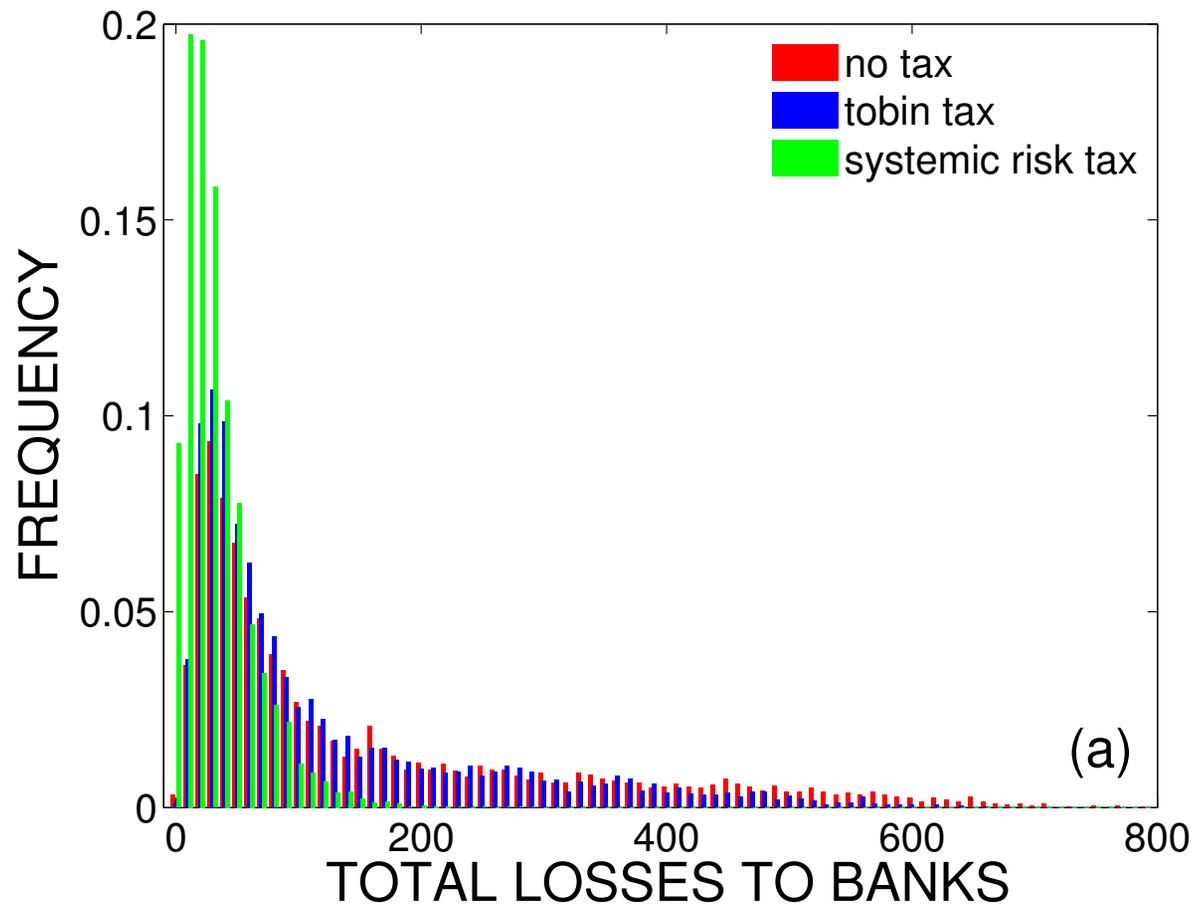
## Austria



## Model

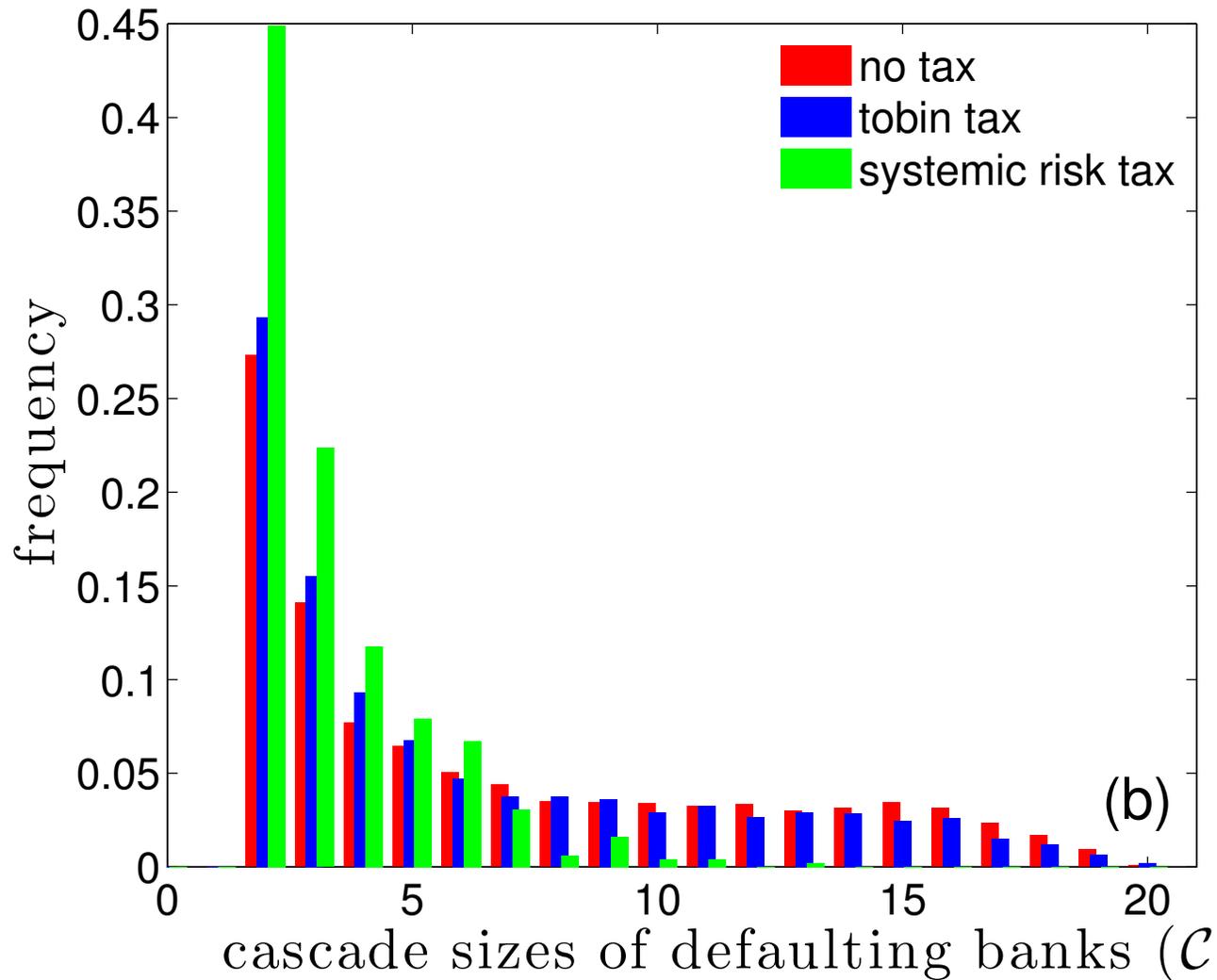


# Model results: Distribution of losses

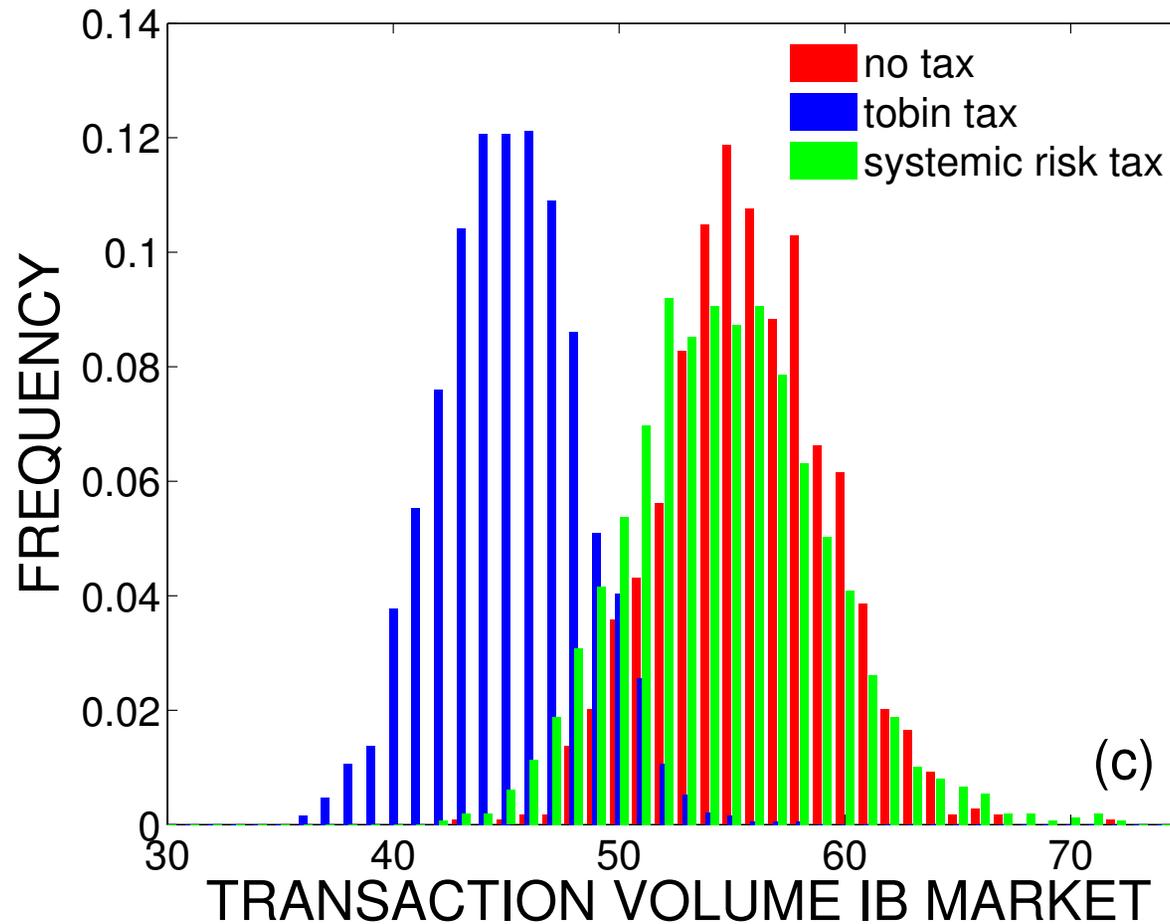


SRT eliminates systemic risk. How?

# Model results: Cascading is suppressed



# Model results: Credit volume



Tobin tax reduces risk by reducing credit volume

Mathematical proof:

SR-free equilibrium under SRT exists

M. Leduc, S. Thurner, J Economic Dynamics and Control 82 (2017) 44

# Reduce SR from overlapping portfolios?

→ see talk of Anton Pichler

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J Economic Dynamics and Control 82 (2017) 44

# Conclusions

- systemic risk is a network property – endogenously created
- can be measured for each institution / transaction
- can be eliminated by SRT (networks don't allow for cascading)
- SRT should **not be payed!** – evasion re-structures networks
- SRT does not reduce credit volume; **re-ordering** transactions
- Basel III as planned does not work – 3 fold works – costly
- SR requires a multi-layer network framework
- SR tax is technically feasible
- SR can be drastically reduced as a optimization problem